erties of the 2_1^- level [as well as in the stripping strength of the reaction[14,15] ^{39}K(^{3}He,d)^{40}Ca]. Gerace and Green[1] quote a value of 67% deformed-state admixture in the 2_1^--state wave function. Using this value, the deformed transition amplitude of the $2_1^- \to 3_1^-$ decay can be calculated in terms of that for the $3_2^- \to 5_1^-$ decay in a manner similar to that used above for the decay of the 5_1^- state. A coherent combination of spherical and deformed amplitudes yields $B(E2; 2_1^- \to 3_1^-) = 34.9e^2$ fm^4.

The strengths of the three transitions $3_2^- \to 5_1^-$, $5_1^- \to 3_1^-$, and $2_1^- \to 3_1^-$ are all underestimated in p-h calculations. Actually, the experimental strengths are consistent with those calculated with the inclusion of spherical and deformed-component admixtures. The results of such a calculation for these and six other transitions between negative-parity levels are summarized in Table II. The calculated transition strengths are in excellent agreement with experiment, which demonstrates the interplay of spherical and deformed components in the $E2$ rates.

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GALACTIC LINE EMISSION FROM 1 TO 10 keV

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We calculate the flux of x rays produced by low-energy cosmic-ray nuclei in H I regions. We consider electron capture to excited states by cosmic-ray nuclei of heavy elements, followed by cascades down to the ground state. It is found that the electron-capture processes may yield appreciable line intensities in the range 1-10 keV in the galactic plane.

Hayakawa[1] originally suggested that low-energy cosmic rays (in the range 1-100 MeV) may be an important heat source in the interstellar medium. Cosmic-ray heating has been extensively studied by Hayakawa, Nishimura, and Takayanagi[2] and more recently by Pikel'ner,[3] Balasubrahmanyan et al.,[4] and Spitzer and Tomasko,[5] who find substantial agreement with the observed properties of the interstellar medium. However, this is at best an indirect argument for the presence of subcosmic rays. Indeed the observed diffuse soft x-ray flux may be of comparable significance as a heating mechanism.[6] In order to distinguish between heating by subcosmic rays and other possible mechanisms, it is clearly of great importance to attempt to observe low-energy cosmic rays. Direct observations at low energies are unreliable because of the substantial degree of solar modulation.[7]

A more promising approach is to investigate the interactions of low-energy cosmic rays with H I regions. Observations yield an upper limit on the heating due to low-energy cosmic rays,[8] in the form of an integral over the cosmic-ray spectrum. Greenberg[9] has discussed the radio emission lines produced by electron cascades following recombination to highly excited states. Proton inner-bremsstrahlung radiation by low-energy cosmic rays interacting with the interstellar medium has been discussed by Hayakawa and Matsuoka,[10] and more recently by Boldt and Serlemitsos.[11] A power-law spectrum is produced, of energy spectral index equal to that of the nonrelativistic proton spectrum at photon energies above about 20 keV. However, the flux falls some two orders of magnitude or more below the observed diffuse background at 1 keV. Note that this result depends only on the heat input assumed for the low-energy cosmic rays.

The fluxes of characteristic x-rays lines from K-series transitions following K-shell ionizations by fast protons in the direction of the galactic center have previously been discussed by Gould and Burbidge[12] and Hayakawa and Matsuoka.[10] We have extended their work to include calculations of the K-series x rays produced by interaction of both low-energy cosmic-ray nuclei and diffuse x rays above 1 keV with heavier ions present in H I regions. The line fluxes were found to amount to less than ~10^{-3} of the observed diffuse flux. A more significant process was found to be electron capture into excited states by cosmic-ray nuclei of heavy elements followed by cascades down to the ground state. In the following we describe our calculation of this process.

In an encounter with a hydrogen atom, a low-energy cosmic-ray nucleus may capture the electron by the charge-exchange process

$$H(1s)+A^{Z+}\rightarrow A^{(Z-1)+}(nl)+H^+.$$

The electron thus captured in an excited state will cascade down to the ground state. If, in this process, the electron finds itself in the $2p$ state, it will emit the analog of the Lyman (Ly) α line and jump down to the ground state. The energy of the line emitted is

$$E_\alpha^{\;Z}=1.02(Z/10)^2\ \text{keV}, \tag{1}$$

where Z is the charge of the cosmic-ray nucleus.

Similarly, if the electron finds itself in the $3p$ state, it will emit the analog of the Ly β line and jump down to the ground state, 88% of the time. The other 12% of the time it will end up in the metastable $2s$ state. The energy of this line is

$$E_\beta^{\;Z}=1.21(Z/10)^2\ \text{keV}. \tag{2}$$

The cross section for Ly α emission may be expressed as

$$\sigma_\alpha^{\;Z}=\sum_{l=0}^{\infty}\sum_{n=l+1}^{\infty}f_{nl}^{\;\alpha}\sigma_{1s-nl}^{\;\;Z}. \tag{3}$$

$f_{nl}^{\;\alpha}$ is the probability that an electron, initially in the state nl, will ultimately make the $2p$-$1s$ transition. $\sigma_{1s-nl}^{\;\;Z}$ is the cross section for a nucleus of charge Z to capture an electron from the ground state of hydrogen into the nl state. The corresponding cross section for Ly β emission is

$$\sigma_\beta^{\;Z}=\sum_{l=0}\sum_{n=l+1}f_{nl}^{\;\beta}\sigma_{1s-nl}^{\;\;Z}. \tag{4}$$

$f_{nl}^{\;\beta}$ is the probability an electron captured into the state nl will ultimately make the $3p$-$1s$ transition.

