erate-density gas,[11] and we foresee applications of this experiment to studies of the radial distribution function in this region ($0.9R_0$ to $1.3R_0$) for fluid xenon at all densities.

In the present measurements[12,13] the local magnetic field was measured by NMR free-precession techniques at a field of 12.2 kG in which the resonant frequency of Xe^{129} was measured in constant applied field H_0. Improved field stability, more stable frequency sources, and the use of signal averaging techniques resulted in the greater precision of these measurements.

The samples were similar to those used in previous investigations,[2,7] consisting of sealed Pyrex containers of (constant) volume about 0.2 cm^3. Densities were determined by weighing and are known to 1%. Higher temperature (pressure) measurements were obtained in this work by keeping the Pyrex containers under pressure in a BeCu pressure vessel to compensate for internal gas pressures. Some of the measurements at lower densities were performed in a 7-kG permanent magnet. The excellent stability of this magnet (drift <1 part in 3×10^8 per min) kept the resonant frequency constant, allowing the signal averager to be used to better advantage. Because of the smaller magnet gap, our pressure vessel could not be used in this magnet; this limited the study to the lower densities at temperatures up to 90°C. The measurements on the 29-amagat sample which were made in this magnet at 7 kG have been scaled to 12.2 kG for presentation on Fig. 2.

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ANOMALOUS RESISTIVITY IN COLLISIONLESS PLASMA SHOCK WAVES*

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The anomalously large resistivity observed in collisionless shock-wave experiments is explained in terms of a drift instability of ion acoustic waves.

A number of experiments[1,2] have detected shock waves traveling perpendicular to an ambient magnetic field $\vec{B}=B_0\hat{n}_z$ and established that the structure of these waves implied a resistivity in the shock front much larger than the resistivity due to electron-ion collisions. This anomalous resistivity was made evident by the width of the shock front, L_s, which far exceeded the classical width.[3]

A model for explaining this enhanced width in terms of ion waves driven unstable by currents in the shock front has been proposed by Sagdeev. Early calculations[4] based on this model produced results which were too large by two orders of magnitude to agree with the experimental resistivity. However, when the results were multiplied by an arbitary factor $A\approx 0.01$ they did give a shock thickness which scaled with density in approximately the same way as did the experiments.

We believe that the Sagdeev idea is the correct one, and that it fails to agree with experiments mainly because the calculations have ignored the ambient magnetic fields and the field gradients. We here include the magnetic field self-consistently and show that it provides an averaging effect which reduces the Sagdeev resistivity by a large factor without radically changing the scaling, and produces a prediction of shock width and anomalous resistivity in agreement with experiments over a very large range of parameters.

To calculate turbulent resistivity and shock thickness, we apply a recently developed theory[5] of the linear and quasilinear behavior of ion-wave instabilities in a magnetically confined plasma to the usual two-fluid model[3] of perpendicular (to $\vec{B}$) low-amplitude collisionless shocks.

In this model, a weak perpendicular magnetic shock wave consists of a plasma in a rising magnetic field $B_z = B_0(x)\hat{\imath}_z$, with a gradient $\epsilon = (1/B) \times (dB/dx)$; the front propagates in the x direction with a speed $u_s \gtrsim B_0(4\pi nM)^{-1/2}$. There is an electric field $E_x \sim -\epsilon B_0^2/4\pi en$ arising from charge separation in the front, and a density gradient $(1/n)(dn/dx) \sim (1/B_0)(dB/dx)$ produced by field compression, where n is the plasma density at the midpoint of the shock, e is the electron charge, and c is the velocity of light. There is also an electric field E_y since $c dE_y/dx = -dB_z/dt$, and a diamagnetic current $j_y = -\epsilon B_0 c/4\pi$ in cgs Gaussian units. This current is produced in part by $\vec{E}\times\vec{B}$ drifts and in part by magnetic gradient drifts.

The time scale of these shocks is such that the electrons gyrate many times as the shock front passes, but the ions are nearly undeflected by the B field. A detailed derivation of the equilibrium distributions has recently been presented[6] elsewhere.

Extending the calculation of Ref. 5 by allowing the instabilities to be two-dimensional, $\delta\vec{E} = \vec{E}_1 \times e^{ikx}e^{iky}e^{i\omega t}$, and including the zero-order electric fields prescribed by the two-fluid equations,[3] we find that this shock front is unstable to low-frequency ($\omega < eB/mc$), short-wavelength (λ smaller than the electron gyroradius), electrostatic, ion-acoustic waves:

$$\omega = -\frac{k(T_e/M)^{1/2}}{(1+k^2\lambda_D^2)^{1/2}} + i\gamma. \qquad (1)$$

The growth rate γ is

$$\gamma = -\frac{(k^2+K^2c_s)^{1/2}}{[1+(k^2+K^2)\lambda_D^2]^{3/2}}\frac{\pi}{2}J_0^2\left(\frac{k\sqrt{Y}}{\Omega}\left[\frac{2T_e}{m}\right]^{1/2}\right) \times (Y-1)e^{-Y}, \qquad (2)$$

where T_e is the electron temperature, M the ion mass, $c_s = (T_e/M)^{1/2}$, $\lambda_D = (T_e/4\pi ne^2)^{1/2}$ the Debye length, and $\Omega = eB_0/mc$. The parameter Y is defined in terms of the parameters of the shock-front fields:

$$Y = \left|\frac{eE_x}{\epsilon T_e}\right| - \frac{m\,\Omega k c_s}{\epsilon T_e K(1+k^2\lambda_D^2)^{1/2}}.$$

The growth rate γ depends on e^{-Y}, the fraction of electrons in the resonance with the wave ω.

