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[9]See also Feldman and Matthews, Ref. 2, part 2, for a general review and further references.

[10]If a, b, and c are any four-vectors (c timelike), we denote by $\theta(a,b;c)$ the angle between $\vec{a}$ and $\vec{b}$ in the frame in which $\vec{c}=0$. (See, e.g., Feldman and Matthews, Ref. 2, part 2, Appendix.)

[11]See, e.g., K. M. Bitar and G. L. Tindle, Phys. Rev. 175, 1835 (1968) or Feldman and Matthews Ref. 2, part 2.

[12]We clearly also have the restriction $\lambda=M_1$, which determines which of the helicity amplitudes of the p_1,p_0 system survives at $t_1=0$.

[13]If the sum over J_2 in (9) is dominated by a single resonance of spin S_2, then $M_1 \leq \min(j_0+j_1, S_2+j_2)$.

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VENEZIANO-TYPE FORM FACTORS FOR THE PION*

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The scalar, pseudoscalar, vector, and axial-vector form factors of the pion are derived from (i) the Veneziano amplitudes for $\pi^+\pi^- \to \pi^+\pi^-$ and $\pi^+\pi^- \to A_1^+\pi^-$, (ii) the hypothesis of the partially conserved axial-vector current, and (iii) the requirement that a chirally conjugate pair of form factors should exhibit similar structures. The resulting form factors feature an infinite sequence of poles corresponding to daughters of specific spin and parity, and definite signature. Various predictions on the clashing-beam production of 2π and on ρ-meson coupling constants are given.

Recently, the Veneziano-type form factors for the pion have been discussed by several authors,[1] but these fail to satisfy a natural requirement that a chirally conjugate pair of sources, like the vector current $V_\mu{}^a$ and the axial-vector current $A_\mu{}^a$, should couple in more or less symmetric ways to an infinite number of particles with specific spin and parity, whose existence is one consequence of the Veneziano model.[2]

To illustrate our point, let us take for example the following off-shell $\pi\pi$ scattering amplitude which is consistent with the condition of partially conserved axial-vector current (PCAC) and which reduces to the Veneziano-Lovelace[3] amplitude on the mass shell:

$$\langle\pi^-(q')|\partial^\lambda A_\lambda{}^{(-)}(0)|\pi^+(p),\pi^-(q)\rangle=\sqrt{2}f_\pi m_\pi{}^2(k^2-m_\pi{}^2)^{-1}\beta_0 B_1{}^{1,1}(s,t), \tag{1}$$

where

$$B_k{}^{i,j}(s,t)\equiv\Gamma(i-\alpha_s)\Gamma(j-\alpha_t)/\Gamma(k-\alpha_s-\alpha_t).$$

Here k is the momentum of the off-shell π^+ meson and $s=(p+q)^2$, $t=(k-p)^2$, and $u=(q-k)^2$. $\alpha_s=\alpha(s)$ represents the degenerate ρ-f trajectory and is given by $\alpha(s)=\frac{1}{2}+b(s-m_\pi{}^2)$ with $b^{-1}=2(m_\rho{}^2-m_\pi{}^2)$. f_π is the pion decay constant and $\beta_0=g_{\rho\pi\pi}{}^2$. Now by continuing to $p\to 0$ by means of the standard soft-pion technique the left-hand side of (1) reduces to the pion matrix element of the so-called Σ term,

$$[A_0{}^{(+)}(x),\partial^\lambda A_\lambda{}^{(-)}(0)]_{x_0=0}=2i\delta^3(x)\Sigma(0).$$

On the right-hand side, we have $s=u=m_\pi{}^2$, $t=k^2$, and using $\alpha(m_\pi{}^2)=\frac{1}{2}$ we obtain[4]

$$\langle\pi^-(q')|\Sigma(0)|\pi^-(q)\rangle\equiv\tilde{\Sigma}(t)=-\frac{m_\pi{}^2}{\sqrt{\pi}}\frac{\Gamma(1-\alpha_t)}{\Gamma(\frac{3}{2}-\alpha_t)}. \tag{2}$$

