## MEASUREMENT OF THE RELATIVE PHASE OF THE $K_L \rightarrow \pi^+\pi^-$ AND $K_S \rightarrow \pi^+\pi^-$ DECAY AMPLITUDES BY "VACUUM REGENERATION"\*†

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The nonexponential  $\pi^+\pi^-$  decay of neutral K's, due to the interference of  $K_S$  and  $K_L$  in this common channel, has been quantitatively studied using a beam which is preponderantly  $K^0$  at t=0. The value  $(40\pm5)^\circ$  is obtained for  $\arg\eta_+$ .  $(\eta_{+-}=\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle)/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^+\pi^-|T|K_L\rangle/\langle\pi^$ 

One of the few parameters entering the phenomenological description<sup>1</sup> of CP nonconservation in the  $K^{0}$  complex is the decay-amplitude ratio

 $\eta_{+-} = |\eta_{+-}| e^{i\varphi_{+}} = \langle \pi^{+}\pi^{-}|T|K_{L} \rangle / \langle \pi^{+}\pi^{-}|T|K_{S} \rangle.$ 

While the experimental value of the magnitude of this ratio has hardly changed since the discovery of CP nonconservation by the Princeton group  $[(2.3\pm0.4)\times10^{-3}$  in the original report<sup>2</sup> versus  $(1.92\pm0.04)\times10^{-3}$  in a recent compilation<sup>3</sup>], much confusing evidence has been accumulated about its phase  $\varphi_{+-}$ .<sup>4-9</sup> This contrasting state of affairs is particularly regrettable because at least certain theories make quantitative predictions for  $\varphi_{+-}$  while no current theory predicts  $|\eta_{+-}|$  precisely.

Most of the previous results are not independent determinations of  $\varphi_{+-}$ , but rather successive interpretations of the same data.<sup>4-9</sup> The difficulties in interpretation arose from the fact that one usually measured the relative phase  $\Phi$  of the pertinent  $K_L$  and  $K_S$  decay amplitudes in interference experiments where the  $K_S$ 's were produced by regeneration in matter. In this type of experiment the quantity  $\Phi = \varphi_{+-} - \varphi_L$  ( $K_S = \rho K_L$ ,  $\varphi_L = \arg \rho$ ), and  $\varphi_L$  was not reliably known.

It is clear that these difficulties are absent when one uses a given  $K^0$  as the source of both the  $K_s$  and the  $K_L$  amplitudes, i.e., when one observes the nonexponential decay of the  $K^0$  into two charged pions implied by CP nonconservation.<sup>10</sup> As a function of proper time t, the  $\pi^+\pi^-$  decay rate of  $K^0$ 's produced at t=0 varies, in fact, as

$$I_{+-}(t) \propto e^{-t\Gamma_{S}} + |\eta_{+-}|^{2} e^{-t\Gamma_{L}} + 2|\eta| e^{-t(\Gamma_{S}+\Gamma_{L})/2} \cos(\Delta m t - \varphi_{+-}), \tag{1}$$

where  $\Gamma_{S(L)} = \text{decay rate of } K_{S(L)}$  and  $\Delta m = |M_L - M_S|$ . For  $\overline{K}^{0}$ 's the sign of the cross term in (1) becomes negative. In practice,  $\overline{K}^{0}$ 's as well as  $K^{0}$ 's are produced incoherently from the same target, say with momentum spectra  $\overline{S}(p_K)$  and  $S(p_K)$ , at the production angle  $\theta$ . One then has, instead of (1),

$$I_{+-}(t) \propto e^{-t\Gamma_{S}} + |\eta_{+-}|^{2} e^{-t\Gamma_{L}} + 2D |\eta_{+-}| e^{-t(\Gamma_{S} + \Gamma_{L})/2} \cos(\Delta m t - \varphi_{+-}), \tag{1'}$$

where  $D \equiv [S(p_K) - \overline{S}(p_K)] / [S(p_K) + \overline{S}(p_K)]$ ; i.e., the interference term is reduced by the "dilution factor"  $D(p_K)$ .

We describe here briefly an experimental observation of (1') leading to a determination of  $\varphi_{+-}$  by "vacuum regeneration." This experiment is quite similar to one reported by a CERN group at the Heidelberg Conference.<sup>11</sup>

Aside from points of instrumentation, the two experiments differ primarily in that our  $\overline{K}^{0}$  contamination is smaller, implying a lesser dilution of the interference  $(D_{av} \approx 0.93 \text{ vs } D_{av} \approx 0.45)$  and a correspondingly increased statistical power. This, together with the larger number of events retained for final analysis, leads to a significantly smaller statistical uncertainty in  $\varphi_{+-}$ .

