tion of a lower limit for the absolute magnitude of the cross section for v-uv photon production. This lower limit is shown in Fig. 2 along with a second estimate of the cross section. The second estimate is obtained by normalizing the value for the photon production cross section at 22 eV in the c.m. system to that obtained by extrapolating McLaren and Hobson's[5] linear function for the excitation cross section. They report shock-tube measurements of the threshold behavior of excitation, in argon-argon collisions, to levels in the $(^2P)4s$ configuration. Their experiment does not distinguish between excitation to metastable states and those which decay by allowing transitions. Therefore, this second estimate may serve as an upper limit to the absolute cross section if it is assumed that the observed photons originate principally from states below 13.8 eV (the energy of the next excited state which may decay by v-uv photon emission).

It is very interesting to compare the photon production cross section with the cross section for negative-charge production[6] in collisions between neutral agron atoms, also shown in Fig. 2. One notes that the photon-production cross section remains below the cross section for ionization as the energy is increased. The most striking feature of the present results is the structure between 60- and 80-eV c.m. energy. This structure is of the same form and over the same interval as that found in the negative-charge production cross section, shown in Fig. 2 as a solid line. The meaning of this similarity between ionization and photon production is not yet clear. It may be the result of an energy resonance for the formation of some excited state, atomic or molecular in character, which decays either by ionization or photon emission or both. In any case, it is apparent that energetic photons, produced in considerable abundance even at low energies, may provide valuable information pertaining to inelastic atomic collisions.

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[4]The quantum efficiency of the tungsten MEM cathode was taken from the Bendix MEM operating manual. At wavelengths of 800 Å the efficiency may typically be 14% or more.

[5]T. I. McLaren and R. M. Hobson, Phys. Fluids 11, 2162 (1968).

[6]The cross sections for negative-charge production used in this comparison have been obtained in the same manner as, and agree well with, measurements reported by H. C. Hayden and R. C. Amme, Phys. Rev. 141, 30 (1966), and J. Chem. Phys. 44, 2828 (1966). However, hydrogen charge-transfer gas was used in the present work to ensure a ground-state neutral argon beam.

MICROWAVE SCATTERING DUE TO ELECTRON PLASMA-WAVE INSTABILITY*

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An experimental analysis is made of the scattering of microwaves by the density fluctuations due to electron plasma-wave instability in a beam-plasma system. Reasonably good (i.e., within an order of magnitude) agreement is obtained with the Shapiro-Drummond-Pines theoretical estimate of the energy associated with the "linearly unstable" electron plasma waves at "quasilinear" steady state.

According to theory, a beam-plasma system will become unstable against a growing plasma oscillation due to a two-stream instability mechanism when the beam density is sufficiently large and the beam velocity is approximately equal to the phase velocity of the plasma wave. Shapiro[1] has shown that even if initially the beam is not a gentle bump, such a "linearly unstable" beam-plasma system will in time reach a "quasilinear" steady state of Drummond and Pines[2] provided the density of beam electrons is very much less than that of the plasma electrons so that the growth rate of the plasma waves due to the beam electrons is less than the frequency of the plasma waves. In this quasisteady state the energy associated with the electron plasma wave under consideration is several orders of magnitude larger than its corresponding thermal equilibrium value (i.e., the value in the absence of the beam). Since the cross section for the scattering of electromagnetic waves from electron plasma waves[3,4] is proportional to the energy of the plasma waves, it is possible to carry out experiments on the scattering of microwaves from the density fluctuations associated with electron plasma-wave instability. Indeed, Böhmer and Raether[5] and Wharton and Malmberg[6] have previously reported experimental observations of microwave scattering from unstable electron plasma waves of beam-plasma systems. However, in these papers no comparison is attempted between quasilinear theory and experiment. It is our aim in this paper to see how well the theoretical predictions of the Shapiro-Drummond-Pines quasilinear theory are borne out by our microwave-scattering experiment.

