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UNIVERSAL ρ COUPLING AND THE ADLER-WEISBERGER THEOREM FOR $\pi^-\rho^+$ AND $\pi^-A_1^+$ IN THE VENEZIANO MODEL*

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We write minimal Veneziano representations, normalized to the t-channel ρ poles, for the $\pi^-\rho^+\to\pi^-\rho^+$ and $\pi^-A_1^+\to\pi^-A_1^+$ invariant amplitudes relevant to the Adler-Weisberger theorem. Using these amplitudes and the results of other authors for $\pi^-\pi^+$ and π^-K^+, we find that the Adler-Weisberger theorem is satisfied for $\pi^-\pi^+$, π^-K^+, $\pi^-\rho^+$, and $\pi^-A_1^+$ elastic scattering if the $\rho\pi\pi$, ρKK, $\rho\rho\rho$, and ρAA charge couplings all have the same (universal) value.

Resonance-saturated and nearby Regge-pole-dominated "bootstrap" studies[1,2] led Veneziano[3] to propose a simple expression for relativistic scattering amplitudes. Predictably, the Veneziano representation has been successful largely in the region from threshold through the first one or two resonances in each channel. One prominent type of application has been the no-satellite Veneziano expression for spinless meson scattering[4] which, supplemented by the Adler partially conserved axial-vector current (PCAC) consistency condition,[5] predicts scattering lengths and mass and coupling relations in agreement with chiral symmetric and/or experimental results. We are encouraged, despite complications present when more than one external particle has nonzero spin,[6] to investigate simple Veneziano prescriptions for kinematically more complicated processes in the low-energy region. Of special interest are the restrictions imposed by the Adler consistency condition[5] and the Adler-Weisberger (A-W) low-energy theorem.[7]

We discuss here the $\pi^-\rho^+$ and $\pi^-A_1^+$ elastic-scattering cases, concentrating on the invariant amplitude[8] $D(s,t,u)$ relevant to the low-energy theorem. The simplest Veneziano expression consistent with conservation laws, properties of well-established particles,[9] and the Adler PCAC condition[5] is written down. It is found that the A-W theorem is satisfied for $\pi^-\pi^+$, π^-K^+, $\pi^-\rho^+$, and $\pi^-A_1^+$ if the $\rho\pi\pi$, ρKK, $\rho\rho\rho$, and ρAA charge couplings[10] have the same value, as required by the ρ universality of gauge-field theory.[11] Under the PCAC and charge-algebra requirements, the universal ρ coupling is directly related to the universality of the trajectory slope in the Veneziano model, as conjectured by Kawarabayashi, Kitakado, and Yabuki.[4]

The Veneziano representation of invariant amplitudes plays a particularly important part in the πA_1 case. There the equality of the ρAA and $\rho\pi\pi$ charge couplings follows only if $G_S^A=0$, [10,12] which is one of two solutions for G_S^A/G_D^A required by consistency between ρ-normalized minimal Veneziano representations for $\pi^-\pi^+\to\pi^-A_1^+$ [13] and $\pi^-A_1^+\to\pi^-A_1^+$ [14] scattering.

Before taking up the πA_1 and $\pi\rho$ cases, we shall briefly present the results from $\pi^-\pi^+$ and π^-K^+ scattering to establish the role played by the t-channel ρ-pole normalization requirement.

The A-W theorems for $\pi^-\pi^+$ and π^-K^+ read[15]

$$\begin{pmatrix}2\\1\end{pmatrix}=-2F_\pi^{\,2}\frac{1}{2E}\frac{d}{dk_0}\begin{pmatrix}T^{\pi^-\pi^+}(k_0,\nu,t=0)\\T^{\pi^-K^+}(k_0,\nu,t=0)\end{pmatrix}\Bigg|_{\nu=0,k_0=0}. \tag{1}$$

