$2S(\Xi_-^-)+S(\Lambda_-^0)-\sqrt{3}S(\Sigma_0^+)=0$. For pc decays, on the other hand, the $\Delta I=\frac{1}{2}$ rule remains valid for $\Omega^-\to\Xi\pi$, $\Sigma\to N\pi$, and $K\to 3\pi$, but we have not been able to derive the same rule for Λ and Ξ decays in the presence of SU(3) breaking.

We close this note with a few remarks: (a) As stated before, the amplitude corresponding to (1) was assumed to satisfy the duality principle that governs hadronic processes. This assumption, though not yet verified, may not be unreasonable since for such quasihadronic processes there is so far no evidence for a fixed pole singularity usually present in nonhadronic amplitudes. Furthermore, we note that the Pomeranchuk singularity is not involved in our case. (b) It is known that for baryon-antibaryon scattering,[5] such as $\Delta\bar{\Delta}\to\Delta\bar{\Delta}$, arguments similar to that which we employed here lead to the problem that the relevant amplitude should vanish. Frankly, we have no clear-cut solution to this problem.[12] We note, however, that our case rather corresponds to meson-baryon scattering, for which we have met no such difficulty.

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[6]In a recent paper Suzuki attempted, from a similar point of view, to explain some properties of pc baryon decays [M. Suzuki, Phys. Rev. Letters 22, 1217, 1413(E) (1969)].

[7]For reactions involving octet spurions we do not meet such situations. The number of conditions that prohibit exotic resonances in certain channels is always smaller than the number of nonexotic amplitudes in its dual channels.

[8]For $K\to 3\pi$ decays we require, as an additional assumption, the factorization property of five-point amplitudes.

[9]We consider, in place of (1), the scattering involving spurions with $I=\frac{3}{2}$, $|Y|=1$, and $|I_Z|=\frac{1}{2}$.

[10]In the present case, there are three independent amplitudes to describe Σ decays since the $\Delta I=\frac{3}{2}$ transition is still effective in Σ^+ decays. However, it can be shown by explicit calculations that the triangle relation is satisfied.

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[12]It is our feeling that the difficulty associated with baryon-antibaryon scattering should be settled on quite different grounds. Several possibilities have so far been suggested to solve this problem. See S. Pinsky, Phys. Rev. Letters 22, 677 (1969); J. E. Mandula, J. Weyers, and G. Zweig, ibid. 23, 266 (1969).

PHOTON-PHOTON SCATTERING CONTRIBUTION TO THE SIXTH-ORDER MAGNETIC MOMENT OF THE MUON*

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We report a calculation of the three-photon–exchange (electron-loop) contribution to the sixth-order anomalous magnetic moment of the muon. Our result, which contains a logarithmic dependence on the muon-to-electron mass ratio, brings the theoretical prediction into agreement with the CERN measurements, within the 1–standard-deviation experimental accuracy.

In the last few years increasingly accurate measurements of the magnetic moment of the muon have been performed at CERN. The most recent value of the anomalous part of the muon g factor is[1] $[a=\frac{1}{2}(g-2)]$

$$a_{\rm exp}=(116\,616\pm31)\times10^{-8}. \quad (1)$$

The theoretical result for a which has been calculated thus far from standard quantum electrodynamics is

$$\alpha/2\pi + 0.765\,78(\alpha/\pi)^2 + 3.00(\alpha/\pi)^3. \quad (2)$$

The second term has been evaluated analytically up to and including terms of order $(\alpha/\pi)^2(m_e/m_\mu)^2$.[2] The last term consists of several parts: an estimate of the sixth-order contributions to the electron g factor $[0.13(\alpha/\pi)^3]$,[3] the contribution to the electron g factor from certain sixth-order Feynman diagrams not contained in the above estimate $[0.055(\alpha/\pi)^3]$,[4] and a calculation of those sixth-order terms which are generated by the insertion of electron loops of second and fourth order into the virtual-photon lines of the second- and fourth-order electromagnetic vertices of the muon $[2.82(\alpha/\pi)^3]$.[5-8]

The latest estimate of the contribution from strong interactions (vacuum polarization due to hadrons) to the muon g factor, based on the Orsay colliding-beam data for $e^+ + e^- \to \rho$, ω, and φ resonances, is[9]

$$\Delta a_{\rm had} = (6.5 \pm 0.5) \times 10^{-8}. \quad (3)$$

If one uses the value[10]

$$\alpha^{-1} = 137.036\,08 \pm 0.000\,26 \quad (4)$$

for the fine-structure constant, one obtains from (2) and (3) the theoretical prediction

$$a = (116\,564 \pm 2) \times 10^{-8}, \quad (5)$$

which disagrees slightly (1.7 standard deviations) with the experimental value (1). The error interval in (5) reflects the uncertainty in the strong-interaction contribution (0.5×10^{-8}), in the value of $\alpha/2\pi$ (0.2×10^{-8}), and in the sixth-order correction described in Ref. 7 (0.6×10^{-8}). It does not take into account the uncertainty in the magnitude of the vacuum-polarization contribution of higher mass hadrons.[11] We have also not included possible weak-interaction corrections[12] to the muon moment which could be expected to be of order 1×10^{-8}.