The present experiment was carried out at the zero-gradient synchrotron (ZGS), using the arrangement shown in Fig. 1. A Hevimet target

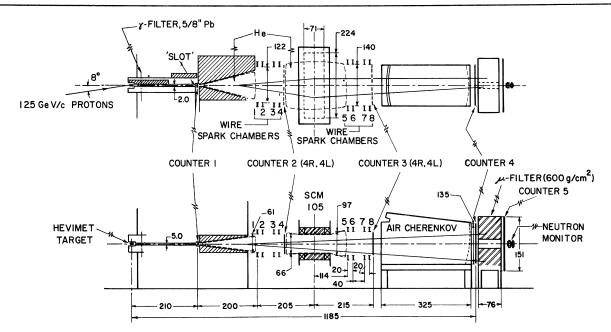


FIG. 1. Plan and side views of the experimental apparatus. The shaded portions of the shielding are of uranium metal.

 $(0.63 \times 2.54 \times 5.08 \text{ cm})$  was bombarded with the external proton beam [12.5 GeV/c, 700-msec]spill,  $(1-2) \times 10^{10}$  protons/pulse], and a neutral beam at 8° was produced through a 210-cm-long collimator (2 cm wide by 5 cm high) swept with a field of 18 kG. The choice of these collimator dimensions was based primarily on two criteria: (a) maximum shielding consistent with the emergence of kaons decaying in the region of strongest interference  $(t \approx 12\tau_s)$ ; and (b) sweeping out of particles with up to 12.5-GeV/c momentum. The collimator was followed by a decay region of 200 cm. While the main shielding wall was of steel, the region directly hit by the proton beam and that surrounding the decay region were of depleted uranium. The direction and momenta of the charged particles were measured with a spectrometer consisting of a large magnet sandwiched between two banks of four wire spark chambers<sup>12</sup> each. One of these chambers (No. 3) was split into independent halves to allow the resolution of ambiguities, and vertical strips (of 7.6 to 12.6 cm width) at the centers of all chambers were kept unpulsed to allow for the passage of the "neutral" beam. In the disposition of the triggering counter, as well as in the use of a Cherenkov counter and an iron muon filter for lepton suppression, our setup is similar to that introduced at CERN by the Vivargent-Winter group.<sup>13</sup> The Cherenkov counter, to be described elsewhere, was operated with air at ambient

density, and had a measured electron rejection efficiency of >98%. The trigger required for " $2\pi$ " events was (I, 2R/4, 3R/4, 2L/4, 3L/4, 4L/2, 4R/2) in anticoincidence with  $\check{C}$  and 5; the fractional notation X/n means that one (and only one) of the *n* independent scintillators consituting counter X is required. Counter 4 consists of four scintillators; by requiring two diagonally opposite quadrants in the signature a mild coplanarity requirement was imposed. While any one scintillator of counters 2 and 3 was acceptable for the trigger, the individual units were electronically tagged, and the identity of those units that fired in a given trigger was recorded on tape together with the spark-chamber coordinates. This information was available off line for the task of event construction.

During the main data-taking run, the " $2\pi$ " trigger rate was 5/pulse; subsequent analysis showed that ~8% of these triggers were potential  $K \rightarrow \pi^+\pi^-$  events. The performance of the chambers was remarkably clean: Whereas a perfect event should produce 32 "sparks" (counting the x and y coordinates separately), the actual spark-number distribution centered around 42 sparks. This corresponds to 1.2 spurious "sparks" per chamber.

In this experiment it is of pivotal importance to make sure (a) that the kaons originate from the target, (b) that they emerge from the collimator without regenerating, and (c) that the  $\pi^+\pi^-$  decays not be contaminated by leptonic events. Meeting these requirements is made possible by the high resolution of the spectrometer, yielding an uncertainty in the reconstructed kaon direction  $\hat{p}_K$  of 1.5 mrad (full width at half-maximum), corresponding to spatial uncertainties of ±1.5 mm in the exit plane of the collimator, and ±3 mm in the plane of the target.

