EXPERIMENTAL CONSEQUENCES OF THE ALGEBRA OF FIELDS

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The commutation rules of the algebra of fields have been applied to predict sum rules for the electric-dipole moment and the magnetic-quadrupole moment of nucleons. In the calculation it was assumed further that the sum rule could be saturated by just one state [$N^{**}(1518)$]. The resulting consistency condition obtained is in striking conflict with experimental results. The identical condition is obtained if one assumes saturation of the sum rule by an arbitrary number of one-particle states. This failure of the consistency condition based on the algebra of fields is to be contrasted with the reasonable success of the corresponding relation based on current algebra and saturation by the same single state.

Lee, Weinberg, and Zumino[1] proposed a set of commutation rules for the vector and axial-vector current components. The resulting algebraic structure, "the algebra of fields," is similar to but not identical with the "current algebra" proposed by Dashen and Gell-Mann.[2] The algebra of fields has a somewhat simpler mathematical structure than does the current algebra (there are no q-number Schwinger terms), and it has also produced some striking experimental predictions. A very interesting attempt by Sugawara[3] to construct a dynamical theory using currents as the primary entities is based on the same algebra of fields. The purpose of this paper is to show that there are experimental difficulties, perhaps serious ones, which follow from the commutation relations of the algebra of fields. It is possible, using the commutation relations of current algebra, to obtain a relation between the magnetic moment and the charge radius of the nucleon (B. W. Lee[4]). Another consistency relation, again based on current algebra for the electric-dipole and magnetic-quadrupole moment, was derived by Bietti.[5] Both these derivations assume in addition to the algebraic structure the saturation of the sum over intermediate states by one (or two) single-particle states. The results obtained agree with experiment. In this note a similar consistency relation is derived based on the algebra of fields, assuming, in addition, saturation by an arbitrary number of single-particle states. The resulting relation appears to be in flagrant conflict with the experimental results.

The algebra of fields is described by

$$[V_0{}^\alpha(x), V_0{}^\beta(y)]_{x_0=y_0} = if_{\alpha\beta\gamma}V_0{}^\gamma(x)\delta^3(\vec{x}-\vec{y}), \quad (1.1)$$

$$[V_0{}^\alpha(x), A_0{}^\beta(y)]_{x_0=y_0} = if_{\alpha\beta\gamma}A_0{}^\gamma(x)\delta^3(\vec{x}-\vec{y}), \quad (1.2)$$

$$[A_0{}^\alpha(x), A_0{}^\beta(y)]_{x_0=y_0} = if_{\alpha\beta\gamma}V_0{}^\gamma(x)\delta^3(\vec{x}-\vec{y}), \quad (1.3)$$

$$[V_0{}^\alpha(x), V_a{}^\beta(y)] = [A_0{}^\alpha(x), A_a{}^\beta(y)] = if_{\alpha\beta\gamma}V_\alpha{}^\gamma\delta^3(\vec{x}-\vec{y}) + iS\partial_a\delta^3(\vec{x}-\vec{y}), \quad (1.4)$$

$$[V_0{}^\alpha(x), A_a{}^\beta(y)] = [A_0{}^\alpha(x), V_a{}^\beta(y)] = if_{\alpha\beta\gamma}A_\alpha{}^\gamma(x)\delta^3(\vec{x}-\vec{y}), \quad (1.5)$$

$$[V_a{}^\alpha(x), V_b{}^\beta(y)] = [V_a{}^\alpha(x), A_b{}^\beta(y)] = [A_a{}^\alpha(x), A_b{}^\beta(y)] = 0. \quad (1.6)$$

Here S is a c number, $f_{\alpha\beta\gamma}$ are the structure constants of SU(3), a and b (or i and j) run over 1, 2, 3. $V_\mu{}^\alpha(x)$ and $A_\mu{}^\alpha(x)$ are the octets of vector- and axial-vector currents. ($\alpha, \beta = 1, \cdots 8$, $\mu = 0, 1, 2, 3$). These currents are the dynamical variables in Sugawara's theory; the construction of a dynamical theory would consist of finding appropriate relations or equations for these entities. In this paper just the consequences of the Eqs. (1.1)-(1.6) will be studied. It should be noted that the crucial difference between the algebra of currents and the algebra of fields (apart from the Schwinger terms) lies in the commutation rule (1.6). In the algebra of currents the space components do not commute; instead,

$$[V_a{}^\alpha(x), V_b{}^\beta(y)] = i\delta_{ab}f_{\alpha\beta\gamma}V_0{}^\gamma\delta(\vec{x}-\vec{y}). \quad (1.7)$$

To obtain the sum rules we introduce the electric-dipole operator $\vec{E}^\alpha$ and the magnetic-quadrupole operator $M_{ij}{}^\alpha$:

$$\vec{E}^\alpha = \int d^3x\, \vec{r}V_0{}^\alpha(x), \quad (2.1)$$

$$M_{ij}{}^\alpha = \int d^3x\, r_i[\vec{r}\times\vec{V}^\alpha]_j. \quad (2.2)$$