The one-term Veneziano forms for $T^{\pi^-\pi^+}$ and $T^{\pi^-K^+}$ are[4]

$$T^{\pi^-\pi^+}(s,t,u)=2f_{\rho\pi\pi}^{\;2}\frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_\rho(s))}{\Gamma(1-\alpha_\rho(t)-\alpha_\rho(s))} \tag{2a}$$

and

$$T^{\pi^-K^+}(s,t,u)=f_{\rho\pi\pi}f_{\rho KK}\frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_{K^*}(s))}{\Gamma(1-\alpha_\rho(t)-\alpha_{K^*}(s))}, \tag{2b}$$

respectively. As usual the trajectories are assumed to be real and linear, i.e., $\alpha(x)=\alpha'x+\alpha(0)$. In writing Eq. (2), it is assumed that no u-channel (exotic) resonances exist. The vanishing of the amplitudes at the Adler limits is assured by the approximately satisfied trajectory conditions $2\alpha_\rho(\mu^2)=1$ and $\alpha_\rho(\mu^2)+\alpha_{K^*}(m_K^{\,2})=1$. The amplitudes have been normalized by matching the ρ pole in the t channel with the respective Born terms. Combining Eqs. (1) and (2), and invoking the equality of the slopes of the ρ and K^* trajectories, we obtain[16,17]

$$f_{\rho\pi\pi}^{\;2}=f_{\rho\pi\pi}f_{\rho KK}=\frac{1}{\pi\alpha'}\frac{1}{2F_\pi^{\,2}}=\frac{2}{\pi}\frac{2m_\rho^{\,2}}{2F_\pi^{\,2}}, \tag{3}$$

which gives the equality $f_{\rho\pi\pi}=f_{\rho KK}$ common to both SU(3) and universal ρ coupling. In other words, the coupling relation is determined only by the isospin factors, the derivatives at $k_0=0$ of the combinations of gamma functions in Eqs. (2a) and (2b) being the same, namely $-\pi\alpha'$.

Considering now the $\pi\rho$ and πA_1 cases, we shall first discuss the features essential in prescribing minimal Veneziano forms for the invariant amplitudes.

The Adler consistency conditions for pion scattering from a spin-1 target of mass m_T are

$$A(m_T^{\,2},\mu^2,m_T^{\,2})-C(m_T^{\,2},\mu^2,m_T^{\,2})=0, \tag{4a}$$

and

$$D(m_T^{\,2},\mu^2,m_T^{\,2})=0. \tag{4b}$$

Considering $\pi\rho$ specifically for a moment, we note that the normal-parity ω trajectory contribution should vanish trivially in (4b),[18] as well as in the limit $s=u=m_T^{\,2}$, $t=0$. It is crucial that this kinematical threshold suppression be properly included, since the ω pole appears at the soft-pion points for degenerate ρ-ω masses. We therefore treat the ω contribution to D on a separate footing[19] and put in the threshold factor by requiring that the ω pole in the s channel has the perturbation-theory residue, which includes the correct p-wave $|\vec{P}_{\text{c.m.}}|^2$ threshold dependence. Only the terms with the abnormal-parity s-channel trajectories contribute to the s wave at threshold, and these terms must vanish by cancellation or by the trajectory condition. For πA_1 scattering, the situation regarding normal and abnormal parity is reversed.

We shall avoid the parity-doubling difficulty[6] by simply excluding those terms which by themselves contain parity-doubling consequences along the leading trajectories. This choice obviates the need for detailed cancellation of terms to ensure pure parity states at each pole.[20]

With the above considerations in mind, we keep the fewest terms necessary to produce maximum allowed Regge behavior, observed low-lying resonances in every channel, and polynomial residues for particle poles. We take advantage of the experimentally unclear situation for the $\pi\rho$ system, particularly in the A_1 mass region,[21] to admit the presence of the two $I=0$ trajectories which are required by duality and absence of $I=2$ resonances to be exchange degenerate with π and A_1.[2]

