tained from (2.11), (2.12′), and (2.13):

$$\frac{1}{6}\langle r_V{}^2\rangle\left(\frac{d^2G_E{}^V(K^2)}{d(K^2)^2}\right)_{K=0} = \frac{1}{4M^2}\left(\frac{dG_M{}^V(K^2)}{dK^2}\right)_{K=0}^2, \qquad (2.17)$$

which gives experimentally $2.7\times10^{-3}=2.8\times10^{-3}$, a much better result. If one retains a sum over all one-particle states one can again use the Wigner-Eckart theorem in (2.7) to express all matrix elements in terms of $\langle p|M_{33}{}^3|n\rangle$, with the result

$$\sum_n c_n|\langle p|M_{33}{}^3|n\rangle|^2=0, \qquad (2.18)$$

where the c_n are positive constants. Hence all the matrix elements $\langle p|M_{33}{}^3|n\rangle$ vanish, and this in turn gives, via (2.8), the previous result (2.14).

It thus appears that if one can attribute any physical significance to the approximate saturation of commutators by single-particle states, the algebra of fields leads to serious discrepancies with experiment, while the current-algebra results seem to be reasonably good.

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HADRONIC MASS QUANTUM

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On the basis of chiral dual dynamics it is shown that the square of the mass of any nonexotic strange or nonstrange meson or baryon of nonvanishing isospin (with the possible exception of $I=1$ baryons) must be an integer multiple of the "quantum" $\frac{1}{2}m_\rho{}^2$. It is found than the strength of SU(3) breaking can only take certain discrete values.

By use of field-current identities, current algebra, and off-shell extensions of the Veneziano model, it has been recently possible to obtain closed expressions for electromagnetic form factors.[1] While extrapolation off the mass shell is by no means unambiguous, all these expressions are of the form

$$G(t)=\frac{\Gamma(1-\alpha_\rho(t))}{\Gamma(\frac{1}{2}n-\alpha_\rho(t))}P(t). \qquad (1)$$

Here $G(t)$ is the isovector electromagnetic form factor (we are for the time being limiting our discussion to $I\neq0$ hadrons); $\alpha_\rho(t)$ the Regge trajectory of the ρ meson,

$$\alpha_\rho(t)=\tfrac{1}{2}+t/2m_\rho{}^2; \qquad (2)$$

n a positive odd integer; and $P(t)$ a polynomial in t, so normalized that

$$\frac{\Gamma(1-\alpha_\rho(0))}{\Gamma(\frac{1}{2}n-\alpha_\rho(0))}P(0)=I_3, \qquad (3)$$

I_3 being the third component of the isospin of the hadron. The various off-the-mass-shell extrapolation procedures used by different authors affect only the detailed form of $P(t)$. The crucial feature of Eq. (1) is the fact that n is odd. This is a direct consequence of the origin of Eq. (1) in the soft-pion limit of the amplitude for $\pi H\to a^{(0)}H$ [$a^{(0)}$ is the $\Delta Y=0$, $\Delta I=1$ axial-vector current; H is the hadron the form factor of which is given by (1)]. In this limit only states of normality opposite to that of H contribute; and because of the quantization condition of Regge trajectories,[2] this leads to n being odd. We shall show that this simple result leads to extremely strong constraints on the hadron spectrum. In particular it relates the masses of baryons to those of mesons and quantizes the scale of SU(3) breaking. Our result is that (A) the square of the mass of any (nonexotic) hadron (be it a meson or a baryon) of nonvanishing isospin (with the possible exception of $I=1$ baryons) must be an integer multiple of $\frac{1}{2}m_\rho{}^2$. By nonexotic we mean any meson obtainable as $q\bar{q}$ and baryon obtainable as qqq, in other words all ($|B|\leq1$) hadrons known at present with the possible exception of

Z^* resonances (if they exist?) or of some of the "subpeaks" of A_2, R, S, T, U, if they should turn out to be $qq\bar{q}\bar{q}$ structures.

