sentative of pure solid He^3.

These measurements will be extended to lower temperatures and higher densities in the near future. We would like to acknowledge the assistance of Mr. L. A. Dietz in taking and analyzing the data.

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POSITRON ANNIHILATION AND ELECTRONIC LATTICE STRUCTURE IN INSULATOR CRYSTALS*

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The dimensions of the electronic lattice structure of the hexagonal quartz crystal and the fcc CaF_2 crystal have been measured by resolving characteristic markings on the angular-correlation curves as observed for the two gamma quanta emitted in the annihilation of positrons with electrons in these crystals.

The angular correlation of the two gamma quanta emitted in the para-annihilation of positrons with electrons in matter is observed in most experiments as the correlation function

$$I(p) = 2\pi \int_p^\infty \rho(\vec{p}')\, p'dp'. \quad (1)$$

The momentum p of the annihilating positron-electron pair in some laboratory direction is equal to $mc\theta$, where θ is the angle between the two emerging gamma quanta. In a crystal with any basis, the density $\rho(\vec{p})$ can be expressed in terms of the square of the Fourier-transformed positron-electron wave function in the unit cell $\rho_{\vec{k}_-}{}^c(\vec{p})$, where $\vec{k}_-$ is the wave vector of the annihilating electrons.[1] For thermalized positrons ($\vec{k}_+ \simeq 0$)

$$\rho(\vec{p}) = \sum_{\vec{k}} \delta_{\vec{p},\vec{k}_-+\vec{K}} \rho_{\vec{k}_-}{}^c(\vec{p}). \quad (2)$$

The sum extends over all reciprocal lattice vectors $\vec{K}$. Equation (1) can then be factorized[2]:

$$I(p) = A(p)F(p). \quad (3)$$

$F(p)$ is determined by the momentum-density dis-

tribution in the unit cell. $A(p)$ is equal to the number of occupied states in the plane which cuts perpendicular to the direction of observation through the periodic zone structure at the distance p from the origin in $\vec{p}$ space. It has the periodicity of the electronic structure.

To date, such structure has been observed only on the correlation curves of rare-earth metals.[2] If the periodicity of $A(p)$ can be resolved for insulators, positron annihilation can elucidate the electronic lattice structure of insulators, of which little is known by direct evidence. It is the purpose of this note to report observations of reciprocal lattice dimensions of the hexagonal quartz crystal and the fcc CaF_2 crystal through characteric modulations of $I(p)$ in accordance with Eq. (3).

The measurements were performed with a standard angular-correlation instrument of resolution $\delta\theta = 0.6$ mrad and a Cu^{64} source strength of 1-6 Ci. Results are shown in Fig. 1 for a synthetic quartz sample in the three crystallographic directions X, Y, and Z. Markings appear on the angular correlation curve. If observed along the X direction they appear (to within ±7%) at $\pm p_X = 2\pi/a$; along the Y direction at $\pm p_Y = 2\pi/3^{1/2}a$, at $\pm 2p_Y$, and at $\pm 3p_Y$; along the Z direction at $\pm p_Z = 2\pi/c$. The lattice constants of the hexagonal quartz are $a = 4.19$ and $c = 5.40$ Å. The modulations mark the values p_X, p_Y, and p_Z that are equal to the dimensions of the first Jones[3] zone in quartz.

Not all crystals of different origin show these markings. Figure 2 compares three types of quartz along the Y direction. Type a, a natural quartz from Brazil, shows none. Type b, from Madagascar, exhibits modulations at $\pm p = p_Y$. Type c, a synthetic quartz from the same source as those studied in Fig. 1, shows in addition modulations at $\pm p = 2p_Y$.

Similar modulations are observed with some samples of the fcc CaF_2 crystal along ⟨111⟩ direc-

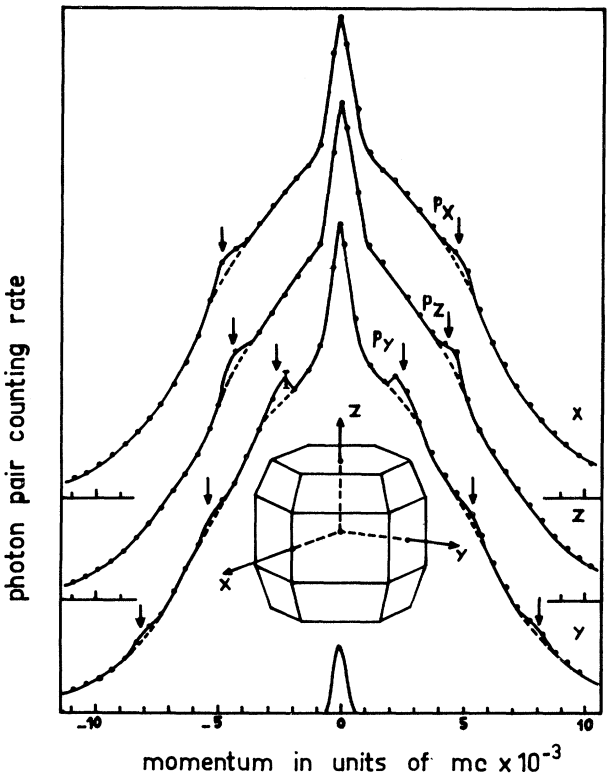

FIG. 1. Angular-correlation curves of a synthetic quartz crystal as observed in the X, Y, and Z crystal directions, respectively. The arrows at p_X, p_Y, and p_Z mark the dimensions of the first Jones zone of the hexagonal quartz crystal, which is sketched in the central figure. The dotted lines are drawn to aid the recognition of the markings characteristic of the electronic structure in this insulator. The instrument resolution is shown at the bottom.