Guided by the preceding remarks, and requiring the Regge behaviors $t^{\alpha_\omega(s)}$, $t^{\alpha_\pi(s)-1}$, $t^{\alpha_A(s)-1}$ for large t with fixed s, and $s^{\alpha_\rho(t)}$ for large s with fixed t,[6,14] we express $D^{\pi^-\rho^+}$ as

$$D^{\pi^-\rho^+}(s,t,u)=2f_{\rho\pi\pi}f_{\rho\rho\rho}\frac{\Gamma(1-\alpha_\rho(t))\Gamma(2-\alpha_\pi(s))}{\Gamma(2-\alpha_\rho(t)-\alpha_\pi(s))}-f_{\rho\pi\pi}f_{\rho\rho\rho}\frac{\Gamma(1-\alpha_\rho)\Gamma(1-\alpha_\pi)}{\Gamma(2-\alpha_\rho-\alpha_\pi)}$$
$$+\alpha' g_{\omega\rho\pi}{}^2\{\tfrac{1}{2}st-\tfrac{1}{4}[s-(m-\mu)^2][s-(m+\mu)^2]\}\frac{\Gamma(2-\alpha_\rho)\Gamma(1-\alpha_\omega)}{\Gamma(3-\alpha_\rho-\alpha_\omega)}, \quad (5)$$

where the s poles in the first two terms contain both $I=1$ and $I=0$ ingredients of the same spin. To determine the coefficients we have required ρ-pole normalization in the t channel, ω-pole normalization in the s channel, and cancellation of the first two terms to satisfy the Adler consistency condition.

In the infinite ρ-momentum limit, the A-W relation for $\pi^-\rho^+$ scattering can be written in the transverse ρ polarization case as

$$2=2F_\pi{}^2\left[\frac{g_{\omega\rho\pi}{}^2}{4}-\frac{1}{2E}\frac{d}{dk_0}D^{\pi^-\rho^+}(k_0,\nu,t=0)\right]_{k_0=0,\nu=0}. \quad (6)$$

Differentiating Eq. (5) and substituting into Eq. (6), we find that the derivative of the ω part in Eq. (5) cancels the first term on the right-hand side of (6), and the theorem reads

$$2=4\pi F_\pi{}^2\alpha' f_{\rho\rho\rho}f_{\rho\pi\pi}. \quad (7)$$

By referring to Eq. (3), we then obtain

$$f_{\rho\rho\rho}=f_{\rho\pi\pi}. \quad (8)$$

The A-W theorem for the longitudinal ρ polarization, which contains no reference to ω, leads to the same results.

Before turning to the πA_1 case, we note that the residue at the pole $\alpha_\omega(s)=2$ in Eq. (5) yields $g_{\omega\rho\pi}{}^2=m_\omega{}^2 g_{A_2\rho\pi}{}^2$ for the $A_2\rho\pi$ coupling. This agrees with results of several authors.[22]

For πA_1 scattering, we assume that the ρ trajectory dominates the s and t channels. For the amplitude $D^{\pi^-A_1^+}$, the ρ-dominated asymptotic Regge behaviors for large t and s are $t^{\alpha_\rho(s)-1}$ and $s^{\alpha_\rho(t)}$, respectively. Allowing the possibility of s-wave $A_1\rho\pi$ coupling, so that a ρ pole can appear in D in the s channel, we write the minimal Veneziano form

$$D^{\pi^-A_1^+}(s,t,u)=2f_{\rho\pi\pi}f_{\rho AA}\frac{\Gamma(2-\alpha_\rho(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(2-\alpha_\rho(s)-\alpha_\rho(t))}-(G_S{}^A)^2\alpha'\frac{\Gamma(1-\alpha_\rho)\Gamma(1-\alpha_\rho)}{\Gamma(2-\alpha_\rho(s)-\alpha_\rho(t))}, \quad (9)$$

where the first term has been normalized to the ρ Born amplitude in the t channel and the second to the (pure s-wave) Born amplitude in s. The terms vanish individually at the Adler point $s=u=m_A{}^2$, $t=\mu^2$ due to the approximately satisfied condition $2=\alpha_\rho(m_{A_1}{}^2)+\alpha_\rho(\mu^2)$.