For each event the vertex position z, the kaon momentum  $\vec{p}_{K}$ , and the invariant "kaon" mass  $M_{2\pi}$  were computed. 20% of the reconstructed " $2\pi$ " events survieved a crude mass cut (448  $\leq M_{2\pi} \leq 548$  MeV) and various stringent geometric cuts (vertex location, traversal of counters and chambers, etc.). The illumination of the collimator exit (slot) and target planes by these events was examined, and further cuts were applied (in a manner clearly suggested by the data !) to meet criteria (a) and (b). Figure 2(a), as an example, shows illumination plots in the narrow (horizontal) direction of the slot for various  $p_{K}$ . The main group (3-4 GeV/c) shows

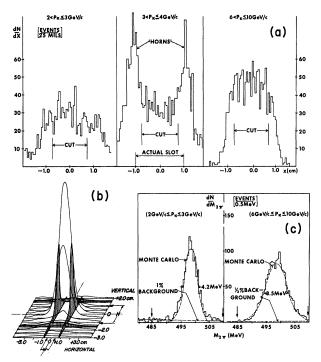


FIG. 2. (a) Histograms of the "slot" illumination by events with  $488 \leq M_{2\pi} \leq 508$  MeV. The illumination in the horizontal direction is shown for three different kaon momentum ranges. (b) Isometric projection of the illumination of the target plane. The shaded areas show the cuts applied to the data. H and W indicate the actual height and width of the target. (c) Histograms of  $M_{2\pi}$  distributions for events surviving all cuts, for two kaon momentum ranges.

clearly two "horns" centered at the positions of the collimator edges. These correspond to coherent regeneration of  $K_s$ 's from  $K_L$ 's hitting the collimator walls at glancing incidence; this hypothesis is verified by the different decay distribution of the events inside and outside the cuts shown in Fig. 2(a), the "horn" events showing an essentially exponential  $K_S$  decay. The events at lower and at higher momenta do not exhibit "horns" clearly, and this for different reasons: At low  $p_K$  the detection efficiency for events decaying near the collimator exit is small, while for high  $p_K$  many  $K_S$ 's survive in the unscattered beam itself. Note that 74% of the events were eliminated by the cuts indicated in Fig. 2(a); those retained were subjected to the target cuts indicated in Fig. 2(b). The target illumination is governed horizontally by the actual target width and vertically by the beam size. The  $M_{2\pi}$  distributions obtained after these cuts, exemplified in Fig. 2(c), were very clean and had widths in excellent agreement with the Monte Carlo predictions. Events in the range 485-510 MeV were after some further minor cuts retained for final analysis. The size of this final sample (9400 events) is in reasonable agreement with that predicted from the total proton flux and estimates of the p-W neutral-kaon production cross sections.

Data reduction. - The events were fitted to Eq. (1') written in terms of  $p_K$  and  $z [t = z(m_K/p_K)]$ , using an efficiency function  $\Omega(p_K, z)$  obtained from a Monte Carlo calculation (which included corrections for spectrometer resolution,  $\pi - \mu$ decays in flight, penetration of the " $\mu$  filter, " simulation of  $2\pi$  events by the decay  $\Lambda \rightarrow p\pi$ , and other small effects), and treating  $S(p_K)$  and  $\overline{S}(p_K)$ as unknown; for this purpose, the parametrizations  $S(p_K) = p_K^{\alpha} \exp\left[-(p_K/p_0)^2\right]$  and  $C(p_K) = \overline{S}(p_K)/$  $S(p_K) = C_0 \exp[-(p_K/p_0')^2]$  were adopted. A maximum-likelihood technique (rather than a  $\chi^2$  minimization) was used in view of the fine  $p_{K}$ -z grid adopted. While the spectral parameters  $\alpha$ ,  $C_0$ ,  $p_0$ , and  $p_0'$  were always left free, two fits for  $\varphi_{+-}$  were made: (a) a free fit, with  $\tau_s$  and  $\eta_{+-}$ treated as unknowns, and (b) a fit using the currently accepted values of these decay parameters. The results of the free fit (a),  $\tau_s = (0.866)$  $\pm 0.023$  × 10<sup>-10</sup> sec and  $\eta_{+-} = (1.94 \pm 0.21) \times 10^{-3}$ , are in excellent agreement with their current accepted values, viz.  $(0.862 \pm 0.006) \times 10^{-10}$  sec and  $(1.92 \pm 0.04) \times 10^{-3}$ .<sup>3</sup> This provides confidence in the  $\Omega(p_{K},z)$  used. Similarly, the spectral parameters ( $\alpha = 0.74$ ,  $C_0 = 0.38$ ,  $p_0 = 2.89$ 

GeV/c, and  $p_0' = 3.08 \text{ GeV}/c$ ) give  $S(p_K)$  and  $C(p_K)$ in substantial agreement with the *p* dependence of the  $K^+$  differential production cross section and the  $K^-/K^+$  ratios of Lundy et al.<sup>14</sup> The phase obtained in this free fit is  $\varphi_{+-} = 0.72$  $\pm 0.09$ . In fit (b) the accepted values of  $\tau_S$  and  $\eta_{+-}$  quoted above were used and these parameters were not varied. The result is  $\varphi_{+-} = 0.69$  $\pm 0.06$ , where the error quoted is statistical and the fit obtained is 38% probable.

