FEEDBACK CONTROL OF COLLISIONAL DRIFT WAVES BY MODULATED PARALLEL-ELECTRON-CURRENT SINK—EXPERIMENT AND INTERPRETATION*

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By modulating a parallel-electron-current sink, the collisional drift wave is stabilized with negative feedback in the unstable regime and destabilized with positive feedback in the stable regime. Upon stabilization, improved plasma confinement is measured.

We report the results of work on feedback control of the collisional drift instability[1,2] by modulation of the electron-current flow along magnetic field lines to an electrostatic probe immersed in the plasma. For optimum stabilization the modulated electron sink is approximately out of phase with the wave and for destabilization, in phase with the wave, in agreement with a linear-theory interpretation: Stabilization is achieved when feedback results in a reduced rate of expansion of electron flow along the field, and destabilization when feedback results in an increased rate. Upon stabilization, the instability amplitude is reduced by a factor of 100 and, concomitantly, improved plasma confinement is observed.

Feedback is essential to the control of discrete systems. Its application to the suppression of instabilities in continuous media[3] such as fluids or plasmas appears to be impractical, in general. However, in confined plasmas, where boundary conditions impose discrete wavelengths so that only a few unstable modes are present with sufficiently long spatial (wavelength) and temporal (oscillation period and growth time) scales, feedback control becomes practical with only a few suppressor elements. Such feedback suppression of plasma instabilities has recently been reported for the $m=1$ flute[4] and ion-cyclotron[5] modes in the OGRA-II mirror device, using sensor and suppressor electrodes at the radial plasma boundary to stabilize these waves, which have finite amplitude at the radial plasma surface.[6] However, for drift instabilities driven by a density gradient located inside the plasma, radial boundary conditions play only a supplementary role and do not affect the basic features of the instability.[7] Therefore, feedback control of radial boundary or surface conditions is expected to be ineffective, and stabilization of these instabilities may have to rely on injecting the feedback signals into the plasma interior; e.g., by modulated electron or ion sources.

Such feedback stabilization is performed on collisional drift waves in the present experiment. In previous work[2] this wave has been identified in detail by comparison with the linear theory of the parametric dependences of ω and $\vec{k}$. As the magnetic field is increased, successive azimuthal modes destabilize when the electron-fluid expansion rate along field lines exceeds the diffusion rate of ions across the field due to ion-fluid viscosity. Below the critical onset magnetic field, critical (i.e., enhanced thermal) fluctuations at the drift-wave frequency appear. Concomitant with instability onset, wave-induced plasma loss occurs and the plasma density decreases. The present experiment is performed in the oscillatory regime near instability onset where the growth rate, and hence the instability amplitude, can be varied so that feedback effects may be studied in detail. The instability amplitude and frequency for different growth rates are measured, as functions of both the feedback phase shift and gain. Simultaneously with the reduction of instability amplitude, improved plasma confinement is determined from the measured increase in plasma density.

The simple feedback system employed to modulate local electron flow uses one sensor and one suppressor probe. The sensor signal, which is the instability amplitude in the unstable regime or the critical-fluctuation amplitude in the stable regime, is amplified, phase-shifted, and capacitatively coupled to the suppressor probe. Small variations of the floating potential primarily vary electron current, hence the suppressor acts as a local electron sink. With appropriate dc bias, the suppressor probe may be used as an ion as well as an electron sink.

The stability theory of localized collisional drift modes with modulated electron and ion flows can be carried out by including sink terms, $-S$, in the linearized continuity equations in a "slab" model[2]:

$$\partial n_{e,i}/\partial t+\nabla_\perp\cdot(n_{e,i}\vec{u}_{0e,i})+\nabla_\perp\cdot(n_0\vec{u}_{e,i})+\nabla_\parallel n_0 u_{e\parallel}=-S_{e,i}. \quad (1)$$

The calculation is made particularly simple by representing the sink terms as $S_{e,i}=\sigma_{e,i}n_{e,i}$, $\sigma=|\sigma|\times\exp(i\theta)$, and assuming $\sigma_{e,i}$ independent of spatial coordinates.[8] Using standard notation and following the procedure of Ref. 2, we obtain the dispersion relation:

$$b\omega^2+\left[b(\omega_e+i\sigma_e)+i\frac{1+2b}{t_\parallel}+\frac{i}{t_\perp}\right]\omega-\frac{1}{t_\parallel t_\perp}[2+t_\parallel\sigma_e+t_\perp\sigma_i]+i\omega_e\left[\frac{1}{t_\perp}-\frac{1}{t_\parallel}+\sigma_i-\sigma_e\right]=0, \tag{2}$$

where $\omega_e=k_yv_d$, $v_d=-(cKT/eB)(1/n_0)(dn_0/dx)$, $b=\frac{1}{2}k_\perp{}^2r_L{}^2$, $r_L=(2KT/M)^{1/2}/\Omega_i$, $1/t_\parallel=k_\parallel{}^2KT/m_e\nu_{ei}$, and $1/t_\perp=\frac{1}{4}b^2\nu_{ii}$.