In the limit of infinite A_1 momentum, the A-W theorem is

$$2=-2F_\pi{}^2\frac{1}{2E}\frac{d}{dk_0}D^{\pi^-A^+}(k_0,\nu,t=0)\Big|_{k_0=0,\nu=0}. \quad (10)$$

Substitution of Eq. (9) into Eq. (10) provides the relation

$$2=2F_\pi{}^2[2(G_S{}^A)^2(\alpha')^2\pi+2\pi\alpha' f_{\rho AA}f_{\rho\pi\pi}]. \quad (11)$$

Comparing Eq. (11) with Eqs. (7) and (3), we see that the universal value of $f_{\rho AA}$ ($f_{\rho AA}=f_{\rho\pi\pi}=f_{\rho\rho\rho}$) follows only if $G_S{}^A=0$. We now show that $G_S{}^A=0$ is one of two solutions required by consistency between minimal Veneziano representations for $\pi^-\pi^+\to\pi^-A_1{}^+$ and $\pi^-A_1{}^+\to\pi^-A_1{}^+$ scattering.

The process $\pi^-\pi^+\to\pi^-A_1{}^+$ has been thoroughly studied[13,14] and yields the relation

$$\tfrac{1}{2}f_{\rho\pi\pi}(G_S{}^A+\tfrac{1}{2}m_\rho{}^2G_D{}^A)=g_{\sigma\pi\pi}g_{A\sigma\pi}. \quad (12)$$

When combined with the result $g_{\sigma\pi\pi}=\frac{1}{2}m_\rho f_{\rho\pi\pi}$ obtained from $\pi\pi$ analysis,[4] Eq. (12) can be written

$$G_S^A+\tfrac{1}{2}m_\rho^2G_D^A=m_\rho g_{A\sigma\pi}. \tag{13}$$

Now the ρ-dominated Regge behaviors for the amplitude $C^{\pi^-A_1^+}$ are $t^{\alpha_\rho(s)}$ and $s^{\alpha_\rho(t)}$ for large t and s, respectively. One of the three independent ρAA couplings leads to a t-channel ρ pole in the amplitude $C^{\pi^-A_1^+}$. We thus write the minimal Veneziano form for $C^{\pi^-A_1^+}$ simply as

$$C^{\pi^-A^+}(s,t,u)=\frac{(G_D^A)^2}{2}\frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_\rho(s))}{\Gamma(1-\alpha_\rho(t)-\alpha_\rho(s))}, \tag{14}$$

where the coefficient is obtained from normalization to the ρ Born term in the s channel. By completely identifying the $\alpha_\rho(s)=1$ pole in Eq. (14) with the σ and ρ Born terms, we obtain the additional relation

$$m_\rho^2g_{A\sigma\pi}^2+(G_S^A)^2+m_\rho^2G_S^AG_D^A-\tfrac{1}{4}(G_D^A)^2m_\rho^4=0. \tag{15}$$

Combining Eqs. (13) and (15) we find

$$G_S^A(G_S^A+m_\rho^2G_D^A)=0. \tag{16}$$

The solution $G_S^A=0$ requires, with Eq. (11), that

$$f_{\rho AA}=f_{\rho\pi\pi}(=f_{\rho\rho\rho}=f_{\rho KK}), \tag{17}$$

the universal result. The solution $G_S^A=-m_\rho^2G_D^A$ leads to a considerable shift of $f_{\rho AA}$ from the $f_{\rho\pi\pi}$ value if A_1 couples appreciably to $\rho\pi$.

We have presented evidence that the Veneziano representation appears to require universal ρ coupling in order to satisfy the A-W low-energy theorem. The ambiguity that occurs in the ρAA charge coupling can only be removed by detailed study of the consequences of each solution to Eq. (16). Construction of scattering amplitudes in addition to those considered here is necessary to pursue this point. This goes beyond the scope of this note. We emphasize that the modest charge-algebra, soft-pion extrapolation, and PCAC assumptions employed here with the Veneziano representation should suffice to implement further study.

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$$T=A\epsilon\circ P\epsilon'\circ P+\tfrac{1}{2}B(\epsilon\circ P\epsilon'\circ Q+\epsilon\bullet Q\epsilon'\circ P)+C\epsilon\circ Q\epsilon'\circ Q+D\epsilon\circ\epsilon',$$

where $P=\frac{1}{2}(p+p')$ and $Q=\frac{1}{2}(q+q')$.