We shall start by proving the result for the N, Σ, and Ξ. Consider the isovector Sachs form factors of the nucleon $G_M^{VN}(t)$ and $G_E^{VN}(t)$. These form factors should obey the conditions

$$G_M^{VN}(4m_N^2) = G_E^{VN}(4m_N^2) = 0. \quad (4)$$

Equations (4) can be obtained from various assumptions. For instance, $SU(6)_W$ immediately leads to Eqs. (4).[3] Alternatively the scaling law[4]

$$G_M^{VN}(t) = \mu^{VN} G_E^{VN}(t) \quad (5)$$

if valid at $t = 4M_N^2$ (?) together with the requirement that the Dirac and Pauli form factors $F_1^{VN}(t)$ and $F_2^{VN}(t)$ be regular at $t = 4M_N^2$ require (4), since $\mu^{VN} \neq 1$.[5] Now, how can Eq. (4) be obeyed by the form (1)? There are only two ways: Either

$$P(4m_N^2) = 0, \quad (6)$$

or

$$\tfrac{1}{2}n - \alpha_\rho(4m_N^2) = -m \quad (m = 0, 1, 2, \cdots). \quad (7)$$

Equation (6) does not hold for any of the proposals of Ref. 1. In fact P is most likely a constant or a first degree polynomial that does not vanish at $t = 4M_N^2$. We therefore opt for Eq. (7). Combining Eqs. (2) and (7) we find

$$m_N^2/(\tfrac{1}{2}m_\rho^2) = m + (n-1)/2. \quad (8)$$

Since n is odd, the right-hand side of Eq. (8) must be an integer. The same proof applies for Σ and Ξ. At this point we compare with experiment. In reality

$$\frac{m_N^2}{\frac{1}{2}m_\rho^2} = 3.01, \quad \frac{m_\Sigma^2}{\frac{1}{2}m_\rho^2} = 4.86, \quad \frac{m_\Xi^2}{\frac{1}{2}m_\rho^2} = 5.95, \quad (9)$$

which are remarkably close to the integers 3, 5, and 6. To apply the reasoning that led us to Eq. (8) for the magnetic $\Sigma^0\Lambda$-transition form factor one has to consider the process $\pi_1 + \Lambda \to \pi_2 + \Sigma$. Depending on whether one lets π_1 or π_2 become soft one would then obtain different expressions for the $\Sigma\Lambda$ form factor unless

$$m_\Sigma^2 = m_\Lambda^2. \quad (10)$$

That we obtain the condition (10) should not come as a surprise as we are using the chiral arguments of Ref. 2 which also imply it. It is therefore clear that the spectrum of $I = 1$ and $I = 0$ baryons from our point of view will appear distorted. As in all considerations based on chiral dynamics, exchange degeneracy or the quark model, the $\Sigma - \Lambda$ mass difference is a mystery. We should therefore not attach too much significance to mass formulas involving $I = 1$ or $I = 0$ baryons. If, nevertheless, one were to take $m_\Sigma^2/(\frac{1}{2}m_\rho^2) = 5$ seriously, then all results stated below for $I = \frac{1}{2}$ baryons only would extend to $I = 1$ and certain $I = 0$ baryons as well.

An important feature of our results so far is that we have fixed the difference

$$\Delta = (m_\Xi^2 - m_N^2)/\tfrac{1}{2}m_\rho^2 = \text{integer}. \quad (11)$$

This relation shows that the intensity of SU(3) symmetry-breaking interaction cannot vary continuously. It can only take discrete values compatible with Eq. (11). In particular SU(3) breaking cannot be arbitrarily small. One either has exact SU(3) symmetry ($\Delta = 0$) or $\Delta \geq 1$. In the quark model since $m_\Xi^2 - m_N^2 > m_\Sigma^2 - m_N^2 > 0$ one would expect $\Delta \geq 2$. Experimentally,

$$\Delta_{\rm exp} = 2.94 \approx 3. \quad (12)$$

Equation (11) presents us with a discrete set of possible scales of SU(3) breaking. Why nature choses the particular scale (12) and why $m_\Xi - m_N > 0$ is left unexplained by our arguments. The value $\Delta = 3$ is very close to the "minimal" quark-model value $\Delta = 2$.