The probabilities $f_{nl}^{\;\alpha}$, $f_{nl}^{\;\beta}$ have been evaluated by Bethe and Salpeter.[13] Bates and Dalgarno[14] have evaluated the capture cross sections for all states through $n=4$, $l=3$. Hiskes[15] has given expressions for σ_{1s-nl} for all states through $n=15$, $l=14$.

The flux of line emission from electron capture is

$$\begin{aligned} J_\alpha^{\;Z}&=\langle N_\mathrm{H}R\rangle(N_Z/N_\mathrm{H})_\mathrm{CR}\int j(E)\sigma_\alpha^{\;Z}(E)dE,\\ J_\beta^{\;Z}&=\langle N_\mathrm{H}R\rangle(N_Z/N_\mathrm{H})_\mathrm{CR}\int j(E)\sigma_\beta^{\;Z}(E)dE. \end{aligned} \tag{5}$$

Here $\langle N_\mathrm{H}R\rangle$ is the effective number of H atoms along a line of sight. From the 21-cm measurements,[16] $\langle N_\mathrm{H}R\rangle\approx 6\times10^{21}\ \mathrm{cm}^{-2}$, corresponding to a line of sight in the galactic plane. This, in fact, is rather conservative. Within ±2.5° of the galactic equator, $\langle N_\mathrm{H}R\rangle$ varies from a minimum of ~$6\times10^{21}\ \mathrm{cm}^{-2}$ in the anticenter direction to a maximum of ~$2\times10^{22}\ \mathrm{cm}^{-2}$ towards the galactic center. For the low-energy cosmic-ray flux, we adopt the 2-MeV flux employed by Field, Goldsmith, and Habing,[8] i.e., $4\pi j(E)=24\delta(E-2\ \mathrm{MeV})$ $\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}\ \mathrm{MeV}^{-1}$. $(N_Z/N_\mathrm{H})_\mathrm{CR}$ is the abundance of nuclei with charge Z relative to protons in the 2-MeV cosmic-ray flux, and is extrapolated from the data of Comstock, Fan, and Simpson.[17]

For $E=2$ MeV and $10\le Z\le 26$, the cross sections $\sigma_\alpha^{\;Z}$ and $\sigma_\beta^{\;Z}$ were estimated from (3) and (4) by including all contributions through the $4f$ state. The resulting cross sections are therefore lower limits to the exact cross sections. They are most accurate for low Z. For $Z\ge 20$ they may be too small by an order of magnitude.

The results appear in Table I. The lines below 1.4 keV have been corrected for interstellar ab-

Table I. $2p$-$1s$ and $3p$-$1s$ transitions following electron capture by cosmic-ray nuclei. E_α=energy of $2p$-$1s$ transition, E_β=energy of $3p$-$1s$ transition, J_α=flux of x rays with energy E_α, J_β=flux of x rays with energy E_β.

Element	E_α (keV)	J_α (cm^{-2} sec^{-1} sr^{-1})	E_β (keV)	J_β (cm^{-2} sec^{-1} sr^{-1})
Ne	1.02	2.0×10^{-1}	1.21	5.7×10^{-2}
Na	1.23	3.1×10^{-2}	1.46	6.7×10^{-3}
Mg	1.47	4.9×10^{-1}	1.74	8.9×10^{-2}
A	1.72	4.3×10^{-2}	2.04	7.8×10^{-3}
Si	2.00	3.7×10^{-1}	2.37	6.7×10^{-2}
P-K	2.29-3.68	8.6×10^{-2}	2.72-4.37	1.5×10^{-2}
Ca-Cr	4.08-5.88	9.3×10^{-2}	4.84-6.97	1.7×10^{-2}
Mn-Ni	6.38-8.00	1.5×10^{-1}	7.56-9.49	2.8×10^{-2}

sorption using the results of Bell and Kingston[18] to provide an effective value of $\langle N_H R\rangle$ corresponding to optical depth unity.