[9]We regard π, ω, ρ, and A_1 (provisionally) as well established enough to exclude parity doubling. An isospin-0 $J^{PC}=1^{+-}$ state at the mass of the A_1 is allowed in our scheme (see Ref. 21).

[10]The couplings which we use in normalizing residues are summarized as follows:

$$\begin{aligned}\mathcal{H}_I={}&f_{\rho\pi\pi}\tilde\rho_\mu\cdot\tilde\pi\times\partial_\mu\tilde\pi+f_{\rho KK}\tilde\rho_\mu\circ\bar K\tfrac{1}{2}\tilde\tau\partial_\mu K+f_{\rho\rho\rho}\tilde\rho_\mu\cdot\tilde\rho_\nu\times\partial_\mu\tilde\rho_\nu+f_{\rho AA}\tilde\rho_\mu\cdot\tilde A_\nu\times\partial_\mu\tilde A_\nu+G_S^A\tilde A_\mu\cdot\tilde\rho_\mu\times\tilde\pi+G_D^A\tilde A_\mu\cdot\tilde\rho_\nu\times\partial_\mu\partial_\nu\tilde\pi\\&+G_S^HH_\mu\tilde\rho_\mu\cdot\tilde\pi+G_D^HH_\mu\tilde\rho_\nu\cdot\partial_\mu\partial_\nu\tilde\pi+g_{\sigma\pi\pi}\sigma\tilde\pi\cdot\tilde\pi+g_{A\sigma\pi}\tilde A_\mu\cdot\partial_\mu\tilde\pi\sigma+ig_{A_2\rho\pi}\epsilon_{\alpha\beta_1\gamma\delta}\partial_\alpha(\tilde A_2)_{\beta_1\beta_2}\cdot\tilde\rho_\gamma\times\partial_\delta\partial_{\beta_2}\tilde\pi\\&+ig_{\omega\rho\pi}\epsilon_{\alpha\beta\delta\gamma}\partial_\alpha\omega_\beta\partial_\delta\tilde\rho_\gamma\cdot\tilde\pi,\end{aligned}$$

where the tildes denote isovectors. The explicit form of the two additional ρAA couplings is never needed. Our $G_S^{A_1}$ and $G_D^{A_1}$ agree with the convention of T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 1067 (1967). Our $G_D^{A_1}$ is twice that of Fayyazuddin and Riazuddin, Phys. Letters 28B, 561 (1969), and "The Veneziano Model for Meson Systems and Its Connection with the Chiral Symmetry" (to be published).

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INTERPRETATION OF RECENT EXPERIMENTAL TESTS OF VECTOR-MESON DOMINANCE*

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There are several experiments involving photon interactions with nucleons or nuclei which are in apparent disagreement with the usual predictions of the vector-meson-dominance model. We present a tentative analysis of these experiments in terms of additional mass dependences arising from the vector-meson amplitudes.

The field-current identity asserts[1]

$$j_\mu^{em}=-\left(\frac{e}{f_\rho}m_\rho^2\rho_\mu^0+\frac{e}{f_\omega}m_\omega^2\omega_\mu+\frac{e}{f_\varphi}m_\varphi^2\varphi_\mu\right). \quad (1)$$

According to (1) any amplitude involving real or virtual photons is a linear combination of vector-meson amplitudes each multiplied by a vector-meson propagator. In the absence of an adequate dynamical model, the simplifying assumption is usually made that the only nontrivial dependence on the photon mass k^2 arises from the propagators and not from the amplitude of the vector meson. These two assumptions are usually referred to as the vector-meson-dominance (VMD) model, and from them various practical applications follow directly. In the present discussion, we shall refer to this as "optimistic" VMD.

There are now various experiments (to be discussed in more detail below) which are in conflict with optimistic VMD. This conflict may be resolved either by changing the structure of the electromagnetic current (1) or by allowing for a mass dependence in the vector-meson amplitudes. It is logically impossible to distinguish these two explanations experimentally without additional theoretical input.

Some modifications of (1) lead to a universal form factor and therefore affect all experiments in the same way. Examples are k^2-dependent photon-meson couplings or additional terms pro-