Here we have used a scaling law[5]

$$\pi f_\pi{}^2\beta_0 b=1 \tag{3}$$

in order to obtain the above normalization. Thus, in spite of our original assumption expressed in (1), namely, that the pseudoscalar source $\partial^\lambda A_\lambda{}^{(-)}(x)$ couples only to the π meson, we obtain the result that its chiral counterpart Σ couples to all 0^+ daughters of the ρ-f trajectory. We consider this asymmetric situation unsatisfactory and, therefore, generalize the PCAC condition (1) so as to take into account all 0^- daughters of the π-A_1 trajectory:

$$\langle\pi^-|\partial^\lambda A_\lambda^{(-)}|\pi^+,\pi^-\rangle=\sqrt{2}\tilde{\pi}(k^2)B_1^{1,1}(s,t), \tag{4}$$

where

$$\tilde{\pi}(t)=\sum_{n=0}^{\infty}f_n m_n^2(t-m_n^2)\beta_n. \tag{5}$$

The summation is tentatively on all the 0^- daughters, and $n=0$ refers to π meson. The meaning of $\tilde{\pi}(t)$ as a pseudoscalar form factor can be made more explicit by taking the σ meson (the 0^+ daughter of ρ meson) pole in the s channel of (4). We find

$$\langle\pi^-(q')|\partial^\lambda A_\lambda^{(-)}(0)|\sigma(q)\rangle=(4bg_{\sigma\pi\pi})^{-1}\tilde{\pi}(t).$$

Again applying the soft-meson technique to (4) we obtain, instead of (2),

$$\tilde{\Sigma}(t)=-f_\pi\tilde{\pi}(t)\pi^{1/2}\Gamma(1-\alpha_t)/\Gamma(\tfrac{1}{2}-\alpha_t). \tag{6}$$

As stated before we want similar structures for the chirally conjugate pair $\tilde{\Sigma}(t)$ and $\tilde{\pi}(t)$, and an obvious way is for $\tilde{\pi}(t)$ to eliminate half the poles and zeros of $\Gamma(1-\alpha_t)/\Gamma(\frac{1}{2}-\alpha_t)$. This is possible assuming that the π-A_1 trajectory is parallel to that of ρ, that is, $\alpha_\pi(t)=\alpha(t)-\frac{1}{2}$. Thus we may take

$$\tilde{\pi}(t)=-\frac{m_\pi^2}{\sqrt{2}f_\pi}\frac{1}{\Gamma(\frac{1}{4})}\frac{\Gamma(\frac{1}{4}-\frac{1}{2}\alpha_t)}{\Gamma(1-\frac{1}{2}\alpha_t)} \tag{7}$$

$$=-\frac{m_\pi^2}{f_\pi}\sum_{n=0}^{\infty}\frac{\Gamma(n+\frac{1}{4})}{\pi\Gamma(\frac{1}{4})n!}\frac{1}{\frac{1}{2}+2n-\alpha_t}. \tag{7$'$}$$

The normalization has been fixed by comparing the expansion (7′) with (5), the first term of which is known. Now introducing (6) into (5) we find

$$\tilde{\Sigma}(t)=-m_\pi^2\frac{\sqrt{\pi}}{\Gamma(\frac{1}{4})}\frac{\Gamma(\frac{1}{2}-\frac{1}{2}\alpha_t)}{\Gamma(\frac{3}{4}-\frac{1}{2}\alpha_t)}. \tag{8}$$

Thus, we have achieved similar structures for $\tilde{\pi}(t)$ and $\tilde{\Sigma}(t)$, both having poles and zeros at every other possible position, that is, separated by $2b^{-1}$.

To obtain the vector form factor, we consider, instead of (4), the matrix element of the axial-vector current $A_\mu^{(-)}$ between π^- and $\pi^+\pi^-$. Such matrix elements were constructed by several authors[6-8] in the case when the axial-vector current was coupled to the π and A_1 mesons only. Here we assume for the moment that the axial-vector current couples to all 0^- and 1^+ daughters of the π-A_1 trajectory. Generalizing the result by Rosner and the author,[8] we write then

$$\begin{aligned}(\sqrt{2}i)^{-1}\langle\pi^-(q')|A_\mu^{(-)}(0)|\pi^+(p),\pi^-(q)\rangle=&\sum_{n=0}^{\infty}f_n\beta_n(k^2-m_n^2)^{-1}B_1^{1,1}(s,t)k_\mu+(q_\mu C_1+k_\mu C_2)B_1^{1,1}(s,t)\\&+\sum_{n=0}^{\infty}G_n^A(k^2-M_n^2)^{-1}(-g_{\mu\lambda}+M_n^{-2}k_\mu k_\lambda)\\&\quad\times\{[\gamma_nB_1^{1,1}(s,t)+\gamma_n'B_2^{1,1}(s,t)](q'-p)^\lambda+\gamma_n(\alpha_s-\alpha_t)B_2^{1,1}(s,t)(q'+p)^\lambda\}\\&+[(\alpha_s-\alpha_t)(q'+p)_\mu C_3+q_\mu C_4+k_\mu C_5]B_2^{1,1}(s,t).\end{aligned} \tag{9}$$

