When combined with the result $g_{\sigma\pi\pi}=\frac{1}{2}m_\rho f_{\rho\pi\pi}$ obtained from $\pi\pi$ analysis,[4] Eq. (12) can be written

$$G_S{}^A+\tfrac{1}{2}m_\rho{}^2G_D{}^A=m_\rho g_{A\sigma\pi}. \tag{13}$$

Now the ρ-dominated Regge behaviors for the amplitude $C^{\pi^-A_1^+}$ are $t^{\alpha_\rho(s)}$ and $s^{\alpha_\rho(t)}$ for large t and s, respectively. One of the three independent ρAA couplings leads to a t-channel ρ pole in the amplitude $C^{\pi^-A_1^+}$. We thus write the minimal Veneziano form for $C^{\pi^-A_1^+}$ simply as

$$C^{\pi^-A^+}(s,t,u)=\frac{(G_D{}^A)^2}{2}\frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_\rho(s))}{\Gamma(1-\alpha_\rho(t)-\alpha_\rho(s))}, \tag{14}$$

where the coefficient is obtained from normalization to the ρ Born term in the s channel. By completely identifying the $\alpha_\rho(s)=1$ pole in Eq. (14) with the σ and ρ Born terms, we obtain the additional relation

$$m_\rho{}^2g_{A\sigma\pi}{}^2+(G_S{}^A)^2+m_\rho{}^2G_S{}^AG_D{}^A-\tfrac{1}{4}(G_D{}^A)^2m_\rho{}^4=0. \tag{15}$$

Combining Eqs. (13) and (15) we find

$$G_S{}^A(G_S{}^A+m_\rho{}^2G_D{}^A)=0. \tag{16}$$

The solution $G_S{}^A=0$ requires, with Eq. (11), that

$$f_{\rho AA}=f_{\rho\pi\pi}(=f_{\rho\rho\rho}=f_{\rho KK}), \tag{17}$$

the universal result. The solution $G_S{}^A=-m_\rho{}^2G_D{}^A$ leads to a considerable shift of $f_{\rho AA}$ from the $f_{\rho\pi\pi}$ value if A_1 couples appreciably to $\rho\pi$.

We have presented evidence that the Veneziano representation appears to require universal ρ coupling in order to satisfy the A-W low-energy theorem. The ambiguity that occurs in the ρAA charge coupling can only be removed by detailed study of the consequences of each solution to Eq. (16). Construction of scattering amplitudes in addition to those considered here is necessary to pursue this point. This goes beyond the scope of this note. We emphasize that the modest charge-algebra, soft-pion extrapolation, and PCAC assumptions employed here with the Veneziano representation should suffice to implement further study.

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$$T=A\epsilon\circ P\epsilon'\circ P+\tfrac{1}{2}B(\epsilon\circ P\epsilon'\circ Q+\epsilon\cdot Q\epsilon'\circ P)+C\epsilon\circ Q\epsilon'\circ Q+D\epsilon\circ\epsilon',$$

where $P=\frac{1}{2}(p+p')$ and $Q=\frac{1}{2}(q+q')$.

[9]We regard π, ω, ρ, and A_1 (provisionally) as well established enough to exclude parity doubling. An isospin-0 $J^{PC}=1^{+-}$ state at the mass of the A_1 is allowed in our scheme (see Ref. 21).

[10]The couplings which we use in normalizing residues are summarized as follows:

