menting current algebra with analyticity and unitarity can be advantageously employed in a variety of problems. Work is continuing along the lines of such a program.

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*Work supported in part by the National Science Foundation.

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PHYSICAL PICTURE FOR HIGH-ENERGY DIFFRACTION SCATTERING*

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Recent theoretical results on high-energy scattering are extended. We give explicit rules to obtain the high-energy behavior of the sums of large classes of Feynman diagrams. This consideration suggests, for high-energy diffraction scattering, a physical picture which is simple and natural. We emphasize that this physical picture has the virtue of yielding correctly all the high-energy results of quantum electrodynamics.

Recently, we obtained[1] the high-energy behavior for all two-body elastic scattering amplitudes in quantum electrodynamics. The original procedure of getting these results from perturbation theory is quite complicated,[2] but, if the justification of certain steps is not required, simplified derivations are possible with the help of the variables $p_0 \pm p_3$.[3] The results contradict both the Regge-pole model[4] and the droplet model[5,6] in their most straightforward interpretations. In this Letter, we present a simple physical picture consistent with, and indeed suggested by, our results from quantum electrodynamics.

A feature of our results is the close similarity between electron-electron scattering, electron-positron scattering, and electron scattering by a Coulomb field. More precisely, although these three processes are very different at low energy, the matrix elements, to the orders considered, are essentially identical in the limit of infinite energy. The same is true for electron Compton scattering and Delbrück scattering. Since these relations must be immediate from any useful physical picture, we first discuss the somewhat simpler case of scattering by a static potential.

When radiative corrections are neglected, the scattering of an electron by a static potential is determined by the Dirac equation. For high energies, this problem was already analyzed[7] many years ago. Let $V(x,y,z)$ be the static potential, where the incident electron moves along the z direction; then for high energies the scattering depends on V only through the integral

$$\int_{-\infty}^{\infty} V(x,y,z)dz.$$

In other words, if $V(x,y,z)$ is replaced by

$$\delta(z)\int_{-\infty}^{\infty} V(x,y,z)dz,$$

the scattering amplitude at infinite energy is not changed. The physical reason for this result is simply that, to an electron traveling near the speed of light, the static potential appears Lorentz contracted into a thin slab. Consequently, the "thickness" of the potential is not an important dimension.

Consider a particle of mass m moving in the z direction with momentum $p \gg mc$. Due to strong or electromagnetic interactions, this particle is sometimes dissociated virtually into n particles,[8] respectively of mass m_l and of momentum $\beta_l p$ in the z direction and $\vec{p}_{\perp l}$ in the xy plane. Of course,

$$\sum_{l=1}^{n}\beta_l = 1,$$

and

$$\sum_{l=1}^{n}\vec{p}_{\perp l} = 0.$$

The invariant mass of this system of n particles is $(c=1)$

$$\{[\sum_{l=1}^{n}(\beta_l^2 p^2 + \vec{p}_{\perp l}^{\,2} + m_l^2)^{1/2}]^2 - p^2\}^{1/2},$$

which is, for large p, approximately

$$[\sum_{l=1}^{n}\beta_l^{-1}(m_l^2 + \vec{p}_{\perp l}^{\,2})]^{1/2}. \quad (1)$$

Note that p does not appear in (1). Thus, by the uncertainty principle, this virtual state of n particles can exist for a finite length of time in its own center-of-mass system (provided that none of the β's is small). By time dilatation, this virtual state can be present for a time proportional to p for large p in the frame of the external potential. On the other hand, the velocity $\vec{v}_l$ of a high-energy particle is

$$v_{zl} = \beta_l p(\beta_l^2 p^2 + \vec{p}_{\perp l}^{\,2} + m_l^2)^{-1/2}$$
$$\sim 1 - (2\beta_l^2)^{-1}(\vec{p}_{\perp l}^{\,2} + m_l^2)/p^2,$$

and

$$\vec{v}_{\perp l} = \vec{p}_{\perp l}(\beta_l^2 p^2 + \vec{p}_l^{\,2} + m_l^2)^{-1/2} \sim (2\beta_l)^{-1} p_{\perp l}/p. \quad (2)$$

Therefore, during the lifetime of the virtual state, the separation of the particles is of the order p^{-1} in the z direction and of the order 1 in the x and y directions.

It is thus seen that, in the frame of the external static potential, there are three distinct scales in the z direction when p is large: p, the distance traveled by each of the n particles during the lifetime of the virtual state; 1, the size of the external potential; and p^{-1}, the separation of the n particles in the z direction. Our simple physical picture is based on the fact that the scale p is the most important one.[9]

More precisely, the physical picture is as follows. The incident particle is visualized as a superposition of n-particle[8] virtual states ($n = 1, 2, 3, \cdots$), and the n particles interact independently and simultaneously with the static potential which is contracted into a thin slab. After the interaction, the new n-particle virtual states recombine to contribute to the scattered states, either the original particle for elastic scattering or more generally another particle for diffraction scattering. It seems appropriate to call this picture the impact picture.