Here the first sum is over all 0^- particles as before and the second sum is over all 1^+ particles with G_n^A defined by $\langle 0|A_\mu^{(-)}|n\text{th }1^+\rangle=\sqrt{2}G_n^A\epsilon_\mu$. The two parameters γ_n and γ_n' correspond to the two possible couplings of 1^+ meson with $\rho\pi$. Namely, they are related to the so-called S- and D-wave coupling constants by[8] $g_S^{(n)}/g_D^{(n)}=(\gamma_n-\gamma_n')/2\gamma_n$. For the moment we will leave these parameters unspecified. The five C_i terms may be called subtraction constants at $k^2=\infty$, and they are necessary in order that (8) satisfies the generalized PCAC condition (4) and (5). In fact, that condition requires that

$$C_1=-C_2=C_3=-\sum_{n=0}^{\infty}G_n^A\gamma_nM_n^2,\quad C_5=\sum_{n=0}^{\infty}G_n^A\gamma_n'M_n^{-2},$$

and

$$C_4+C_5=2\sum_{n=0}^{\infty}f_n\beta_n b. \tag{10}$$

The pion form factor is obtained by applying again the soft-pion technique to $\pi^+(p)$ and taking the limit $p\to 0$. Then the left-hand side of (9) gives $-f_\pi^{-1}\langle\pi^-(q')|V_\mu^{(3)}|\pi^-(q)\rangle=f_\pi^{-1}F_\pi(t)(q+q')_\mu$. In the same limit, the right-hand side still contains a gauge-noninvariant term

$$\sum_{n=1}^{\infty}f_n\beta_n n[t-m_n^2]^{-1}k_\mu.$$

However, comparison of (7′) and (5) shows that for any finite t, the above summation is only of the order of m_π^2/m_ρ^2 relative to the $n=0$ term. Thus, assuming that our whole procedure is valid only to the lowest order in m_π^2, we will neglect this term against gauge-invariant terms. We obtain then

$$F_\pi(t)=\pi^{1/2}f_\pi\frac{\Gamma(1-\alpha_t)}{\Gamma(\frac{3}{2}-\alpha_t)}\{-C_1bt+\tfrac{1}{2}C_4-\tfrac{1}{2}\sum_{n=0}^{\infty}G_n{}^A(t-M_n{}^2)^{-1}(\gamma_n'-2\gamma_n bt)\}. \tag{11}$$

We notice that if $C_1\neq 0$ we have $F_\pi\to t^{1/2}$ $(t\to\infty)$. This may not be of too much concern as our infinitely rising linear trajectory is certainly an idealization. However, it appears that we will not have the kind of reciprocity as shown in (7) and (8) as long as $C_1\neq 0$. Hence, we will assume that $C_1=0$. We can further choose $C_4=0$ without any contradiction. From (10), we then have

$$C_5=2\sum_{n=0}^{\infty}f_n\beta_n b=2\pi^{-1}f_\pi^{-1}. \tag{12}$$

The last step follows from (3) since the sum for $n\geq 1$ contributes only a term of order m_π^2/m_ρ^2 relative to the $n=0$ term. Although $C_1=0$ does not necessarily require $\gamma_n=0$, we will assume this in the following as a simplest possibility. Then

$$F_\pi(t)=\pi^{1/2}f_\pi\frac{\Gamma(1-\alpha_t)}{\Gamma(\frac{3}{2}-\alpha_t)}\tilde{A}(t), \tag{13}$$

where

$$\tilde{A}(t)=\tfrac{1}{2}\sum_n G_n\gamma_n'(t-M_n{}^2)^{-1}. \tag{14}$$

If we take out the σ pole in the s channel of (8), we find

$$(\sqrt{2}i)^{-1}g_{\sigma\pi\pi}\langle\pi^-(q')|A_\mu{}^{(-)}(0)|\sigma(q)\rangle=(q+q')_\mu\tilde{A}(t)+(k_\mu\text{ term}). \tag{15}$$

Thus $\tilde{A}(t)$ and $F_\pi(t)$ are a pair of chirally conjugate form factors. Now we may take

$$\tilde{A}(t)=-\frac{1}{\pi f_\pi}\frac{\Gamma(\frac{3}{4})}{\sqrt{\pi}}\frac{\Gamma(\frac{3}{4}-\frac{1}{2}\alpha_t)}{\Gamma(1-\frac{1}{2}\alpha_t)}, \tag{16}$$

where the normalization has been fixed by (12). From (13) and (16) we have[9]

$$F_\pi(t)=\frac{\Gamma(\frac{3}{4})}{\sqrt{2\pi}}\frac{\Gamma(\frac{1}{2}-\frac{1}{2}\alpha_t)}{\Gamma(\frac{5}{4}-\frac{1}{2}\alpha_t)}. \tag{17}$$