The value adopted here for  $\Delta m$  corresponds to the critical survey of Carnegie<sup>15</sup> but includes his own new result. A change in  $\Delta m$  affects  $\psi_{+-}$ according to

$$\delta\varphi_{+-}=295^{\circ}\frac{\Delta m/\hbar-0.538\times10^{10}}{\Delta m/\hbar},$$

i.e., the current 3.5% uncertainty in  $\Delta m$  induces an uncertainty of ±11.5° in  $\psi_{+-}$ .

Figure 3 shows the data points so treated as to isolate the cosine term in (1'), together with a curve corresponding to the  $\varphi_{+-}$  obtained. While this distribution is illustrative of the statistical level of our experiment, it must be emphasized that it is the result of, rather than the basis for, the fit (b) described above.

Systematic errors. - More stringent cuts in the slot and target illumination distributions than those shown in Figs. 2(a) and 2(b) were imposed

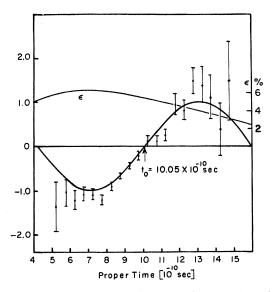


FIG. 3. Data points as a function of proper time t, so treated—using the parameters of the constrained fit (b)—as to isolate the interference term. The smooth curve through the points is  $\cos(\Delta m t/\hbar - \varphi_{+-})$ , with  $\varphi_{+-}$ = 40° and  $\Delta m/\hbar = 0.538 \times 10^{10} \sec^{-1}$ . The curve labeled  $\epsilon$  gives the detection efficiency of our apparatus.

as a test for contamination by coherently regenerated events. The fits remained good and gave  $\varphi_{+-}$ 's within the statistical error quoted above. Similarly, as a test for residual lepton contamination, the final mass cuts [see Fig. 2(c)] were opened up to ±30 MeV. Again,  $\varphi_{+-}$  was unaffected within the quoted error. Regeneration in the  $\gamma$  filter (see Fig. 1)<sup>16</sup> contributes at most an error of ±1°. For the present, the overall systematic error is estimated as 2.8° and our result, allowing for this, is

 $\varphi_{+-} = (40 \pm 5)^{\circ},$ 

using  $\Delta m/\hbar = 0.538 \times 10^{10} \text{ sec}^{-1}$ .

We would like to thank R. J. Powers and D. Stowell for active assistance in several phases of the experiment. To our colleagues at CERN, especially C. Rubbia, P. Darriulat, K. Winter, and M. Vivargent, we are indebted for much helpful exchange of information. We express our gratitude to J. Horton, M. Neumann, T. Nunamaker, and T. Shea for lending us their remarkable technical skills, and D. Cosgrove and R. Rihel who were responsible for the  $K^0$  facility at Argonne National Laboratory. Finally, we would like to thank W. K. H. Panofsky and his staff for the hospitality extended to one of us (DAJ) at Stanford Linear Accelerator Center.

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<sup>2</sup>J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters <u>13</u>, 138 (1964).

<sup>3</sup>N. Barash-Schmidt <u>et al.</u>, Rev. Mod. Phys. <u>41</u>, 109 (1969).

<sup>4</sup>V. L. Fitch, R. F. Roth, J. S. Russ, and W. Vernon, Phys. Rev. Letters <u>15</u>, 73 (1965),  $\varphi_{+-} = (45 \pm 40)^{\circ}$ .

<sup>5</sup>M. Bott-Bodenhausen et al., Phys. Letters 24B, 438 (1967),  $\varphi_{+-} = (81 \pm \frac{47}{4})^{\circ}$ .

<sup>6</sup>M. Bott-Bodenhausen <u>et al.</u>, in <u>Topical Conference</u> on Weak Interactions, CERN, Geneva, Switzerland, <u>14-17 January 1969</u> (CERN Scientific Information Service, Geneva, Switzerland, 1969), p. 329,  $\varphi_{+-} = (53 \pm 12)^{\circ}$ .

<sup>7</sup>R. E. Mischke et al., Phys. Rev. Letters <u>18</u>, 138

<sup>\*</sup>Research supported at the University of Chicago by National Science Foundation Grant No. Gp 9093.

<sup>&</sup>lt;sup>†</sup>Paper submitted by D. A. Jensen to the Department of Physics, The University of Chicago, in partial fulfillment of the requirement for the Ph.D. degree.

