up and running of this experiment. We also wish to acknowledge the important contributions made to this experiment by the staffs of the BNL On-Line Data Facility, the BNL Instrumentation Division, and the Physics Design Groups at BNL and Carnegie-Mellon University. We are particularly grateful to A. Abrahamson, B. Bihn, R. Rothe, and J. Smith for their valuable assistance throughout the experiment.

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# HARD-PION EFFECTIVE-RANGE FORMULA FOR THE PION FORM FACTOR*

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Using chiral SU(2) current algebra and partially conserved axial-vector current to generate a set of Ward identities we have derived a relation which can be interpreted as a dynamical equation for the off-shell pion form factor. We solve the equation on shell in an effective-range approximation for the pion form factor, and from unitarity we deduce the $p$-wave $\pi\pi$ phase shift. Our results are in excellent agreement with all relevant data.

When $\rho$ dominance of the pion form factor is written in its simplest form,

$$F(t)=m_\rho^2/(m_\rho^2-t), \tag{1}$$

it is evident that the analyticity of $F$ is not correctly given. Equation (1) cannot be confronted with the data[1,2] in the timelike region until the instability of the $\rho$ is incorporated. These deficiencies are readily overcome, as Gounaris and Sakurai have shown.[3] They employ an analytic function having the correct $\pi\pi$ branch cut by writing

$$F(t)=f(0)/f(t), \quad f(t)=[\cot\delta_{11}(t)-i]P^3/\sqrt{t} \tag{2}$$

in which $P^2=\frac{1}{4}(t-4m_\pi^2)$ and $\delta_{11}$, the $T=J=1$ $\pi\pi$ phase shift, is given by an effective-range formula. In this paper we show how hard-pion methods may be used to generate an effective-range formula for the form factor directly, without reference to the $\pi\pi$ phase shift. A significant gain in predictive power is thus achieved because once $F$ is given, $\delta_{11}$ is among the output.

We begin with the three-point functions of the vector and axial-vector currents of chiral SU(2):

$$\begin{aligned} W_\lambda{}^{abc}(q,p)&=\int dxdy\, e^{-iqx}e^{ipy}\langle 0|T\partial_\mu A_\mu{}^a(x)\partial_\nu A_\nu{}^b(y)V_\lambda{}^c(0)|0\rangle,\\ W_{\nu\lambda}{}^{abc}(q,p)&=\int dxdy\, e^{-iqx}e^{ipy}\langle 0|T\partial_\mu A_\mu{}^a(x)A_\nu{}^b(y)V_\lambda{}^c(0)|0\rangle,\\ W_{\mu\nu\lambda}{}^{abc}(q,p)&=\int dxdy\, e^{-iqx}e^{ipy}\langle 0|TA_\mu{}^a(x)A_\nu{}^b(y)V_\lambda{}^c(0)|0\rangle. \end{aligned} \tag{3}$$

$a$, $b$, and $c$ are isospin indices. The first of these, for $c=3$, gives the off-shell pion form factor, extrapolated in the momenta $q$, $p$, and $k=p-q$. Schnitzer and Weinberg have shown[4] how hard-pion current algebra relates these to one another and to the propagators through a set of Ward indentities. Moreover, they have obtained a description of all the interrelated physical processes in terms of one parameter by employing meson dominance in all three channels, $(q,a)$, $(p,b)$, and $(k,c)$. Among their results is

$$F(t)=\frac{1+\delta}{4}+\frac{3-\delta}{4}\frac{m_\rho{}^2}{m_\rho{}^2-t}, \quad t=-k^2, \tag{4}$$

where their parameter has the value $\delta\simeq-\frac{1}{2}$ in order to fit both $A_1$ and $\rho$ decay.