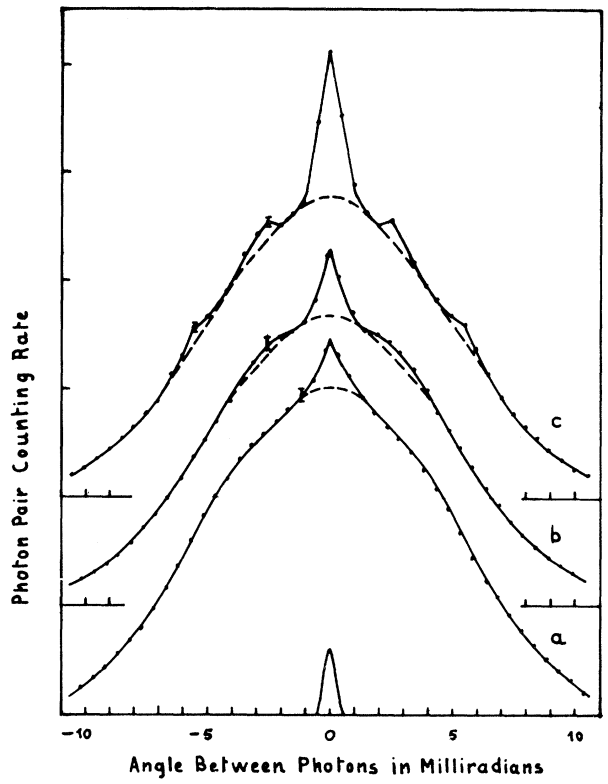

FIG. 2. Angular-correlation curves for three types of quartz crystals as observed along the Y crystal direction. Type a is natural quartz from Brazil, type b natural quartz from Madagascar, and type c stems from the same source as the synthetic crystals investigated in Fig. 1. The instrument resolution is shown at the bottom.

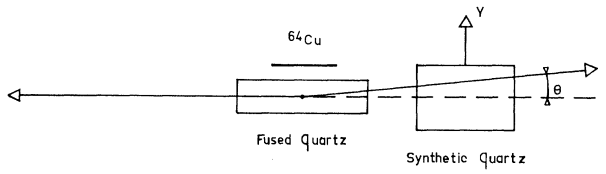

FIG. 3. Experiment in search for gamma-ray diffraction in synthetic quartz crystal along the *Y* direction.

tion at $\pm p = 3.5\pi/a$, where $a = 5.46$ Å is the lattice constant. This is equal to the dimension of the first Brillouin zone in this direction. With other crystals of different origin but cut in the same direction no markings on the angular correlation curve have been resolved.

The narrow components in Figs. 1 and 2 comprise (1-5)% of all annihilations and indicate that positrons annihilate with electrons of small binding energy ($\simeq$0.2 eV). Since quartz crystals show no long positron lifetime component, these annihilations can be indicative of Ps formation in defects with high pickoff rates or, equivalently, of positron-electron pair annihilations in A_-' or A_+' centers.[4] The results of the study of the narrow component will be reported elsewhere.

We investigated the possibility that the periodic structure at multiples of $\pm p_Y$ is caused by gamma-ray diffraction in the crystal in a transmission experiment.[5] Gamma rays emerging from positron annihilations in a sample of fused (amorphous) quartz, or a MgO pill, traversed, before detection, a type-*c* quartz crystal oriented in the same direction in which curve *Y* in Fig. 1 was measured (Fig. 3). The absence of any structure in the resulting correlation curve is taken as a proof that gamma-ray diffraction is not important in these experiments.

The question remains why the lattice structure produces markings in the angular-correlation curves of some crystals but not in others, as illustrated in Fig. 2. One estimates that the effect of continuously distributed or extended lattice defects on $A(p)$ is to suppress these markings and, in conserving sum rules, to contribute instead a weak (essentially logarithmic) change near $p = 2\pi/d$, where d is the mean defect spacing. This corresponds to angles so small that the effect cannot be resolved. Moreover, if one averages over a resolution $mc\delta\theta > 2\pi/d$, the effect is washed out such that it is unlikely to stimulate a narrow component. On the other hand, one can advance arguments to the effect that point defects at lattice sites, e.g., vacancies, can accentuate the markings to make them more readily resolvable than in perfect crystal. If this is so, optimum conditions should exist for real crystals to exhibit the periodicity of $A(p)$ in experimental angular correlation curves. Studies to find such conditions are currently underway.

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