In the theoretical analysis of this problem, one considers an electromagnetic wave of frequency $\omega_i = 2\pi f_i$ and wave vector $\vec{k}_i$ incident on a plasma of electron number density n_0 in which there exists an electron plasma wave of frequency $\omega_p = 2\pi f_p$, wave vector $\vec{k}_p$, and energy density $\bar{\mathcal{E}}(\vec{k}_p, \omega_p)$. Let $\omega_s = 2\pi f_s$ and $\vec{k}_s$ be the frequency and wave vector, respectively, of the scattered radiation such that the energy and momentum conservation relations

$$f_s = f_i \pm f_p \tag{1}$$

and

$$\vec{k}_s = \vec{k}_i \pm \vec{k}_p, \tag{2}$$

respectively, are satisfied. Then one can show that the differential cross section per electron $d^2\sigma(\vec{k}_p, \omega_p)/d\Omega d\omega$ which represents the fraction of the incident electromagnetic wave energy that is scattered by a single plasma electron into a unit frequency and solid-angle range around (ω_p, Ω_{k_p}) is given by[4]

$$\frac{d^2\sigma(\vec{k}_p, \omega_p)}{d\Omega d\omega} = \sigma_T(\theta_{k_p})\left(\frac{k_p^2}{4\pi n_0 e^2}\right)\bar{\mathcal{E}}(\vec{k}_p, \omega_p)\delta(\omega - \omega_p), \tag{3}$$

where $\sigma_T(\theta_{k_p}) = (e^2/mc^2)^2[1-\frac{1}{4}\sin^2 2\theta_{k_p}]$ is the Thomson cross section[7] for a scattering angle of θ_{k_p}.

For our experiment, measurements made with a conventional electrostatic energy analyzer indicate that the normalized velocity distribution function of the plasma electrons in the direction of the beam may be approximated by[8]

$$f_-(v) \approx \frac{(1-a)}{(2\pi)^{1/2}v_0} e^{-v^2/2v_0^2} + \frac{a}{(2\pi)^{1/2}v_1} e^{-v^2/2v_1^2}, \tag{4}$$

where $\kappa T_0/e = mv_0^2/e \approx 26$ eV, $\kappa T_1/e = mv_1^2/e \approx 680$ eV, and $a \approx 7.7\times10^{-4}$. Similar measurements indicate that the equivalent temperature[9] of the beam electrons $\kappa T_b/e = mv_b^2/e \approx 60$ eV at a beam voltage of $V_D = mv_d^2/2e = 3.9$ kV, where v_d is the drift velocity of the beam electrons and $2v_b$ is the velocity spread of the beam around v_d. Since for our experiment $v_b/v_d < (n_b/n_0)^{1/3} \ll 1$, where n_b and n_0 are the densities of the beam electrons and the plasma electrons, respectively, our beam is, according to Shapiro,[1] initially a low-density monoenergetic beam. When

$$\omega_p \approx k_p v_d, \tag{5}$$

and when the ratio of the growth rate due to the beam electrons to the damping rate due to the plasma electrons[10] is

$$\alpha \approx \frac{\sqrt{3}}{2}\left(\frac{n_b}{2n_0}\right)^{1/3}\frac{1}{a}\left(\frac{8}{\pi}\right)^{1/2}\left(\frac{v_1}{v_d}\right)^3 e^{v_d^2/2v_1^2} > 1, \tag{6}$$

such a beam-plasma system is unstable, and in the early stages of its time evolution the beam becomes smeared out to such an extent[1,11] that the eventual quasisteady state of the system is the quasilinear steady state of Drummond and Pines.[2,12] At this quasilinear steady state the energy density of the electron plasma waves per unit wave-number interval is (to within an order of magnitude) given by[3,13]

$$\sum\nolimits_{k_{p\perp}}\mathcal{E}(k_p, \omega_p) \approx \pi(\delta k_{p\perp}/2\pi)^2\bar{\mathcal{E}}(k_p, \omega_p) = j\,\tfrac{1}{4}(\tfrac{1}{2}\pi)^{1/2} n_b m\omega_p^2/k_p^3, \tag{7}$$

where $\perp$ refers to the component perpendicular to $\vec{v}_d$; $j \approx 1$ according to the Drummond-Pines estimate,[2] $j \approx (8\pi)^{1/2}$ according to Shapiro's estimate,[1] and[3]

$$\sin\delta \approx (\delta k_{p\perp}/k_p) \approx (\ln\alpha/R)^{1/2}(v_1/v_d). \tag{8}$$