$$\begin{aligned}\mathcal{H}_I={}&f_{\rho\pi\pi}\tilde\rho_\mu\cdot\tilde\pi\times\partial_\mu\tilde\pi+f_{\rho KK}\tilde\rho_\mu\circ\bar K\tfrac{1}{2}\tilde\tau\partial_\mu K+f_{\rho\rho\rho}\tilde\rho_\mu\cdot\tilde\rho_\nu\times\partial_\mu\tilde\rho_\nu+f_{\rho AA}\tilde\rho_\mu\cdot\tilde A_\nu\times\partial_\mu\tilde A_\nu+G_S{}^A\tilde A_\mu\cdot\tilde\rho_\mu\times\tilde\pi+G_D{}^A\tilde A_\mu\cdot\tilde\rho_\nu\times\partial_\mu\partial_\nu\tilde\pi\\&+G_S{}^HH_\mu\tilde\rho_\mu\cdot\tilde\pi+G_D{}^HH_\mu\tilde\rho_\nu\cdot\partial_\mu\partial_\nu\tilde\pi+g_{\sigma\pi\pi}\sigma\tilde\pi\cdot\tilde\pi+g_{A\sigma\pi}\tilde A_\mu\cdot\partial_\mu\tilde\pi\sigma+ig_{A_2\rho\pi}\epsilon_{\alpha\beta_1\gamma\delta}\partial_\alpha(\tilde A_2)_{\beta_1\beta_2}\cdot\tilde\rho_\gamma\times\partial_\delta\partial_{\beta_2}\tilde\pi\\&+ig_{\omega\rho\pi}\epsilon_{\alpha\beta\delta\gamma}\partial_\alpha\omega_\beta\partial_\delta\tilde\rho_\gamma\cdot\tilde\pi,\end{aligned}$$

where the tildes denote isovectors. The explicit form of the two additional ρAA couplings is never needed. Our $G_S^{A_1}$ and $G_D^{A_1}$ agree with the convention of T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 1067 (1967). Our $G_D^{A_1}$ is twice that of Fayyazuddin and Riazuddin, Phys. Letters 28B, 561 (1969), and "The Veneziano Model for Meson Systems and Its Connection with the Chiral Symmetry" (to be published).

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INTERPRETATION OF RECENT EXPERIMENTAL TESTS OF VECTOR-MESON DOMINANCE*

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There are several experiments involving photon interactions with nucleons or nuclei which are in apparent disagreement with the usual predictions of the vector-meson-dominance model. We present a tentative analysis of these experiments in terms of additional mass dependences arising from the vector-meson amplitudes.

The field-current identity asserts[1]

$$j_\mu^{em}=-\left(\frac{e}{f_\rho}m_\rho^2\rho_\mu^0+\frac{e}{f_\omega}m_\omega^2\omega_\mu+\frac{e}{f_\varphi}m_\varphi^2\varphi_\mu\right). \quad (1)$$

According to (1) any amplitude involving real or virtual photons is a linear combination of vector-meson amplitudes each multiplied by a vector-meson propagator. In the absence of an adequate dynamical model, the simplifying assumption is usually made that the only nontrivial dependence on the photon mass k^2 arises from the propagators and not from the amplitude of the vector meson. These two assumptions are usually referred to as the vector-meson-dominance (VMD) model, and from them various practical applications follow directly. In the present discussion, we shall refer to this as "optimistic" VMD.

There are now various experiments (to be discussed in more detail below) which are in conflict with optimistic VMD. This conflict may be resolved either by changing the structure of the electromagnetic current (1) or by allowing for a mass dependence in the vector-meson amplitudes. It is logically impossible to distinguish these two explanations experimentally without additional theoretical input.

Some modifications of (1) lead to a universal form factor and therefore affect all experiments in the same way. Examples are k^2-dependent photon-meson couplings or additional terms proportional to the vector-meson sources in Eq. (1). The elegant Novosibirsk and Orsay colliding-beam experiments[2] fail to indicate that they play an important role in the range $0 \leq k^2 \leq k_V^2$. We shall see later that it is impossible to account for the experiments under consideration with a universal k^2-dependent modification of VMD. Rather than take the drastic step of introducing new currents unrelated to the known conservation laws, we shall discuss the experiments under the assumption that the main deviations for small k^2 are due to a k^2 dependence of the vector-meson amplitudes.

In pion photoproduction, this dependence follows from a dynamical model which is in agreement with experiment.[3] The deviations from optimistic VMD for photoproduction are as much as a factor of 2 and do not support a further extension of VMD outside the considered range, even for rough estimates of cross sections, until one has a good dynamical model for the mass dependence.