Using the commutation rules (1.1)-(1.6), it is straightforward to obtain

$$[E_i{}^\alpha, E_j{}^\beta] = if_{\alpha\beta\gamma}\int d^3x\, r_i r_j V_0{}^\gamma, \quad (2.3)$$

$$[M_{ij}{}^{\alpha}, M_{lm}{}^{\beta}]=0, \tag{2.4}$$

$$[E_i{}^{\alpha}, M_{jk}{}^{\beta}]=if_{\alpha\beta\gamma}\int d^3x\, r_i r_j[\vec{r}\times\vec{V}^{\gamma}]_k. \tag{2.5}$$

Specializing to the case $\alpha=1$, $\beta=2$, and $i=j=k=l=m=3$ one obtains

$$[E_3{}^1, E_3{}^2]=i\int z^2 V_0{}^3 d^3x, \tag{2.3$'$}$$

$$[M_{33}{}^1, M_{33}{}^2]=0, \tag{2.4$'$}$$

$$[M_{33}{}^1, E_3{}^2]=i\int z^2(\vec{r}\times\vec{V}^3)_3 d^3x. \tag{2.5$'$}$$

It is important to note that (2.3′) and (2.5′) are exactly the same as the corresponding expressions derived using current algebra. However, (2.4′) is characteristically different; its counterpart in the algebra of currents is

$$[M_{33}{}^1, M_{33}{}^2]=i\int z^2(x^2+y^2)V_0{}^3 d^3x. \tag{2.4$''$}$$

Now take the expectation value of (2.3′)-(2.5′) between proton states at rest and insert a complete set of states. The right-hand sides of the expressions obtained can be interpreted in terms of derivatives of the isovector electric and magnetic form factors of the nucleon[5] (see also Hand[6]) as

$$\sum_n\{\langle p|E_3{}^1|n\rangle\langle n|E_3{}^2|p\rangle-\langle p|E_3{}^2|n\rangle\langle n|E_3{}^1|p\rangle\}=i\int z^2\langle p|V_0{}^3(x)|p\rangle d^3x=-\tfrac{1}{2}\left[\frac{dG_E{}^V(K^2)}{dK^2}\right]_{K^2=0}=\tfrac{1}{12}\langle r_V{}^2\rangle, \tag{2.6}$$

$$0=\sum_n\{\langle p|M_{33}{}^1|n\rangle\langle n|M_{33}{}^2|p\rangle-\langle p|M_3{}^{22}|n\rangle\langle n|M_{33}{}^1|p\rangle\}, \tag{2.7}$$

$$\sum_n\{\langle p|M_{33}{}^1|n\rangle\langle n|E_3{}^2|p\rangle-\langle p|E_3{}^2|n\rangle\langle n|M_{33}{}^1|p\rangle\}=i\int d^3x\,\langle p|z^2(xV_2{}^3-yV_1{}^3)|p\rangle=\frac{i}{2M}\left[\frac{dG_M{}^V(K^2)}{dK^2}\right]_{K^2=0}. \tag{2.8}$$

$\langle r_V{}^2\rangle$ is the charge radius of the isovector form factor. Note that the current algebra merely replaces (2.7) by

$$\sum_n\{\ \}=i\int z^2(x^2+y^2)\langle p|V_0{}^3|p\rangle d^3x=2\left[\frac{d^2G_E{}^V(K^2)}{d(K^2)^2}\right]_{K^2=0}. \tag{2.7$'$}$$

Here $G_E{}^V$ and $G_M{}^V$ are the (Sachs) form factors defined by

$$G_{E,M}{}^V(K^2)=G_{E,M}{}^p(K^2)-G_{E,M}{}^n(K^2), \tag{2.9}$$

G_E and G_M are (both for protons and neutrons) defined as

$$G_E(K^2)=F_1(K^2)-\frac{K^2}{4M^2}\frac{F_2(K^2)}{\mu}G_m(K^2)=\mu F_1(K^2)+F_2(K^2). \tag{2.10}$$

F_1 and F_2 are the "usual" form factors, M the nuclear mass, and μ the nuclear magneton. Now assume that the sums in (2.6)-(2.8) could be saturated by one term; since the $N^{**}(1518)$ has the correct quantum numbers, it is a reasonable choice. Then, using the Wigner-Eckart theorem, the matrix elements can all be expressed in terms of $\langle N^{**}|E_{33}{}^3|p\rangle$ and $\langle N^{**}|M_{33}{}^3|p\rangle$. One obtains

$$|\langle N^{**}|E_{33}{}^3|p\rangle|^2=-\tfrac{1}{2}[dG_E{}^V(K^2)/dK^2]_{K^2=0}, \tag{2.11}$$

$$|\langle N^{**}|M_{33}{}^3|p\rangle|^2=0, \tag{2.12}$$

$$\langle N^{**}|E_3{}^3|p\rangle\langle p|M_{33}{}^3|N^{**}\rangle=(i/2M)[dG_M{}^V(K^2)/dK^2]_{K^2=0}. \tag{2.13}$$