A physical picture is of no use unless it can be used as the basis for some calculation. In the present instance, noncovariant perturbation theory[10] can be directly applied with the additional rule that each intermediate state consists either entirely of particles before scattering or entirely of particles after scattering. The justification of this rule is straightforward and similar to our consideration on exponentiation.[1,11]

Let us use Delbrück scattering as an illustration. Here the incident particle is a photon, and the most important virtual state consists of an electron-positron pair ($n=2$). We can thus draw the diagram of Fig. 1 where the momentum trans-

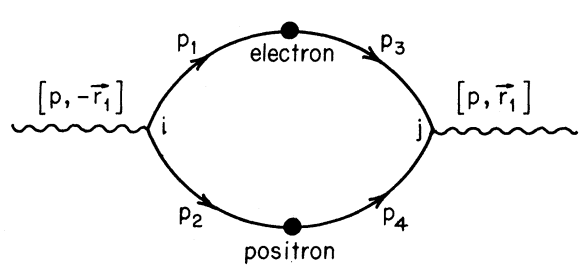

FIG. 1. Perturbation diagram for Delbrück scattering.

fer is $2\vec{r}_1$ and a black round dot indicates interaction with the static Coulomb field. Since we are using noncovariant perturbation theory, all the electron and positron lines are considered to be on the mass shell. We give explicitly the z component and the transverse (or xy) component of the p's as follows:

$$\vec{p}_1 = [\beta p, \vec{p}_\perp],$$

$$\vec{p}_2 = [(1-\beta)p, -\vec{p}_\perp - \vec{r}_1],$$

$$\vec{p}_3 = [\beta p, \vec{p}_\perp - \vec{q}_\perp + \vec{r}_1],$$

and

$$\vec{p}_4 = [(1-\beta)p, -\vec{p}_\perp + \vec{q}_\perp]. \qquad (3)$$

Since $p_k^2 = p_{k0}^2 - \vec{p}_k^2 = m^2$ for $k = 1, 2, 3, 4$, it is trivial to find

$$p_{10} \sim \beta p + (\vec{p}_\perp^2 + m^2)/2\beta p,$$

$$p_{20} \sim (1-\beta)p + [(-\vec{p}_\perp - \vec{r}_1)^2 + m^2]/2(1-\beta)p,$$

$$p_{30} \sim \beta p + [(\vec{p}_\perp - \vec{q}_\perp + \vec{r}_1) + m^2]/2\beta p,$$

and

$$p_{40} \sim (1-\beta)p + [(-\vec{p}_\perp + \vec{q}_\perp)^2 + m^2]/2(1-\beta)p. \qquad (4)$$

With this kinematic information, the matrix element for Delbrück scattering is, for very large p and $\vec{r}_1 \neq 0$,

$$\mathfrak{M}^{(\mathrm{D})} \sim -ie^2(2\pi)^{-5}\int d^2p_\perp \int d^2q_\perp \int_0^1 p\,d\beta (p_{10}+p_{20}-E_i)^{-1}(p_{30}+p_{40}-E_i)^{-1} S_-(\vec{r}_1 - \vec{q}_\perp) S_+(\vec{r}_1 + \vec{q}_\perp)$$
$$\times \mathrm{Tr}\left[\gamma_i \frac{\not{p}_1 + m}{2p_{10}} \gamma_0 \frac{\not{p}_3 + m}{2p_{30}} \gamma_j \frac{-\not{p}_4 + m}{2p_{40}} \gamma_0 \frac{-\not{p}_2 + m}{2p_{20}}\right], \qquad (5)$$

where

$$E_i \sim p + \vec{r}_1^2/2p \qquad (6)$$

is the energy of the incident photon, and

$$S_\pm(\vec{q}) = \int dx dy\, e^{i(q_1 x + q_2 y)} \exp[\mp ie \int_{-\infty}^{\infty} dz\, V(x,y,z)] \qquad (7)$$

with

$$V(x,y,z) = Ze(x^2+y^2+z^2)^{-1/2}. \qquad (8)$$

From (3), (4), (6), (7), and (8), it is straightforward to verify that (5) yields (4.7) of Ref. 11 in terms of impact factors.[1,2] When $n > 2$, more complicated impact factors are needed. This is the hierarchy of impact factors discussed before.[12]

The generalization of the impact picture to the diffraction scattering of two high-energy particles is immediate. The only difference is that the potential as generated by one particle and experienced by the other is Lorentz contracted. Thus, if p is the momentum of each of the particles in the center-of-mass system, there are in the z direction only two scales, p and p^{-1}, instead of three. Since for the present purpose the most important scale is p, the absence of the scale 1 is of no consequence.[13] Straightforward application of this impact picture to electron Compton scattering and photon-photon scattering at high energies yields, in perturbation theory,[14] our previous results.[1,2] The derivation of the impact picture from quantum field theory is to be submitted to The Physical Review.