We shall be concerned mostly with photoproduction off nucleons. Let us write the amplitudes $F_{\gamma a}$ for photoproduction of a $(=\omega,\rho,\varphi,\gamma,\pi,\cdots)$ in terms of the production amplitudes F_{Va} $(V=\rho,\omega,\varphi)$:

$$F_{\gamma a}(t)=\frac{e}{f_\rho}F_{\rho a}(t)C_{\rho a}(t)+\frac{e}{f_\omega}F_{\omega a}(t)C_{\omega a}(t)+\frac{e}{f_\varphi}F_{\varphi a}(t)C_{\varphi a}(t); \qquad (2)$$

here the factors C represent the effect of extrapolating from the vector-meson to the photon mass, and the s and helicity dependences are suppressed. The discussions of the ambiguities in applying VMD to polarized photoproduction[4] mean that the values of the C's depend also on the choice of frame used in defining the polarization. If such dependence is important, we shall understand (2) to be difined in the helicity frame.

From the model in Ref. 3 of the isovector part of (2), we may evaluate $C_{\rho\pi}^{\perp,\parallel}(t)$ for transverse photons, polarized perpendicular ($\perp$) or parallel ($\parallel$) to the reaction plane. Some typical values for $C_{\rho\pi}^{\perp,\parallel}(t)$ averaged over the helicity amplitudes are shown in Table I. The deviation of $C_{\rho\pi}^{\perp,\parallel}(t)$ from 1 has been traced back in Ref. 3 to the mass dependence of the scalar amplitudes A_i^V, which make up the amplitudes $F_{\gamma\pi}(s,t)$. If these A_i^V are calculated by fixed-t dispersion relations then mass dependences apart from the propagator term $(m_\rho^2-k^2)^{-1}$ arise from the fact that the dispersion relations are crossing symmetric under the exchange of s and u. Further effects arise from the effective coupling constants $F^{\rho NN}$ which enter the nucleon form factor. The results in Ref. 3 follow from very general concepts like analyticity, crossing symmetry, superconvergence relations, and mass dependence of coupling constants. Therefore they should not be accidental and can serve in the following discussion as a motivation.

Table I. Example for k^2 dependence in pion photoproduction represented by the factors $C_{\rho\pi}^{\perp,\parallel}$ [see Eq. (2)]. E_γ, photon energy in the laboratory system.

t $[m_\pi^2]$	$C^\perp$	$C^\parallel$	$C^\perp$	$C^\parallel$	$C^\perp$	$C^\parallel$
	$E_\gamma=4$ GeV		$E_\gamma=8$ GeV		$E_\gamma=16$ GeV	
−1	1.26	0.40	1.34	0.54	1.38	0.61
−5	1.36	0.30	1.45	0.32	1.49	0.33
−10	1.52	0.11	1.63	0.12	1.67	0.12

During the past year several authors reconsidered the consequences of VMD for photon interactions with nuclei.[5] The main features of these treatments are the following: (i) Optimistic VMD is applied only to the individual two-body interactions. (ii) Two contributions illustrated in Fig. 1 are expected to be important: The first is a "one-step" amplitude M_1 in which the photon directly produces the final particle on one of the nucleons. The second is a "two-step" amplitude M_2 in which the photon makes a (nearly real) vector meson on one nucleon. This meson propagates to a second nucleon where it interacts in the same manner as the incident photon in the one-step process. (iii) At high energies the interference between the two amplitudes leads to optimistic VMD applied to the whole nucleus,

$$\frac{d\sigma_{\gamma a}}{dt}=\alpha\left[\frac{f_\rho^2}{4\pi}\right]^{-1}\frac{d\sigma_{\rho a}}{dt}. \qquad (3)$$

In contrast to what would be expected from a one-step process alone this cross section shows the features of strong absorption for the photon, such as A dependence. The photon of course is not strongly absorbed because of the factor α in (3). This result comes about due to a near cancellation of the two amplitudes brought about by the fact that their size is related through the VMD assumption. (iv) At low energies (below or not too far above vector-meson threshold), the two-step amplitude becomes negligible, and the strong absorption features for the photon disappear. The physical reason for this is that the wave number of the vector meson differs signif-