The Bessel function J_0^2 contains finite gyroradius effects. When the instability has a wavelength smaller than the electron gyroradius the orbiting of the electrons averages out the δE fields and the growth rate γ is reduced. This reduces γ below the $B=0$ value, over most of the k spectrum. The factor $Y-1$ shows the competition between Landau damping (the 1 term) and the density gradients (the Y term) which tend to destabilize the system. We see that $Y>1$ for instability and $\gamma_{\max} \sim eB/c(mM)^{1/2}$. Wave numbers for which $\gamma \gtrsim \frac{1}{2}\gamma_{\max}$ lie in the range $\Omega(m/2T_e)^{1/2} < k < 1/\lambda_D$ and $K \sim k4\pi n_0 m\,\Omega c_s/\epsilon B^2 < k$. The growth of waves at the expense of currents in the shock front implies a change $\partial f_0/\partial t$ in the plasma distribution f_0.

Since the growth rate (γ) is less than the oscillation frequency of the waves (ω_0), a quasilinear theory[7] can be used to calculate the slow change (time scale $1/\gamma$) in the average electron distribution $[f(t) = f_0(t) + \delta f e^{i\omega t}e^{i\vec{k}\cdot\vec{r}},\ f_0(t) = \langle f\rangle_{\rm av}]$. This theory produces $\partial f_0(t)/\partial t$ in terms of the energy in the waves E_ω, the growth properties γ, and the frequency and wave-number range of the unstable waves. The structure of these perpendicular shock waves is influenced by the moment $\int v_y(\partial f_0/\partial t)d\vec{v}$, and the result can be written in terms of an effective resisitivity

$$\nu_{\rm eff} \equiv \frac{\int v_y(\partial f_0/\partial t)d\vec{v}}{\int v_y f_0 dv} = \frac{eB}{c(mM)^{1/2}}\left(\frac{\omega_e^2\beta_e}{\epsilon^2c^2}\right) \times \frac{E_\omega}{nT_e}\ln\frac{4\pi nmc^2}{B^2}, \qquad (3)$$

where $\beta_e \equiv 4\pi nT_e/B^2$, E_ω is the total electrostatic energy in the unstable waves, and ω_e is the plasma frequency $\omega_e = (4\pi n_0e^2/m)^{1/2}$.

To use Eq. (3) we estimate (justified by the discussion below)

$$E_\omega/nT_e \sim \tfrac{1}{3}, \tag{4}$$

$$\omega_e^2\beta_e/c^2\epsilon^2 \sim 2, \tag{5}$$

and replace this combination of parameters by a constant A which is of order unity. Then

$$\nu_{\rm eff} = \frac{eB}{c(mM)^{1/2}} A \ln\frac{4\pi nmc^2}{B^2},$$

$$A \equiv \frac{E_\omega}{nT_e}\frac{\omega_e^2\beta_e}{c^2\epsilon^2} \sim 1. \tag{6}$$

This estimate is well below the early estimate of Sagdeev,[4] because of the averaging effect of B_0 evident from the Bessel functions J_0^2 in the stability analysis above.

The estimate of E_ω in Eq. (4) comes from the fact that only electrons with velocity $v \geq (2T_e/m)^{1/2}$ are resonant with the unstable waves, which limits the energy available for the instability. The collection of parameters in Eq. (5) is of order unity (experimentally). Theoretically, Eq. (5) represents the condition that the veloctiy of electron currents j_y/ne not exceed the electron thermal speed $(2T_e/m)^{1/2}$. It has long been surmised[2-4] that very early in the shock front a strong instability will preheat the electrons until the electron thermal speed exceeds the velocity of the electron current [e.g., until Eq. (5) is satisfied]. This involves both electron heating (β_e increasing) and shock broadening [$\epsilon = (1/B)(dB/dx)$ being reduced].

The shock thickness[3] is related to dispersion properties of magnetosonic waves (scale $\lambda \sim c/\omega_e$) and to resistivity. When $\nu_{\rm eff} \ll eB_0/c(mM)^{1/2}$ the shock thickness is $L_s \sim c/\omega_e$. When $\nu_{\rm eff} > eB_0/c(mM)^{1/2}$, L_s becomes proportional to $\nu_{\rm eff}$. Using Eq. (6) we predict for the value of L_s and its scaling with density and temperature

$$L_s = (c/\omega_e)[1 + A\ln(4\pi nmc^2/B^2)]. \tag{7}$$

The criterion $4\pi nmc^2/B^2 \gg 1$ for anomalous resistivity is in agreement with experiment over a range of several orders of magnitude in this parameter. Table I shows a comparison, with the constant A estimated by $A = \frac{3}{4}$.

Table I. Shock width, experimental versus theoretical.

Reference	$4\pi nmc^2/B^2$	$L_s^{\rm experiment}$ (mm)	$L_s^{\rm theoretical}$ (mm)
1	30	0.8	1.0
2	1.5×10^3	3-4	4.5
2	4×10^3	1.4	1.1

There are, of course, possible improvements and additions to the above theory. For example, if the ion dynamics are included in the quasilinear theory the energy spectrum would be somewhat modified and Eq. (7) would contain a weak dependence on ion mass M.

In conclusion, we have performed a calculation which makes quantitative the model proposed by Sagdeev to explain anomalous resistivity in weak perpendicular collisionless shock waves. By properly including magnetic effects we find that they reduce the production and coupling of electrons to ion waves, and produce for the first time a calculated resistivity which is in quantitative agreement with experimental results.

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