Here, for the distribution function[14] of Eq. (4), the number of e foldings R through which the wave amplitude has gone before reaching the

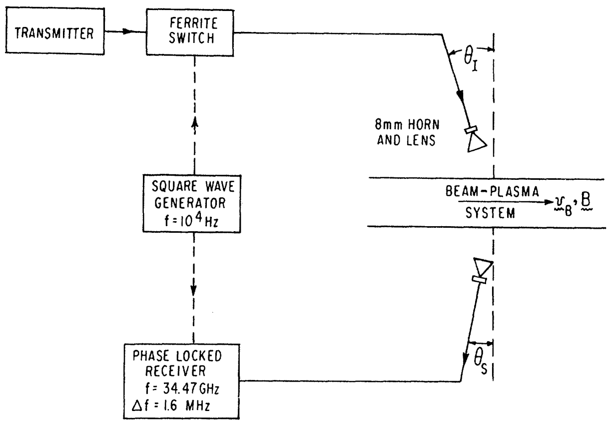

FIG. 1. Schematic diagram of the experimental arrangement.

steady-state spectrum is given by[3]

$$R \approx \tfrac{1}{2}\ln\left[j\left(\frac{\pi}{2}\right)^{3/2}\left(\frac{v_d^2}{v_1^2\ln\alpha}\right)\left(\frac{n_b}{k_p^3}\right)\left(\frac{v_d^2}{v_1^2}\right)\right]. \qquad (9)$$

The beam-plasma system under consideration has cylindrical symmetry about $\vec{v}_d$, hence, for microwave scattering with a pair of transmitter and receiver horns each of which has an angular resolution of $\pm\Delta\theta$ such that $\Delta\theta < \sin\delta \approx \delta$, it is relatively easy to show from Eqs. (3) and (7) that the "average" cross section per plasma electron [i.e., the frequency integral of Eq. (3) over the entire plasma resonance line and the solid-angle integral over the instrumental angular resolution $\Delta\theta \ll \theta_{k_p}$] is given by[15]

$$\sigma(\vec{k}_p, \omega_p) \approx \sigma_T(\theta_{k_p}) jR\pi\left(\frac{\pi}{2}\right)^{1/2} \times\left(\frac{v_d^2}{v_1^2\ln\alpha}\right)\left(\frac{n_b}{k_p^3}\right)\left(\frac{\sin\Delta\theta}{\sin\delta}\right)^2, \qquad (10)$$

where $\Delta\theta$ is in radians.

Figure 1 is a schematic block diagram of the experimental arrangement. The incident microwave power ($P_I = 6.0$ mW) was modulated in the line by a ferrite switch (at 10^4 Hz) and the detector was synchronized to this modulation frequency. The horn and lens combination was designed to give a plane-parallel beam. Half-power points appeared at about $\pm 3°$ from the center; that is $\Delta\theta = 3\pi/180$. The diameters of the cylindrical plasma column and the beam were about 10 and 1.9 cm, respectively, and the length of the beam-plasma system was about 200 cm. The scattering volume was approximately a 2 cm cube centered around the geometric center of the beam-plasma system. In these experiments the cyclotron frequency was 1.8 GHz.