For an electron sink, the stability criteria in the limiting cases are

(a) $1/t_\perp>1/t_\parallel+|\sigma_e|\cos\theta$ if $\omega_e\gg1/t_\parallel$ (long-wavelength limit), (3)

(b) $2/t_\parallel t_\perp>\omega_e(2b\omega_e+|\sigma_e|\sin\theta)$ if $\omega_e\ll1/t_\parallel$ (short-wavelength limit). (4)

These criteria have been found independently by Furth and Rutherford.[9] The additional effect of momentum sources on resistive drift waves has been discussed by Chen and Furth.[10] We note that the criteria (3) and (4) for $\sigma_e=0$ reduce to the conventional stability criteria of the collisional drift wave.[2]

Several important points are revealed by the stability criteria (3) and (4), and the dispersion relation, Eq. (2). First, by adjusting feedback gain $|\sigma_e|$ and phase θ the growth rate γ can be controlled, and thus the wave can be stabilized or destabilized. The optimum feedback phase for stabilization is 180° and 270° in the long- and short-wavelength limits, respectively. Second, the instability frequency varies when feedback is applied.[11]

The effect of feedback control by modulated electron sinks can be further clarified through the linearized complex amplitude equation,[2]

$$b\frac{\partial}{\partial t}\left(\frac{n}{n_0}+\frac{e\varphi}{KT}\right)=\frac{1}{t_\parallel}\left(\frac{n}{n_0}-\frac{e\varphi}{KT}\right)-\frac{1}{t_\perp}\left(\frac{n}{n_0}-\frac{e\varphi}{KT}\right)+\frac{S_e}{n_0}. \tag{5}$$

Here the destabilizing term $(1/t_\parallel)(n/n_0)$ is the rate of expansion of electron fluid along the field due to the perturbing pressure gradient. In the short-mean-free-path[2] limit ($\omega_e\gg1/t_\parallel$), the drift frequency is small ($\partial/\partial t\sim\gamma$) and $|n/n_0|$ becomes much larger than $|e\varphi/KT|$. Clearly, for stabilization the electron sink must be out of phase with the wave in order to reduce the rate of expansion. In the limit of long mean free path ($1/t_\parallel\gg\omega_e$), $|n/n_0|$ and $|e\varphi/KT|$ become comparable and the drift frequency approaches ω_e, i.e., $\partial/\partial t\sim i\omega_e$. Therefore, for stabilization $S\approx in|\sigma|\sin\theta$, i.e., the electron sink is 270° out of phase with the wave.

The experiment was performed on the Princeton Q-1 device.[2] The magnetically confined, fully ionized plasma is produced by surface ionization of potassium atom beams incident on 3.2-cm-diam incandescent, electron-emitting tungsten plates located at both ends of the 128-cm-long plasma column. Ion and electron temperatures are assumed equal to the ionizer-plate temperature. The sensor used in the feedback experiment was a 0.25-mm-diam by 0.61-mm-long Langmuir probe; the suppressor was a similar Langmuir probe, a 0.1-cm^2 disk (oriented perpendicular to the field), or, for the present data, a 0.25-mm-diam by 3-cm-long wire oriented along the magnetic field. The sensor signal was passed through a broad-band audio amplifier and a continuously variable phase shifter. Detector and suppressor probes were generally located in the region of maximum wave amplitude; i.e., axially in the midplane between ionizer plates, and radially at $r\approx7.5$ mm $\approx\frac{1}{2}r_{\text{plasma}}$. However, feedback suppression could be achieved for any radial, azimuthal, or axial locations of sensor and suppressor probes within the plasma provided the phase shift was adjusted for the relative probe positions and for the instability mode number.

Simultaneous measurements of the instability amplitude, equilibrium center-density change, and frequency are shown in Fig. 1 for constant feedback gain as a function of the phase delay θ of the applied suppressor voltage for different magnetic fields, i.e., different growth rates. Here θ is referenced to the spontaneous density oscillation at the location of the suppressor. Stabilization occurs with negative feedback, $\theta\approx180°$, and destabilization with positive feedback, $\theta\sim0°$. With stabilization, the plasma density increases, while positive feedback enhances plasma losses. Figure 1 also shows that the frequency decreases toward stabilization, and that stabilization is achieved over a range of feedback phase shift centered near 180°. This range widens with in-