From this we can draw the following conclusions concerning the colliding-beam production of 2π: (i) There is no ρ' (1^- daughter of an f meson) production. (ii) The cross section should vanish at $\alpha(t)=\frac{5}{2}$ or at the incident energy of ~1500 MeV. (iii) The peak value[10] at $t=m_\rho^2$ is $|F_\pi(m_\rho^2)|^2=8(m_\rho/\pi\Gamma_\rho)^2$. In terms of the ρ-γ and ρ-$\pi\pi$ coupling constants f_ρ and $g_{\rho\pi\pi}$ this corresponds to the value $g_{\rho\pi\pi}/f_\rho=2\sqrt{2}/\pi\sim 0.9$ instead of 1 in the ρ-dominance model. If we further use the value of $g_{\rho\pi\pi}$ as determined from (3), we obtain[11] $f_\rho^2/4\pi=(m_\rho/4f_\pi)^2\sim 4$. (iv) The cross section for the next resonance [$\alpha(t)=3$, the 1^- daughter of a g meson ~1650 MeV] should be down by a factor $|\Gamma(\frac{3}{4})/\Gamma(-\frac{1}{4})|^2\sim\frac{1}{16}$ relative to ρ production, excluding the width factor and the phase-space factor. Finally, the rms radius is given by

$$\langle r^4\rangle=\tfrac{3}{2}m_\rho^{-2}[\psi(1)-\psi(\tfrac{1}{4})]\sim(0.6\times 10^{-13}\text{ cm})^2.$$

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[4]Two remarks about Eq. (2). First $\Sigma(0)=-m_\pi^2$. This result is the same as is obtained from SU(3) mass splitting according to Gell-Mann–Oakes–Renner [M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968)]. This point was first noticed independently by Griffith [R. W. Griffith, to be published]. Second, Eq. (2) can be reproduced from the Omnès phase representation if we choose $\delta=\pi$ for $n<\alpha(t)<n+\frac{1}{2}$ $(n=1, 2, 3, \cdots)$ and 0 otherwise. Such an unphysical oscillating phase shift would simply indicate that the use of the Omnès representation in this case could not be justified. In fact, the resonant states should be considered as quark-antiquark states rather than the elastic π-π channel.

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[9]The normalization of F_π has followed automatically. This is based on our use of the scaling law (3). That is, the normalization condition on F_π is equivalent to the modified Kawarabayashi-Suzuki-Fayyazuddin-Riazuddin relation (3).

[10]We have added an imaginary part $m_\rho\Gamma_\rho$ to the ρ pole, where Γ_ρ is the width of ρ.

[11]We do not quote here any experimental number for comparison because there are so many conflicting experimental results and analyses that we can almost always find a set of data fitting the theory.

INCOHERENT PHOTOPRODUCTION OF ρ^0 MESONS FROM COMPLEX NUCLEI AND COMPARISON WITH VECTOR-DOMINANCE PREDICTIONS*

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Vector-meson dominance predicts a characteristic energy dependence of incoherent ρ photoproduction from complex nuclei. Measurements of this process were made which do not conform to these predictions.

The implications of vector-meson dominance for photon interactions with complex nuclei have been discussed by several authors.[1] A particular feature of these investigations is a transition from the low-energy region (≲4 GeV), where the nucleus acts as a transparent object to the photon, to the high-energy region (≳20 GeV), where the photon appears to be strongly absorbed because its ability to interact decreases as it proceeds through the nucleus. The reason for this decrease is that, as the photon wave propagates through the nuclear matter, a coherent vector-meson wave builds up and approaches a magnitude such that for subsequent interactions there is exact cancellation between the following two amplitudes: one produced by the original photon wave (a "one-step" process), the other produced by the vector-meson wave (a "two-step" process). At low energy the two-step process becomes negligible.

We have studied the energy dependence of ρ^0 photoproduction from complex nuclei at a fixed value of square of momentum transfer, $|t|=0.1$ GeV^2. At this value of $|t|$ the "coherent" forward production has dropped to a negligible value for the nuclei used.[2] It is conventional in this incoherent region to quote the cross section as $A_{\rm eff}$, the effective number of nucleons contributing. Naturally, this is the ratio of the cross section from the given nucleus to the one-nucleon cross section (assuming that the proton and neutron cross sections are equal). Formulas for calculating $A_{\rm eff}$ are given in the papers of Ref. 1, particularly that of Gottfried and Yennie. Quali-