Now let us generalize our arguments to higher baryons. All baryons with $I = \frac{1}{2}$ or $\frac{3}{2}$ lie (i) on one of the N or Ξ trajectories or their daughters, or (ii) on a trajectory degenerate with one of the trajectories mentioned under (i), or (iii) on a trajectory of opposite normality to those mentioned under (i) and (ii). The universal slope of all Regge trajectories is[2] $(2m_\rho^2)^{-1}$. Therefore, two particles lying on the same trajectory must be spaced by an integer multiple of $2m_\rho^2$ and therefore of $\frac{1}{2}m_\rho^2$. Therefore all baryons lying on the N or Ξ trajectories will obey

$$m^2/\tfrac{1}{2}m_\rho^2 = \text{integer} \quad (13)$$

(here we use m as a generic notation for the mass of the hadron referred to). All trajectories mentioned under (i) and (ii) can support paricles only at masses where either the N or the Ξ trajectory supports a particle and therefore obey (13). The trajectories mentioned under (iii) are spaced from either the N or the Ξ trajectory by half an integer.[2] Therefore they support particles at (masses)^2 that differ from those on the N or Ξ trajectories by an integer multiple of m_ρ^2 and therefore of $\frac{1}{2}m_\rho^2$. This proves our result for baryons. Similar conclusions could be reached for $I=1$ baryons, though, as we mentioned earlier, at a lower level of reliability.

For mesons of $I=1$ Eq. (13) is a straightforward consequence of the mass relations of Ref. 2. All we have left are mesons of $I=\frac{1}{2}$. For any nonexotic $I=\frac{1}{2}$ meson K^* there exists an $I=1$ meson ρ^* (or π^*) such that

$$m_{K^*}^2-m_{\rho^*}^2=m_K^2-m_\pi^2. \quad (14)$$

Our result would then be proved for all $I=\frac{1}{2}$ mesons if

$$(m_K^2-m_\pi^2)/(\tfrac{1}{2}m_\rho^2)=1 \quad (15)$$

To prove Eq. (15) we use the fact that in the quark model the Gell-Mann–Okubo (GMO) splittings of baryons and mesons are related. As we have fixed the scale of the former, the scale of the latter must also be fixed. The relationship between meson and baryon mass splittings has been found some years ago[6] to be

$$(m_K^2-m_\pi^2)/(m_\Xi^2-m_N^2)=\tfrac{1}{3}. \quad (16)$$

Combining Eqs. (11), (12), and (16) we find indeed Eq. (15). This completes the proof of the statement (A) made at the beginning of this paper. To get an idea of the accuracy of our relation (15) let us quote from experiment

$$m_K^2/\tfrac{1}{2}m_\rho^2=0.82. \quad (17)$$

We now consider the case of $I=0$ hadrons. $I=0$ mesons belonging to SU(3) octets or ideal nonets have their masses fixed from GMO relations once the masses of all nonexotic $I\geq\frac{1}{2}$ mesons are known. Examples are

$$m_\omega^2=m_\rho^2, \quad m_\varphi^2=m_\rho^2+2m_K^2, \cdots. \quad (18)$$

$I=0$ baryons that belong to SU(3) octets have masses fixed through GMO relations. SU(3)-singlet baryons lie on trajectories that are exchange degenerate with $I=1$ trajectories covered by statement (A).[7] As such their masses are determined [because of relations of the type (10), the spectrum of $I=0$ baryons, much like that of $I=1$ baryons, will be distorted]. A similar argument can be made for SU(3)-singlet mesons, though the corresponding exchange degeneracies are notoriously broken.[8] Our discussion now covers <u>all</u> nonexotic hadrons. Exotic hadrons (if they exist?) can be discussed along similar lines. As a word of caution, our derivations have used the chiral arguments of Ref. 2. All difficulties[9] such as $m_\Sigma^2=m_\Lambda^2$, $m_B^2=m_{A_1}^2$, $m_\eta^2=m_\pi^2$, etc., that go with Ref. 2 are carried over to our work. What we describe here is more like the breakdown from U(3)⊗U(3) to SU(2)⊗SU(2). In short our relations are best for $I=\frac{1}{2}$ and $\frac{3}{2}$ baryons and $I=\frac{1}{2}$ and $I=1$ mesons. Distortions arise in the $I=0$ meson spectrum and in the $I=1$ and $I=0$ baryon spectra.

We now have to say a few words about the isoscalar electromagnetic form factors. We could derive formulas similar to (1) for these as well. The starting point, however, would have to be the process $KH\to a^{(1)}H$ ($a^{(1)}$ is now the $\Delta Y=1$, $\Delta I=\frac{1}{2}$ axial-vector current). Therefore, the analog of Eq. (1) would have to be derived from a <u>soft K-meson</u> rather than a soft-pion theorem. Its reliability would be correspondingly weaker and since m_K^2 is just our "quantum," such arguments could hardly serve any useful purpose. Similarly, strangeness-changing weak vector form factors involve $\pi H\to a^{(1)}H$ (or $KH\to a^{(0)}H$) amplitudes. With broken SU(3) symmetry, the corresponding Veneziano representations involve many satellite terms to insure factorization and universality. We shall therefore not discuss them here.