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<sup>8</sup>C. Alff-Steinberger <u>et al.</u>, Phys. Letters <u>21</u>, 595 (1966),  $\varphi_{+-} - \varphi_{F} = (80.5 \pm 8)^{\circ}$ . Using this result C. Rubbia and J. Steinberger, Phys. Letters <u>23</u>, 167 (1966), obtain  $\varphi_{+-} = (34 \pm 13)^{\circ}$ , and in a later interpretation [Phys. Letters <u>24B</u>, 531 (1967)],  $\varphi_{+-} = (84 \pm 17)^{\circ}$ .

<sup>3</sup>S. Bennett <u>et al.</u>, Phys. Letters <u>27B</u>, 239 (1968),  $\varphi_r = (-29.6 \pm 4.2)^\circ$ ; revised version in <u>Topical Confer</u>-<u>ence on Weak Interactions, CERN, Geneva, Switzer-</u> <u>land, 14-17 January 1969</u> (CERN Scientific Information Service, Geneva, Switzerland, 1969),  $\varphi_r = (-49.9 \pm 5.4)^\circ$ . Using the results of Alff-Steinberger <u>et al.</u> (Ref. 8) and P. Darriulat <u>et al.</u>, unpublished, the following is obtained for  $\varphi_{+-}$ : J. Steinberger, in <u>Topical Conference</u> <u>on Weak Interactions, CERN, Geneva, Switzerland,</u> <u>14-17 January 1969</u> (CERN Scientific Information Service, Geneva, Switzerland, 1969), p. 291,  $\varphi_{+-} = (37.1 \pm 7)^\circ$ .

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<sup>16</sup>Note that the lead  $\gamma$  filter used here was thinner (1.6 vs 7 cm) than the one in Ref. 11.

## UNIVERSAL $\rho$ COUPLING AND THE ADLER-WEISBERGER THEOREM FOR $\pi^-\rho^+$ AND $\pi^-A_1^+$ IN THE VENEZIANO MODEL\*

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We write minimal Veneziano representations, normalized to the *t*-channel  $\rho$  poles, for the  $\pi^-\rho^+ \rightarrow \pi^-\rho^+$  and  $\pi^-A_1^+ \rightarrow \pi^-A_1^+$  invariant amplitudes relevant to the Adler-Weisberger theorem. Using these amplitudes and the results of other authors for  $\pi^-\pi^+$  and  $\pi^-K^+$ , we find that the Adler-Weisberger theorem is satisfied for  $\pi^-\pi^+$ ,  $\pi^-K^+$ ,  $\pi^-\rho^+$ , and  $\pi^-A_1^+$  elastic scattering if the  $\rho\pi\pi$ ,  $\rho KK$ ,  $\rho\rho\rho$ , and  $\rho AA$  charge couplings all have the same (universal) value.

Resonance-saturated and nearby Regge-pole-dominated "bootstrap" studies<sup>1,2</sup> led Veneziano<sup>3</sup> to propose a simple expression for relativistic scattering amplitudes. Predictably, the Veneziano representation has been successful largely in the region from threshold through the first one or two resonances in each channel. One prominent type of application has been the no-satellite Veneziano expression for spinless meson scattering<sup>4</sup> which, supplemented by the Adler partially conserved axial-vector current (PCAC) consistency condition,<sup>5</sup> predicts scattering lengths and mass and coupling relations in agreement with chiral symmetric and/or experimental results. We are encouraged, despite complications present when more than one external particle has nonzero spin,<sup>6</sup> to investigate simple Veneziano prescriptions for kinematically more complicated processes in the low-energy region. Of special interest are the restrictions imposed by the Adler consistency condition<sup>5</sup> and the Adler-Weisberger (A-W) low-energy theorem.<sup>7</sup>

We discuss here the  $\pi^-\rho^+$  and  $\pi^-A_1^+$  elastic-scattering cases, concentrating on the invariant amplitude<sup>8</sup> D(s,t,u) relevant to the low-energy theorem. The simplest Veneziano expression consistent with conservation laws, properties of well-established particles,<sup>9</sup> and the Adler PCAC condition<sup>5</sup> is written down. It is found that the A-W theorem is satisfied for  $\pi^-\pi^+$ ,  $\pi^-K^+$ ,  $\pi^-\rho^+$ , and  $\pi^-A_1^+$  if the  $\rho\pi\pi$ ,  $\rho KK$ ,  $\rho\rho\rho$ , and  $\rho AA$  charge couplings<sup>10</sup> have the same value, as required by the  $\rho$  universality of gauge-field theory.<sup>11</sup> Under the PCAC and charge-algebra requirements, the universal  $\rho$  coupling is directly re-