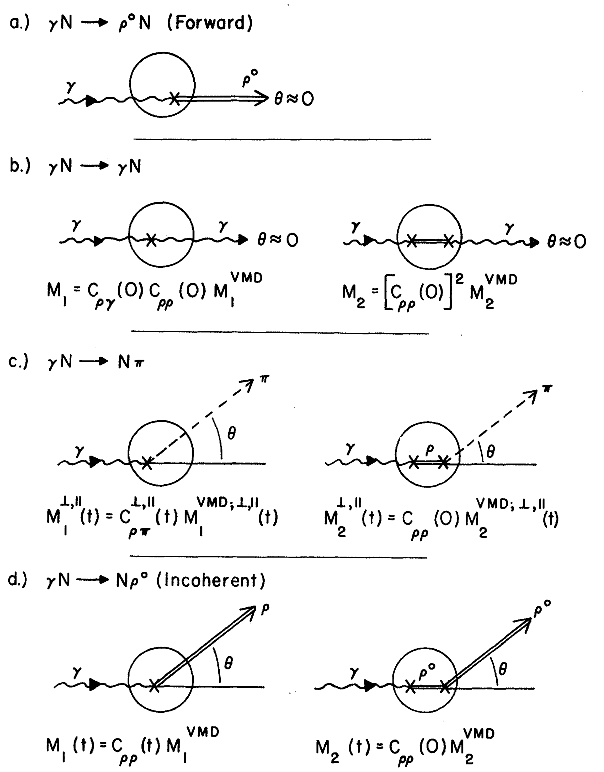

FIG. 1. Illustration of one- and two-step amplitudes M_1 and M_2 for various nuclear reactions. The amplitudes $M_{1,2}^{VMD}$ represent the result of applying vector-meson dominance without k^2 dependence; the effect of mass dependence is contained in the factors C_{jk} which are defined in Eq. (2).

icantly from that of the photon and a form factor suppresses the two-step amplitude. (v) The transition energy is governed by the form factor just mentioned, and turns out to be in the range 5-10 GeV for the ρ-meson contribution. (vi) This general feature of a transition from no photon absorption effects at low energies to strong absorption effects at high energies should be insensitive to detailed optical-model calculations.

The experiments which have been performed to test these expectations disagree rather strongly with them. Thus the optimistic view of VMD does not appear to work for nuclei. We shall discuss these experiments in terms of their implications for the k^2 dependence of various amplitudes, i.e., the deviations of the C's from 1. For simplicity, we shall assume that the ρ meson is more important than the isoscalar mesons; taking the other mesons into account would not change the qualitative picture which will emerge. At this time we shall not attempt a quantitatively precise analysis of the data. We

Table II. Results of various ρ^0-photoproduction experiments (see Ref. 6).

	Energy (GeV)	$\sigma_{\rho N}$ (mb)	$C_{\rho\rho}(0)$
Deutsches Elektronen Synchrotron	4.5	$\sim 31 \pm 5$	~ 1
Cornell	6.2	$\sim 38 \pm 3$	$\sim\frac{2}{3}$
Stanford Linear Accelerator Center	8.8	$\sim 30^{+6}_{-4}$	$\sim\frac{2}{3}$

ignore effects smaller than 10%, but regard larger effects as meaningful. The value of f_ρ from the Orsay experiment[2] will be used.

(1) Coherent ρ^0 photoproduction.[6]—Only the one-step amplitude contributes to this process. The two-step amplitude would correspond to a real two-body scattering rather than an optical-potential interaction and would be associated with the small incoherent ρ^0 production. In this experiment the A dependence yields the total ρ-nucleon cross section $\sigma_{\rho N}$. The forward differential cross section on hydrogen[7] may then be used to obtain the k^2 dependence for this process,

$$\left.\frac{d\sigma_{\gamma p}}{dt}\right|_{t=0} = [C_{\rho\rho}(0)]^2 \alpha \left[\frac{f_\rho^{\,2}}{4\pi}\right]^{-1} \frac{\sigma_{\rho N}^{\,2}}{16\pi}. \tag{4}$$

It is clear from the experimental results, presented in Table II, that no experimental consensus has yet emerged. These disagreements should be resolved as soon as possible. In the subsequent discussion, we shall use the Cornell results $C_{\rho\rho}(0) \sim \frac{2}{3}$ since the hydrogen point was measured directly rather than inferred from the normalization.