To sum up, we have used as input (a) information derived from chiral dynamics and the Veneziano model[10] [Eq. (1)], and (b) information from what may be called the quark model [Eqs. (14) and (16)].[11] As a result we have found that all nonexotic $I>0$ hadrons, with the possible exception of $I=1$ baryons, must have masses whose squares are integer multiples of $\frac{1}{2}m_\rho^2$. Beyond relations like (9), that can be viewed as expressing the common parentage of all hadrons (mesons and baryons alike), our most surprising conclusion is that within our frame of assumptions SU(3) symmetry breaking <u>cannot</u> be arbitrarily small. We have <u>not</u> explained <u>why</u> SU(3) is broken. All we have shown is that <u>if</u> SU(3) is broken, the breaking can occur only with certain discrete intensities as prescribed by Eq. (11). In nature SU(3) is broken and this prescription is rather closely obeyed and at that with what seems to be a "minimal" value of the parameter Δ [Eq. (11)]. In the "primordial SU(3)-symmetric world" the ρ mass presumably does not vanish.[12] This idealized world therefore "carried in itself" the scale of its own destruction.

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[1]Y. Oyanagi, to be published. Similar ideas with different and more general off-shell extrapolation procedures have been considered by R. Arnowitt and collaborators (private communication) and by H. Suura (private communication). Equations of the form (1) have been phenomenologically postulated by R. Jengo and E. Remiddi, to be published; P. di Vecchia and F. Drago, to be published; P. H. Frampton, to be published.

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[4]Contrary to common belief, scaling law does not follow from $SU(6)_W$. A simple counterexample is the following. Imagine the electromagnetic form factors of the nucleons to be dominated by the $I=0$ and $I=1$ members from two vector meson nonets M and M' of unequal masses m and m'; $SU(6)_W$ then yields (in an obvious notation)

$$G_E{}^P(t)=\frac{gfm^2}{m^2-t}+\frac{(1-gf)m'^2}{m'^2-t},$$

$$G_M{}^P(t)=\frac{2m_N}{m}\,\frac{gfm^2}{m^2-t}+\frac{2m_N}{m'}\,\frac{m'^2}{m'^2-t},$$

which violate the scaling law.

[5]di Vecchia and Drago (Ref. 1) have used Eq. (4) in their phenomenological analysis.

[6]P. G. O. Freund, Nuovo Cimento 39, 769 (1965), and Phys. Rev. Letters 16, 291 424(E) (1966). Roughly speaking, in the quark model Eq. (16) follows from $2(m_{K^*}-m_\rho)=m_\Xi-m_N$, $\frac{3}{2}(m_{K^*}+m_\rho)=m_\Xi+m_N$, and $m_{K^*}{}^2-m_\rho{}^2=m_K{}^2-m_\pi{}^2$. In the quark model also, $m_\Sigma{}^2=m_\Lambda{}^2$. Equation (16) actually fixes the normalization of the F part of the baryon mass splittings with respect to the meson mass splittings.

[7]See, e.g., R. H. Capps, Phys. Rev. Letters 22, 215 (1969).

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[10]It may be worthwhile pointing out that our result is stable with respect to the introduction of satellite terms in Eq. (1) as the latter will also have a half-odd integer appearing in the denominator Γ function. The argument gets weakened only to the extent that one has to exclude accidental cancellations between the leading and satellite terms at $t=4m_N{}^2$ as the method by which the threshold constraints (4) are implemented. Our arguments would also remain valid if the contribution of every second pole and every second zero to $G(t)$ were to be discarded by dividing the arguments of both Γ functions in Eq. (1) by 2.

[11]It is likely that part, or all, of the quark model information used might be replaced by considerations of strong interaction dynamics. Our result (A) is after all completely within the spirit of a bootstrap.

[12]An alternative is that in the limit of SU(3) symmetry $m_\rho{}^{\text{symm}}=0$ and $m_{A_2}{}^{\text{symm}}$ sets the slope of Regge trajectories.