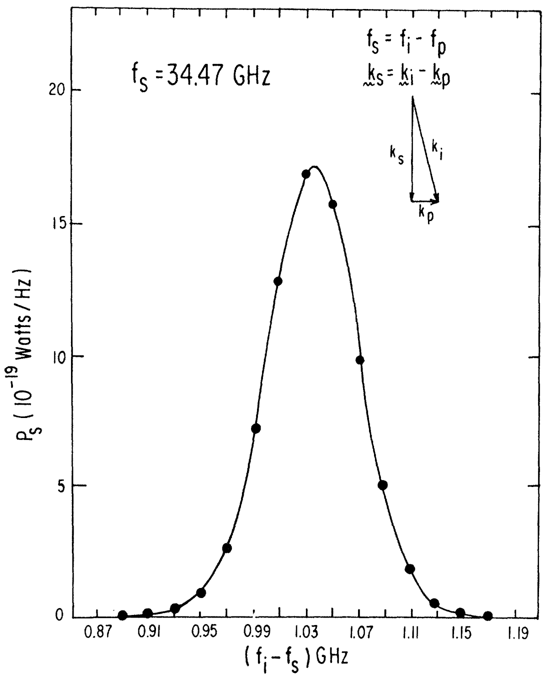

FIG. 2. Scattered power as a function of frequency. Experimental conditions are $n_0 = 1.34\times10^{10}$ cm^{-3} (i.e., $f_p = 1.04$ GHz), beam voltage = 3.9 kV, incident power = 6 mW, beam current = 47 mA ($n_B = 2.8\times10^7$ cm^{-3}), receiver bandwidth = 1.6 MHz, and $\theta_I = 14°$, $\theta_S = 0°$. The gas used was xenon at 6.4×10^{-6} Torr.

We first measured the scattered power at the plasma frequency as a function of the beam electron density n_b keeping all the other parameters fixed. This scattered power at ω_p first increased steadily with n_b, reached a maximum value, and then decreased as n_b was further increased. Figure 2 shows the frequency power spectrum measured at this maximum.

For the data shown in Fig. 2, the beam and plasma parameters were kept fixed (at this maximum value of the scattered power vs n_b), the receiver frequency was held at 34.47 GHz (±0.8 MHz), and the transmitter frequency was varied in steps of about 20 MHz, keeping the incident power level at a constant value of 6.0 mW for the purpose of scanning the scattered frequency-power spectrum. From the integrated area under the frequency-power spectrum of Fig. 2, we get the experimental value $\sigma(k_p, \omega_p) = 9.4\times10^{-19}$ cm^2. Since $\theta_{k_p} = \theta_I + \theta_S = 14°$, $\sigma_T(\theta_{k_p}) = 7.5\times10^{-26}$ cm^2. From Eqs. (5) and (6) we get $k_p = 1.77$ cm^{-1} and $\ln\alpha \approx 7.28$. Since $\Delta\theta = \pi/60$, the theoretical Eqs. (8), (9), and (10) yield $\sin\delta \approx 0.258$, $R \approx 9.51$, $\sigma(k_p, \omega_p) \approx 9.35\times10^{-19}$ cm^2, and $\sigma(k_p, \omega_p)/\sigma_T(\theta_{k_p}) \approx 1.25\times10^7$ for the Drummond-Pines estimate [i.e., for $j = 1$]; and for Shapiro's estimate [i.e., for $j=(8\pi)^{1/2}$] we get $\sin\delta\approx 0.248$, $R\approx 10.32$, $\sigma(k_p, \omega_p)\approx 5.52\times 10^{-18}$ cm^2 and $[\sigma(k_p, \omega_p)/\sigma_T(\theta_{k_p})]=7.36\times 10^7$.

The estimated uncertainty in the experimental value of $\sigma(k_p, \omega_p)$ is approximately a factor of 4. There are two sources that contribute to this factor of 4: (1) The correction that has been applied for the horn-to-horn insertion loss accounts for one-half of this uncertainty; and (2), typically the pair of angles θ_I, θ_S, corresponding to maximum scattered signal, differed by approximately 2° each from the corresponding pair of angles θ_I, θ_S predicted by Eqs. (1), (2), and (5), and consequently the integrated area under the frequency-power spectrum for the two cases differed by a factor of as much as 2. For the results discussed above, this latter factor of 2 is such as to double the experimental value quoted above.

That is, our microwave scattering experiment yields

$$\log_{10}\frac{\sigma(k_p, \omega_p)}{\sigma_T(\theta_{k_p})}=7.10^{+0.6}_{-0.3},$$

while Drummond-Pines quasilinear estimate yields

$$\log_{10}\frac{\sigma(k_p, \omega_p)}{\sigma_T(\theta_{k_p})}=7.10,$$

and Shapiro's estimate is

$$\log_{10}\frac{\sigma(k_p, \omega_p)}{\sigma_T(\theta_{k_p})}=7.87.$$

Thus we believe that our experimental result is in reasonably good agreement with the Shapiro-Drummond-Pines theoretical estimate of $\sigma(\vec{k}_p, \omega_p)$ based on their quasilinear theory.