(2) Absorption of photons by nuclei.[8]—Referring to Fig. 1(b), we see that the ratio of the two-step to the one-step amplitude is modified by the factor $C_{\rho\rho}(0)/C_{\rho\gamma}(0)$ with respect to VMD. The experiment at 16 GeV is compatible with $\frac{2}{3}$ for this ratio. Thus if we use $\frac{2}{3}$ for $C_{\rho\rho}(0)$, we obtain $C_{\rho\gamma}(0) \sim 1$. That is, forward Compton scattering is related to forward photoproduction of ρ's by VMD without significant mass dependence. This agrees with an analysis by Guiragossián and Levy,[9] who used the total photon cross section and the optical theorem to estimate $F_{\gamma\gamma}(0)$.

(3) Photoproduction of π-mesons.[10]—The measurements have been made at 8 and 16 GeV and show no evidence for the energy dependence expected from VMD. The two polarizations must be treated separately and the ratio of M_2 to M_1 is modified by the factor $C_{\rho\pi}^{\perp,\parallel}(t)/C_{\rho\rho}(0)$. The

present experiment does not distinguish polarization effects but yields an effective ratio of $\sim\frac{1}{2}$. Since $\sigma^{\perp}$ dominates the cross section, we see that this result is in agreement with the results of Tables I and II [$C_{\rho\pi}{}^{\perp}(t)\sim\frac{4}{3}$ and $C_{\rho\rho}(0)\sim\frac{2}{3}$]. Since the ratio is drastically different for the two polarizations, it would be interesting to study the energy dependence as a function of the photon polarization; for example, it should be possible to adjust the plane of polarization so as to obtain a big energy dependence in the transition region.

(4) Incoherent ρ^0 photoproduction.[11]—Measurements have been made at 4 and 8 GeV. The cross section shows some shadowing effect but lacks any energy dependence. This time the ratio of the two-step to the one-step amplitudes is are modified by the factor $C_{\rho\rho}(0)/C_{\rho\rho}(t)$. Using $C_{\rho\rho}(0)\sim\frac{2}{3}$, the experiment is compatible with $C_{\rho\rho}(t)\sim 1$ for $-t\sim 0.12$ GeV^2. This rapid t dependence is surprising. It implies a strongly different t dependence between the ρ-scattering and ρ-photoproduction cross sections. Physically, it corresponds to a peripheral absorption of ρ mesons which has no counterpart in the photoproduction of ρ's. It is hard to see how this picture could be entirely wrong. As we stressed before, one would expect a strong energy dependence of the cross section if $C_{\rho\rho}(t)=C_{\rho\rho}(0)$. This conclusion is independent of the precise details of the optical model.

What is the possible physical significance of this peripheral absorption? Perhaps it arises from the circumstance that the ρ is an unstable particle which can be decomposed by the presence of a nucleon which need not be too close. In a real sense it may have a large internal spatial extension due to the fact that its state has a significant two-pion component in addition to a "bare" ρ^0 state.

The optical model using optimistic VMD involves the near cancellation of rather large terms at high energies and results in a large energy dependence of the cross section independent of the details of the calculations. In the present analysis we have disturbed this cancellation by our particular assumption of the k^2 dependence and thereby reduced the energy dependence. In spite of the nonuniqueness of our choice of C's we want to stress—and this is our main point—that any final analysis which retains the field-current identity will have to include some significant k^2 dependence of the vector-meson amplitudes. The specific effects of k^2 dependence could be checked by electroproduction experiments.

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