For comparison, (2.7′) yields

$$|\langle N^{**}|M_{33}{}^3|p\rangle|^2=2[d^2G_E{}^V(K^2)/d(K^2)^2]_{K^2=0}. \tag{2.12$'$}$$

Again (2.12) is the characteristic expression for the algebra of fields. Equations (2.11), (2.12), and (2.13) yield directly

$$\left(\frac{dG_M{}^V(K^2)}{dK^2}\right)_{K=0}=0, \tag{2.14}$$

or

$$\left(\frac{dG_M{}^p(K^2)}{dK^2}\right)_{K=0}=\left(\frac{dG_M{}^n(K^2)}{dK^2}\right)_{K=0} \tag{2.15}$$

(theoretically), while experimentally

$$\left(\frac{dG_M{}^p(K^2)}{dK^2}\right)_{K=0}=-0.30\pm0.002,$$

$$\left(\frac{dG_M{}^n}{dK^2}\right)_{K=0}=+0.20\pm0.008. \tag{2.16}$$

This is a clear conflict. It is to be contrasted to the consistency condition in current algebra obtained from (2.11), (2.12′), and (2.13):

$$\frac{1}{6}\langle r_V{}^2\rangle\left(\frac{d^2G_E{}^V(K^2)}{d(K^2)^2}\right)_{K=0}=\frac{1}{4M^2}\left(\frac{dG_M{}^V(K^2)}{dK^2}\right)_{K=0}^2, \qquad (2.17)$$

which gives experimentally $2.7\times10^{-3}=2.8\times10^{-3}$, a much better result. If one retains a sum over all one-particle states one can again use the Wigner-Eckart theorem in (2.7) to express all matrix elements in terms of $\langle p|M_{33}{}^3|n\rangle$, with the result

$$\sum_n c_n|\langle p|M_{33}{}^3|n\rangle|^2=0, \qquad (2.18)$$

where the c_n are positive constants. Hence all the matrix elements $\langle p|M_{33}{}^3|n\rangle$ vanish, and this in turn gives, via (2.8), the previous result (2.14).

It thus appears that if one can attribute any physical significance to the approximate saturation of commutators by single-particle states, the algebra of fields leads to serious discrepancies with experiment, while the current-algebra results seem to be reasonably good.

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HADRONIC MASS QUANTUM

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On the basis of chiral dual dynamics it is shown that the square of the mass of any nonexotic strange or nonstrange meson or baryon of nonvanishing isospin (with the possible exception of $I=1$ baryons) must be an integer multiple of the "quantum" $\frac{1}{2}m_\rho{}^2$. It is found than the strength of SU(3) breaking can only take certain discrete values.

By use of field-current identities, current algebra, and off-shell extensions of the Veneziano model, it has been recently possible to obtain closed expressions for electromagnetic form factors.[1] While extrapolation off the mass shell is by no means unambiguous, all these expressions are of the form

$$G(t)=\frac{\Gamma(1-\alpha_\rho(t))}{\Gamma(\frac{1}{2}n-\alpha_\rho(t))}P(t). \qquad (1)$$

Here $G(t)$ is the isovector electromagnetic form factor (we are for the time being limiting our discussion to $I\neq0$ hadrons); $\alpha_\rho(t)$ the Regge trajectory of the ρ meson,

$$\alpha_\rho(t)=\tfrac{1}{2}+t/2m_\rho{}^2; \qquad (2)$$

n a positive odd integer; and $P(t)$ a polynomial in t, so normalized that

$$\frac{\Gamma(1-\alpha_\rho(0))}{\Gamma(\frac{1}{2}n-\alpha_\rho(0))}P(0)=I_3, \qquad (3)$$

I_3 being the third component of the isospin of the hadron. The various off-the-mass-shell extrapolation procedures used by different authors affect only the detailed form of $P(t)$. The crucial feature of Eq. (1) is the fact that n is odd. This is a direct consequence of the origin of Eq. (1) in the soft-pion limit of the amplitude for $\pi H\to a^{(0)}H$ [$a^{(0)}$ is the $\Delta Y=0$, $\Delta I=1$ axial-vector current; H is the hadron the form factor of which is given by (1)]. In this limit only states of normality opposite to that of H contribute; and because of the quantization condition of Regge trajectories,[2] this leads to n being odd. We shall show that this simple result leads to extremely strong constraints on the hadron spectrum. In particular it relates the masses of baryons to those of mesons and quantizes the scale of SU(3) breaking. Our result is that (A) the square of the mass of any (nonexotic) hadron (be it a meson or a baryon) of nonvanishing isospin (with the possible exception of $I=1$ baryons) must be an integer multiple of $\frac{1}{2}m_\rho{}^2$. By nonexotic we mean any meson obtainable as $q\bar{q}$ and baryon obtainable as qqq, in other words all ($|B|\leq1$) hadrons known at present with the possible exception of