To obtain a form factor with the desired $\pi\pi$ cut in $t$ we must avoid SW's use of $\rho$ dominance in the $(k,c)$ channel. Accordingly we factor out, to begin with, only the pion poles and write

$$W_\lambda{}^{abc}(q,p)=i\epsilon_{abc}\frac{F_\pi{}^2m_\pi{}^4}{(p^2+m_\pi{}^2)(q^2+m_\pi{}^2)}F_\lambda(q,p), \tag{5}$$

$$W_{\nu\lambda}{}^{abc}(q,p)=\epsilon_{abc}\frac{F_\pi m_\pi{}^2}{q^2+m_\pi{}^2}\left[F_{\nu\lambda}(q,p)+\frac{F_\pi}{p^2+m_\pi{}^2}p_\nu F_\lambda(q,p)\right], \tag{6}$$

$$W_{\mu\nu\lambda}{}^{abc}(q,p)=i\epsilon_{abc}\left\{F_{\mu\nu\lambda}(q,p)+F_\pi\left[\frac{p_\nu}{p^2+m_\pi{}^2}F_{\mu\lambda}(p,q)+\frac{q_\mu}{q^2+m_\pi{}^2}F_{\nu\lambda}(q,p)\right]\right.$$
$$\left.+\frac{F_\pi{}^2}{(p^2+m_\pi{}^2)(q^2+m_\pi{}^2)}q_\mu p_\nu F_\lambda(q,p)\right\}. \tag{7}$$

In (5) the pion decay constant $F_\pi=94$ MeV, and $F_\lambda(q,p)=F(t)Q_\lambda$ for $p^2=q^2=-m_\pi{}^2$, where $Q=p+q$. The Ward identities then yield

$$F_\pi{}^2[F_\lambda(q,p)-Q_\lambda]=q_\mu p_\nu F_{\mu\nu\lambda}(q,p)+\tfrac{1}{2}\int\frac{dx\,\rho_V(x)}{x}\frac{(p^2-q^2)k_\lambda-k^2Q_\lambda}{x+k^2}. \tag{8}$$

Given the assumptions of current algebra and partially conserved axial-vector current this equation is exact. It can be interpreted on shell as a dynamical equation for $F(t)$.

Our knowledge of the term $q_\mu p_\nu F_{\mu\nu\lambda}$ is limited. We can obtain an effective-range solution to Eq. (8) if as a first approximation we adopt SW's expression[5] for $q_\mu p_\nu F_{\mu\nu\lambda}$. Putting both pions on the mass shell, $p^2=q^2=-m_\pi{}^2$, we find

$$F(t)-1=\frac{t}{2F_\pi{}^2}\int\frac{dx}{x}\frac{\rho_V(x)}{x-t}\left[1-\frac{1+\delta}{4}\frac{x}{m_\rho{}^2}\right], \tag{9}$$

where $t=-k^2$, and for generality we do not fix the SW parameter $\delta$. In the $\pi\pi$ region of the variable $t$, the vector-current spectral function is

$$[\rho_V(t)]_{\pi\pi} = \frac{1}{6\pi^2}|F(t)|^2\frac{P^3}{\sqrt{t}}. \tag{10}$$

Equations (9) and (10) imply

$$\mathrm{Im}F = \frac{1}{a_{11}}|F|^2\frac{P^3}{\sqrt{t}}\left(1-\frac{1+\delta}{4}\frac{t}{m_\rho^2}\right), \tag{11}$$

where

$$a_{11} = 12\pi F_\pi^2. \tag{12}$$

Equation (11) is the basis of our effective-range formula for $F(t)$; with the condition $F(0)=1$ it gives a representation for $F(t)$ in terms of just one unknown,

$$F(t) = a_{11}\left[a_{11} + bt + g(t)\left(1-\frac{1+\delta}{4}\frac{t}{m_\rho^2}\right) - g(0)\right]^{-1}, \tag{13}$$

where

$$g(t) = \frac{2}{\pi}\frac{P^3}{\sqrt{t}}\ln\frac{t^{1/2}+2P}{2m_\pi} - i\frac{P^3}{\sqrt{t}}, \quad g(0) = -m_\pi^2/\pi. \tag{14}$$