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[1]V. D. Shapiro, Zh. Eksperim. i Teor. Fiz. 44, 613 (1963) [translation: Soviet Phys.—JETP 17, 416 (1963)].

[2]W. E. Drummond and D. Pines, Nucl. Fusion: Suppl. Pt. 3, 1049 (1962); E. Frieman and P. Rutherford, Ann. Phys. (N.Y.) 28, 134 (1964).

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[4]S. Ichimaru, Ann. Phys. (N.Y.) 20, 78 (1962).

[5]H. Böhmer and M. Raether, Phys. Rev. Letters 16, 1145 (1966). Other related references may be found in this paper and in C. B. Wharton and J. H. Malmberg, Phys. Fluids 11, 2655 (1968).

[6]See Wharton and Malmberg, Ref. 5.

[7]Here we have used the fact that for plane-polarized transverse electromagnetic waves the angle between the polarization vectors of the incident and scattered waves is equal to the scattering angle θ_{k_p}.

[8]Measurements with Langmuir probes give electron temperatures of approximately 10 eV but the final result [Eq. (7)] is insensitive to the "temperature" of the low-energy part of the distribution (see Ref. 10). The plasma is produced by electron cyclotron resonance in a linear mirror machine. The microwave power is coupled in at one end through a slotted cylinder [see, e.g., G. Lisitano et al., Rev. Sci. Instr. 39, 295 (1968)], and the electron beam was injected at the opposite end of the machine.

[9]Strictly speaking we should not call this a beam temperature, but it is a measure of the beam spread. The electron gun was a conventional diode with the filament at $-V_D$ and the anode grounded. The temperature of the filament was about 2000°K and the dc voltage across the filament leads was about 10V. This value of beam spread of $T_b\approx 60$ V is due primarily to the gun geometry and not to the filament voltage or temperature.

[10]Here we have used the fact that for our experimental conditions $a/(1-a)>>(v_1/v_0)^3\exp[v_d^2/2v_1^2-v_d^2/2v_0^2]$. The symbols α of Eq. (6), $\delta k_{p_\perp}$ of Eq. (8), and R of Eq. (9) have the same meaning as they have in Ref. 3.

[11]H. Böhmer and M. Raether, in "Nonlinear Effects in Plasmas" (Gordon and Breach Publishers, Inc., London, England, to be published).

[12]This flattening of the beam distribution has been observed by I. F. Karchenko et al., Nucl. Fusion: Suppl. Pt. 3, 1101 (1962).

[13]For our conditions, since $(\omega_p/k_p)\approx v_d>>v_1>>v_0$, it is relatively easy to show that the result of Eq. (7) obtained with the temporal quasilinear coupled pair of equations also holds true as the asymptotic solution of the spatial quasilinear coupled pair of equations. See Reviews of Plasma Physics, edited by M. A. Leontovich (Consultants Bureau, New York, 1967), Vol. 3, and V. S. Imshennik and Yu. I. Morozov, Zh. Tekh. Fiz. 31, 640 (1961) [translation: Soviet Phys.—Tech. Phys. 6, 464 (1961)]. Also, this result of Eq. (7) is very insensitive to initial $f_-(v)$ (see Ref. 1).

[14]Using $f_-(v)$ of Eq. (4) in Eq. (80) of Ref. 4, we get $\mathcal{E}_0(\vec{k}_p, \omega_p)\approx mv_1^2$ in the absence of the beam. This result is necessary to derive Eq. (9).

[15]The last factor $(\sin\Delta\theta/\sin\delta)^2$ in Eq. (10) arises in the following way: For a given $k_p\approx\omega_p/v_d$, the incident microwave power is scattered by all the $\delta k_{p_\perp}\propto\sin\delta$ of Eq. (8), while the receiver horn only collects the power that is scattered by a $\delta k_{p_\perp}\propto\sin\Delta\theta$. Hence, for our scattering geometry the fraction of the scattered microwave power that is collected by the receiver horn is $\propto(\sin\Delta\theta/\sin\delta)^2$.