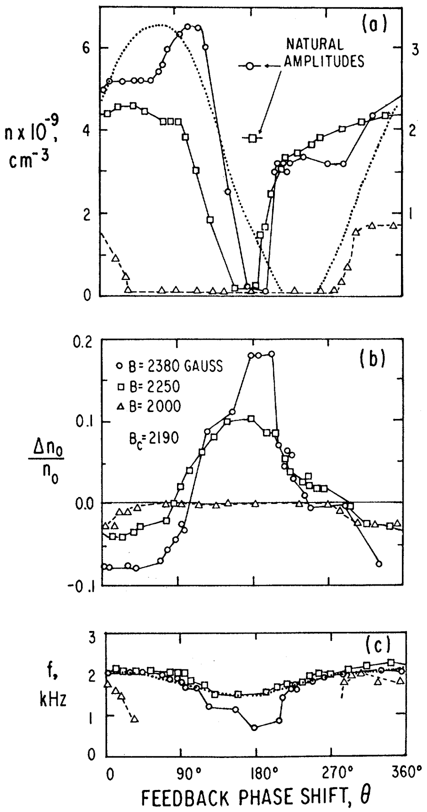

FIG. 1. (a) Instability amplitude, (b) change of density at center of plasma column, and (c) instability frequency after subtraction of Doppler shift due to radial electric field, as functions of feedback phase shift in both stable and unstable regimes ($B_c = 2190$ G). The suppressor probe was displaced azimuthally 180° from the detector. The instability is an $m = 2$ azimuthal mode. Potassium plasma, $T = 2700$°K, $n_0 = 7\times10^{10}$ cm^{-3}, $\nabla n_0/n_0 = -1.25$ cm^{-1}, and $\lambda_{\|} = 256$ cm. Dotted curves in (a) and (c) are, respectively, theoretical growth rate and frequency according to Eq. (2) for $\sigma_i = 0$, $\sigma_e = 2.8$ msec^{-1}, and $B = 1.1B_c$. As in Ref. 2, the critical magnetic field has been scaled by 1.5.

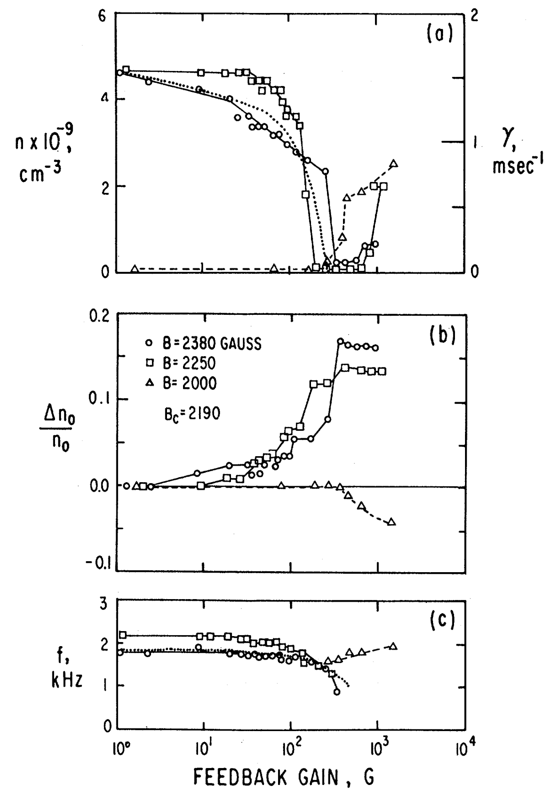

FIG. 2. (a) Instability amplitude, (b) change of density at center of plasma column, and (c) instability frequency as functions of feedback gain G. Conditions as in Fig. 1. Dotted curves in (a) and (c) are, respectively, theoretical growth rate and frequency according to Eq. (2) for $\theta = 180°$. The theoretical gain is relative, referenced to the experimental stabilization point.

creasing feedback gain and with decreasing instability growth rate. Also shown in Fig. 1 are the growth rate γ and frequency f calculated from Eq. (2) using the experimental plasma parameters. The observed optimum phase shift, frequency reduction, and feedback-phase-shift range, as affected by growth rates, are in qualitative agreement with Eq. (2). We note that no other instabilities were driven unstable. Similar measurements and agreement as functions of feedback gain at optimum feedback phase shift are shown in Fig. 2.

Figure 3(a) shows the measured radial distribution of equilibrium density n_0, the instability amplitude n, and the required minimum suppressor voltage V. Suppression is achieved with lowest feedback voltage at the radial location of largest wave amplitude. Beyond the plasma boundary, $r = 16$ mm, suppression could not be achieved with as much as 300 V, differing from the results of Ref. 6. Figure 3(b) shows the plasma density in the center of the column as a function of magnetic field strength for constant input flux. Starting with low magnetic field, confinement increases with increasing B. When the drift wave destabilizes, at $B = B_c = 2190$ G, the density decreases abruptly. However, with feedback the density increases continuously beyond B_c indicating that all the instability-induced losses are recovered.