We conclude with a comparison of our impact picture with the droplet model and the Regge model. There is clearly a similarity in spirit to the droplet model,[5,6] but we believe that our impact picture has the following three virtues: It has a field-theoretic basis; it gives the correct answers in the case of quantum electrodynamics; and transverse momenta of the constituting particles are properly taken into account.

Although not as obvious, there is also a close connection to the Regge-pole model. This is going to be discussed in detail in a forthcoming series of papers, and we give here only a brief comment. Regge poles can be obtained from field theory by considering ladder diagrams in the t channel,[15] due to the appearance of numerous powers of $\ln s$, where $s \sim 4p^2$ is the square of the center-of-mass energy. That these powers of $\ln s$ do appear[16] implies that p is the most important, but not the only important, scale. In this respect, the impact picture shows features of the Regge-pole model not present in the droplet model. However, when the next largest scale is taken into account in the impact picture, Regge poles are still not obtained for a number of reasons, one of which has already been discussed in some detail.[16]

We are greatly indebted to Professor R. P. Feynman and Professor C. N. Yang for the most instinctive and helpful discussions.

*Work supported in part by the U. S. Atomic Energy Commission.

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[8]These particles may or may not be the elementary particles and resonances we know now. For example, they may be members of a set of more "fundamental" particles.

[9]Aside from the explicit results of Refs. 1-3, we do not know any compelling reason why this is so.

[10]By noncovariant perturbation theory, we mean the perturbation theory easily found in most elementary textbooks on quantum mechanics, i.e., the perturbation theory before Feynman invented his rules. Although not manifestly covariant, noncovariant perturbation theory is still consistent with special relativity.

[11]H. Cheng and T. T. Wu, "Impact Factor and Exponentiation in High-Energy Scattering Processes" (to be published).

[12]H. Cheng and T. T. Wu, "Form Factor and Impact Factor in High-Energy Scattering" (to be published).

[13]The difference in scales is, however, interesting and instructive. For example, the corrections to Delbrück scattering and electron Compton scattering are respectively of order p^{-1} and p^{-2} (neglecting logarithms). This is easily understood as the ratio between the most important scale p and the next largest scales, 1 and p^{-1}, respectively.

[14]Note that there are two sets of energy denominators, one set for each incident particle.

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OMEGA PRODUCTION IN $\pi^+d \to \pi^+\pi^-\pi^0pp$ AT 4.19 GeV/c*

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We report on ~340 events of the reaction $\pi^+n \to \omega^0p$ at a beam momentum of 4.19 GeV/c. The differential cross section shows neither a broad dip for $t < -0.2$ (GeV/c)^2 nor a dip in the region $t \approx -0.6$ (GeV/c)^2. There is a 2-standard-deviation dip in ρ_{00} between $t = -0.2$ and $t = -0.3$ (GeV/c)^2 and a small negative value of Reρ_{10} for $t < -1.0$ (GeV/c)^2. The dip in ρ_{00} is consistent with the vanishing, in that region of t, of an exchanged trajectory with unnatural parity. A simple Regge-model calculation with $\rho + B$ exchange does not reproduce the data.

In a bubble-chamber study of the reaction

$$\pi^+d \to \pi^+\pi^-\pi^0pp \tag{1}$$

at a beam momentum of 4.19 GeV/c, we have identified 338 events which correspond to the reaction

$$\pi^+n \to \omega^0p \tag{2}$$

with the other final-state proton participating as a spectator to the strong interaction. Reaction (2) is particularly interesting because G-parity conservation limits the low-lying exchanges to the ρ and B trajectories. The data are not compatible with ρ-exchange dominance of the reaction. The differential cross section varies smoothly, with no dip apparent at $t \approx -0.6$ (GeV/c)^2 or in the forward direction [$t < -0.2$ (GeV/c)^2]. However, a dip is suggested in ρ_{00} for the ω^0 at $t \approx -0.25$ (GeV/c)^2.

We have measured about 21 000 four-prong events on film taken in the Lawrence Radiation Laboratory 72-in. deuterium bubble chamber to obtain about 2700 events fitting Reaction (1). Each event contains a stopping track whose projected length is greater than 1 mm, which we identify as the spectator to Reaction (2). The remaining proton typically possesses a much higher momentum. Events were classified as belonging to Reaction (1) if this hypothesis possessed the smallest one-constraint χ^2 (and no four-con-