It appears from Table I that it would be feasible to look for the $2p$-$1s$ transitions following electron capture by low-energy cosmic-ray nuclei. In our calculation of the line fluxes for the $2p$-$1s$ and $3p$-$1s$ transitions we have used cross sections which in fact are lower limits to the exact cross sections. Measurements of line intensities (or null results) would therefore yield upper limits to the flux of cosmic-ray nuclei with $10 \leq Z \leq 28$ at 2 MeV/nucleon. It should be noted that these lines will be Doppler broadened. Since $\Delta E/E \approx 0.13$, the lines with $E \gtrsim 2$ keV will begin to overlap, making them more difficult to resolve. For $E \lesssim 2$ keV the strongest lines are the Ne α, Mg α, and Si α lines with lesser contributions from the Ne β, Na α, Ar α, Mg β, and Si β lines. The three main lines do not overlap. The ratio of the flux (in photons cm^{-2} sec^{-1} sr^{-1}) in the lines to the flux in the background in an energy interval $\Delta E = 0.13E$ centered about each line center E is 0.92 for Ne α, 2.13 for Mg α, and 1.57 for Si α. With the exception of possible lines in the iron group (Mn-Ni) these should be the most easily observable lines. For $E \gtrsim 2$ keV, the ratio of the flux in lines to the flux in the diffuse background is ~0.07. If the high value of $\langle N_H R\rangle = 2\times10^{22}$ cm^{-2}, corresponding to a line of sight with $|b^{II}| \lesssim 2.5°$, $|l^{II}| \lesssim 60°$, is used, the flux in lines above ~2 keV is higher by about a factor of 3 than the values that appear in Table I. Note that for energies below ~2 keV, $\langle N_H R\rangle$ corresponding to unit optical depth should be employed, so that the flux in lines below 2 keV increases less rapidly than the flux in lines above 2 keV.

It is of interest to know how these results are modified if a more realistic spectrum (say, an extrapolation of the demodulated spectrum of high-energy cosmic rays) is used. A detailed investigation of this is in progress but some general remarks may be made. The spectrum of cosmic rays of energies (per nucleon) less than 2 MeV is extremely uncertain. Even in the range 2-30 MeV, different theories of solar modulation require demodulation factors which differ by as much as ~10^3. To obtain a more realistic low-energy spectrum, we have used the demodulation proposed by Gloeckler and Jokipii,[7] who suggest that the interstellar spectrum remains steep down to at least 15 MeV/nucleon. At sufficiently low energies, ionization losses tend to produce a positive slope. Since direct observations do not provide an unambiguous determination of the turnover energy, we have imposed the restriction that the interstellar hydrogen ionization due to low-energy cosmic rays should not exceed ~10^{-15} ionization per H atom per second. This limit is based on direct observations of atomic hydrogen.[8] We find that the turnover energy must be ~10 MeV, which determines the amount of matter traversed by the cosmic-ray protons. If the heavier cosmic rays have also traversed a similar amount of matter, their spectra will turn over at a higher energy. This enables us to estimate the contribution to the x-ray line fluxes by cosmic rays above 2 MeV/nucleon. The fluxes calculated in this manner are less than the previously calculated fluxes, because the cross sections σ_α and $\sigma_\beta{}^Z$ fall off so rapidly with increasing energy ($\sigma \sim E^{-6}$). The steep energy dependence of the cross sections suggests that contributions from the continuation of the spectrum below 2 MeV/nucleon may dominate. However, below ~2 MeV the heavier cosmic-ray nuclei ($Z \gtrsim 10$) begin accreting electrons more rapidly than they lose them by collisional ionization. Thus, the effective cross section for producing a given line does not continue to rise as one proceeds to lower energies. Most of the contribution to a given line comes from a fairly narrow range of energies around ~2 MeV.

Recently, evidence for a galactic component of the diffuse x-ray background in the 1.4- to 18-keV band has been reported.[19] Although the detector is inadequate to resolve lines from this flux, the data below ~2 keV can perhaps be explained by unresolved Mg α and Si α lines. However, the reported flux in the 2- to 18-keV band exceeds our predicted flux by a factor of ~5. Due to the lower abundances of the heavier cosmic-ray nuclei the flux from lines above ~2 keV is quite small.

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π^-p ELASTIC SCATTERING AT 2.51, 2.76, AND 3.01 GeV/c NEAR $t \simeq -3$ (GeV/c)^2*

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Differential cross sections for the elastic scattering of negative pions from hydrogen have been measured over a limited range of squared four-momentum transfer (t) in the vicinity of $t \simeq -3$ (GeV/c)^2 for incident pion momenta of 2.51, 2.76, and 3.01 GeV/c. These measurements confirm the existence of a minimum in the differential cross section in this region of incident momentum and scattering angle. The minimum occurs at a smaller value of t [$t \simeq -2.6$ (GeV/c)^2] than has been observed at higher momenta.

Previous measurements of π^-p elastic scattering at 5.90 GeV/c clearly show a minimum in the differential cross section at a squared value of four-momentum transfer (t) of $t \simeq -3$ (GeV/c)^2.[1] These measurements stimulated the observation[2] that this same structure exists at a similar value of t in lower momenta measurements.[3,4] Many two-body particle reactions at high incident momenta are characterized by differential cross sections which have structure as a function of the kinematic variables.[5] Various models have been proposed to interpret these structures[6]; for some reactions, considerable progress has been made.[7] In attempting to understand π^-p elastic