With the present feedback circuit arrangement, stabilization of the $m = 2$ azimuthal mode can be achieved up to $B \approx 1.2B_c$. Farther in the unstable regime, and in the higher-mode regime, large feedback amplification results in the destabiliza-

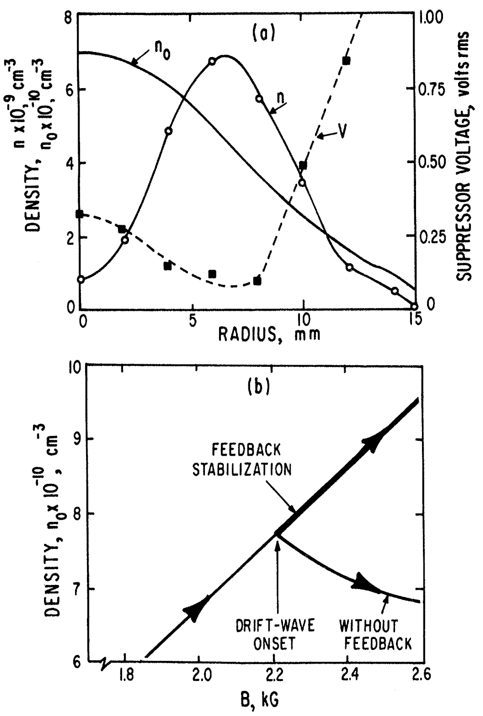

FIG. 3. (a) Radial distribution of plasma density n_0, instability amplitude n, and minimum required suppressor voltage for stabilization, V; (b) plasma density at center of column as a function of $\boldsymbol{B}$ with and without feedback stabilization of drift wave.

tion of oscillations at different frequencies and modes. For example, upon feedback suppression of the $m = 3$ mode, the $m = 2$ mode destabilizes. Such destabilization is consistent with linear-theory results.[2]

The present work demonstrates that drift instabilities, driven by a density gradient located inside the plasma, can be feedback controlled with injection of feedback signals into the plasma interior. Upon stabilization, improved plasma confinement is observed.

Additional experimental and theoretical data will be reported elsewhere.

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[1]A. B. Mikhailovskii and O. P. Pogutse, Dokl. Akad. Nauk SSSR 156, 64 (1964) [translation: Soviet Phys. —Doklady 9, 379 (1964)].

[2]H. W. Hendel, B. Coppi, F. W. Perkins, and P. A. Politzer, Phys. Rev. Letters 18, 439 (1967); T. K. Chu, H. W. Hendel, and P. A. Politzer, Phys. Rev. Letters 19, 1110 (1967); H. W. Hendel, T. K. Chu, and P. A. Politzer, Phys. Fluids 11, 2426 (1968).

[3]J. R. Melcher, Phys. Fluids 9, 1973 (1966).

[4]V. V. Arsenin, V. A. Zhilkov, and V. A. Chuyanov, Plasma Physics and Controlled Nuclear Fusion Research (International Atomic Energy Agency, Vienna, Austria, 1969), Vol. II, p. 515.

[5]V. V. Arsenin, V. Kh. Likhtenshtein, and V. A. Chuyanov, Zh. Eksperim. i Teor. Fiz.—Pis'ma Redakt. 8, 69 (1968) [translation: JETP Letters 8, 41 (1968)].

[6]During preparation of this manuscript two experiments on feedback stabilization of unidentified "drift-type" electrostatic modes have been reported: R. R. Parker and K. I. Thomassen, Phys. Rev. Letters 22, 1171 (1969); B. E. Keen and R. V. Aldridge, Phys. Rev. Letters 22, 1358 (1969). The first work was performed on a reflex discharge and the second on a hollow-cathode arc discharge. In both cases, sensor and suppressor were located at the plasma boundary as in Refs. 4 and 5.

[7]T. K. Chu, B. Coppi, H. W. Hendel, and F. W. Perkins, Phys. Fluids 12, 203 (1969).

[8]Constant $\sigma_{e,i}$ represents uniformly distributed sinks whose strengths are proportional to local instability amplitude. For feedback consisting of local sinks only, by constructing a quadratic form consisting of positive definite integrals it can be shown that the present results retain the basic features.

[9]H. P. Furth and P. H. Rutherford, to be published.

[10]F. F. Chen and H. P. Furth, to be published.

[11]J. B. Taylor has employed a perturbation technique to calculate effects of feedback on frequency and growth rates for plasma instabilities in general (unpublished).