Equation (13) has been constructed to have the analyticity prescribed locally in (11). We determine the one parameter $b$ by requiring

$$\mathrm{Re}F^{-1}(m_\rho^2) = 0; \tag{15}$$

$\rho$ dominance takes this form in our method. We find

$$b = -\frac{1}{m_\rho^2}\left[a_{11} + \frac{m_\pi^2}{\pi} + \frac{3-\delta}{4}\frac{2}{\pi}\frac{P_\rho^3}{m_\rho}L_\rho\right], \quad L_\rho = \ln[(m_\rho + 2P_\rho)/2m_\pi]; \tag{16}$$

so that the parametrization of $F(t)$ is now complete. Near $t = m_\rho^2$ Eq. (13) becomes

$$F(t) = \frac{a_{11}}{-\lambda[m_\rho^2 - t - i\Gamma m_\rho(P/P_\rho)^3(m_\rho/\sqrt{t})]} \tag{17}$$

in which the $\rho$ width is

$$\Gamma = -\frac{1}{\lambda}\frac{3-\delta}{4}\frac{P_\rho^3}{m_\rho^2}, \tag{18}$$

where

$$\lambda = \left(\frac{d}{dt}\mathrm{Re}\frac{a_{11}}{F}\right)_{m_\rho^2}.$$

Numerically, with $m_\rho = 765$ MeV, $\Gamma = 124$ MeV for $\delta = -\frac{1}{2}$. If we compare (18) with

$$\Gamma = \frac{2}{3}\frac{f_{\rho\pi\pi}^2}{4\pi}\frac{P_\rho^3}{m_\rho^2},$$

we obtain, within 5%, the Kawarabayashi, Suzuki, Riazuddin, Fayyazuddin (KSRF) relation,[6-8] modified by the factor $\frac{1}{4}(3-\delta)$:

$$2F_\pi^2 = \frac{3-\delta}{4}\frac{m_\rho^2}{f_{\rho\pi\pi}^2}. \tag{19}$$

In Fig. 1(a) we plot $|F|^2$ in the timelike region and indicate the colliding-beam data[1,2]; in Fig. 1(b) we plot $F$ in the spacelike region and compare with the results from electroproduction experiments.[9] The pion charge radius $r_\pi$ is determined from

$$r_\pi^2 = 6F'(0) = \frac{6}{m_\rho^2}\left\{1-\frac{m_\rho^2}{\pi a_{11}}\left[\frac{1}{3}-\frac{3-\delta}{4m_\rho^2}\left(m_\pi^2 + 2\frac{P_\rho^3}{m_\rho}L_\rho\right)\right]\right\}. \tag{20}$$

Numerically, for $\delta=-\frac{1}{2}$, $r_\pi$ is only 0.3 % smaller than the $\rho$-dominance value: $r_\pi=(6/m_\rho^2)^{1/2}=0.635$ F.

Our method of obtaining $F(t)$ avoids the use of $\delta_{11}$ as input. We can now predict $\delta_{11}$ by appealing to unitarity. We use

$$\mathrm{Im}F=F^*e^{i\delta_{11}}\sin\delta_{11} \tag{21}$$

in the $\pi\pi$ region, refer back to Eq. (11), and conclude[10] that

$$\cot\delta_{11}=\mathrm{Re}\left(\frac{a_{11}}{F}\right)\frac{\sqrt{t}}{P^3}\left(1-\frac{1+\delta}{4}\frac{t}{m_\rho^2}\right)^{-1}. \tag{22}$$

In Fig. 2 we plot the phase shift, compared with the results of Arnowitt et al.[11] and of Brown and Goble.[8] The $p$-wave scattering length is obtained from

$$[(P^3/\sqrt{t})\cot\delta_{11}]_{t=4m_\pi^2}=\left[a_{11}+\frac{m_\pi^2}{\pi}\left(1-\frac{3-\delta}{2}\frac{P_\rho}{m_\rho}L_\rho\right)\right]\left(1+\frac{3-\delta}{4}\frac{m_\pi^2}{P_\rho^2}\right)^{-1}; \tag{23}$$

the soft-pion[12] value for this quantity is $a_{11}$ itself ($a_{11}=17m_\pi^2$). Numerically, we obtain the value $14.9m_\pi^2$ for $\delta=-\frac{1}{2}$. This is to be compared with Olsson's result,[13] $(15\pm1.2)m_\pi^2$, deduced from a forward-dispersion sum rule, and with the hard-pion result of Ref. 11, $14.5m_\pi^2$.

To summarize, hard-pion current algebra and analyticity have been employed in a complementary fashion to obtain the pion form factor. The method can be construed as dynamical in that Eqs. (9) and (10) lead to an integral equation, with a cutoff $\Lambda$, for $F(t)$. The exact solution of this cutoff problem is our effective-range formula (13) wherein $8\pi b=-\ln(\Lambda/m_\pi^2)$. The determination of $F(t)$ has been cast in a form independent of the $\pi\pi$ $p$-wave phase shift; therefore, the method is quite different from that of Ref. 3. Once $F$ is known, unitarity is invoked to yield $\delta_{11}$. Our $p$-wave scattering length differs from the soft-pion result and reproduces, via a very simple method, the value obtained through means[11] which are technically more involved. We suggest that such techniques as we have used for comple-

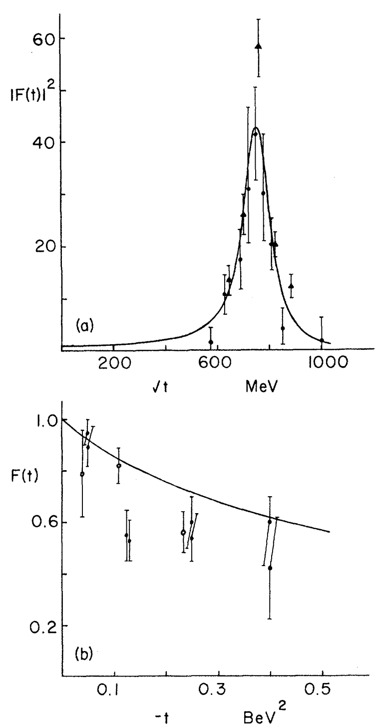


FIG. 1. Results of the form-factor calculation: (a) in the timelike region, (b) in the spacelike region. The SW value $\delta=-\frac{1}{2}$ is taken for the curves. For $\delta=-1$ the peak in (a) drops by 25%. In (a) the closed circle refers to Ref. 1; the closed triangle to Ref. 2.

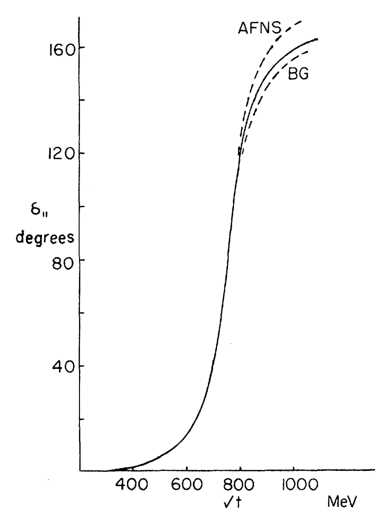


FIG. 2. The $p$-wave $\pi\pi$ phase shift. BG refers to Brown and Goble, Ref. 8; AFNS, to Arnowitt et al., Ref. 11.

menting current algebra with analyticity and unitarity can be advantageously employed in a variety of problems. Work is continuing along the lines of such a program.

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# PHYSICAL PICTURE FOR HIGH-ENERGY DIFFRACTION SCATTERING*

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Recent theoretical results on high-energy scattering are extended. We give explicit rules to obtain the high-energy behavior of the sums of large classes of Feynman diagrams. This consideration suggests, for high-energy diffraction scattering, a physical picture which is simple and natural. We emphasize that this physical picture has the virtue of yielding correctly all the high-energy results of quantum electrodynamics.

Recently, we obtained[1] the high-energy behavior for all two-body elastic scattering amplitudes in quantum electrodynamics. The original procedure of getting these results from perturbation theory is quite complicated,[2] but, if the justification of certain steps is not required, simplified derivations are possible with the help of the variables $p_0 \pm p_3$.[3] The results contradict both the Regge-pole model[4] and the droplet model[5,6] in their most straightforward interpretations. In this Letter, we present a simple physical picture consistent with, and indeed suggested by, our results from quantum electrodynamics.

A feature of our results is the close similarity between electron-electron scattering, electron-positron scattering, and electron scattering by a Coulomb field. More precisely, although these three processes are very different at low energy, the matrix elements, to the orders considered, are essentially identical in the limit of infinite energy. The same is true for electron Compton scattering and Delbrück scattering. Since these relations must be immediate from any useful physical picture, we first discuss the somewhat