Also not included in the above error estimate is the contribution from the sixth-order diagrams containing photon-photon scattering subdiagrams (Fig. 1). This contribution has heretofore not been calculated and was assumed to be small. To examine the validity of such an assumption we have carried out an explicit calculation of this contribution. Our result turns out to be surprisingly large:

$$\Delta a_{\text{ph-ph}} = (18.4 \pm 1.1)(\alpha/\pi)^3 = (23.0 \pm 1.4) \times 10^{-8}. \quad (6)$$

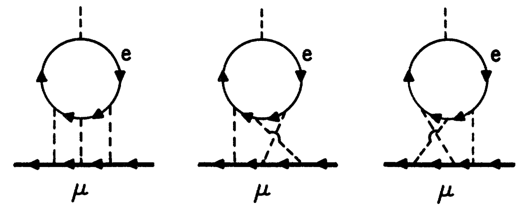

FIG. 1. Feynman diagrams containing subdiagrams of photon-photon scattering type. The heavy, thin, and dotted lines represent the muon, electron, and photon, respectively. There are three more diagrams obtained by reversing the direction of the electron loop.

This leads us to a revised theoretical prediction

$$a_{\rm theo} = (116\,587 \pm 3) \times 10^{-8} \quad (7)$$

and

$$a_{\rm expt} - a_{\rm theo} = (29 \pm 34) \times 10^{-8} = (250 \pm 290)\ \text{ppm}. \quad (8)$$

Thus the addition of the photon-photon scattering contribution essentially eliminates the discrepancy mentioned above. The theoretical error in (7) includes the uncertainty due to the numerical integration of the contribution (6) (1.4×10^{-8}). This error could be reduced if necessary. We wish to emphasize that, with the inclusion of the photon-photon scattering contribution (6), all of the Feynman diagrams from quantum electrodynamics which contribute to the difference of the muon and electron magnetic moments through sixth order have been calculated or bounded.[7]

The largeness of the contribution (6) is closely related to a logarithmic dependence on the muon-to-electron mass ratio. In fact, in the limit of large m_μ/m_e the result (6) can be expressed in the form

$$\Delta a_{\text{ph-ph}} = [(6.4 \pm 0.1)\ln(m_\mu/m_e) + \text{const}] \times (\alpha/\pi)^3. \quad (9)$$

Thus earlier arguments[5] indicating a cancellation among the diagrams of Fig. 1 for the logarithmic terms are disproved.

We have calculated all integrands contributing to the logarithmic term in (9) by hand. The complete integrand was obtained by two separate, dissimilar methods with the help of REDUCE,[13] an algebraic computation program developed by Hearn. The numerical integration over the Feynman parameters is carried out using a program written by G. Sheppey at CERN[14] and improved by one of us (A.J.D.). The term ±1.1 in (6) is our estimate of error in the numerical integration over seven Feynman parameters. In extracting the coefficient of the term $\ln(m_\mu/m_e)$ in (9), integration over two of the parameters is done analytically; the term ±0.1 in (9) is the estimated error in the integration over the remaining parameters. Both of these errors may be reduced if necessary.

Our method of calculation also allows us to obtain the correction to the g factor of the electron which arises from diagrams similar to those of Fig. 1. Details of both the muon and electron calculations will be published shortly.

The near agreement of theory and experiment in Eq. (8) can be used to obtain an interesting bound on the electromagnetic coupling to the entire spectrum of hadrons. The hadronic contribution to the muon g factor can be written as

$$\Delta a_{\rm had} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^\infty \sigma_{e_+e_-}(s)G(s)ds, \tag{10}$$

where

$$G(s) = \int_0^1 dz \frac{z^2(1-z)}{z^2+(1-z)(s/m_\mu^2)} \tag{11}$$

and $\sigma_{e_+e_-}(s)$ is the total e^+e^- annihilation cross section into hadrons at the center-of-mass energy $E_{\rm c.m.} = E_+ + E_- = \sqrt{s}$. We assume that the contribution to $\Delta a_{\rm had}$ for s up to and including the φ resonance in $\sigma_{e_+e_-}$ is given by (3). Then for the higher mass hadrons ($s \geq s^* \gtrsim m_\varphi^2$) we define

$$\Delta a_{\rm had}{}^* = \frac{m_\mu^2}{12\pi^3}\int_{s^*}^\infty ds \frac{\sigma_{e^+e^-}(s)}{s} \times\left[1+O\left(\frac{m_\mu^2}{s^*}\ln\frac{s^*}{m_\mu^2}\right)\right]. \tag{12}$$

If we regard the difference (8) as a sort of upper limit for $\Delta a_{\rm had}{}^*$, we obtain

$$\int_{s^*}^\infty \sigma_{e^+e^-}(s)ds/s < 8.2\mu{\rm b}.$$

In other words, higher mass hadrons can at most give three or four times the contributions of the ρ, ω, and φ to $\Delta a_{\rm had}$. A reduction of the experimental error for the muon moment could not only further confirm the quantum-electrodynamic corrections, but it would also further bound the cross section for e^+e^- annihilation.

We are greatly in debt to Dr. A. C. Hearn for providing his algebraic computation program REDUCE and his assistance in its use. One of us (T.K.) would like to thank Dr. K. C. Wali for the kind hospitality at Argonne National Laboratory and Dr. S. D. Drell for the kind hospitality at the Stanford Linear Accelerator Center, where part of this work was done. Two of us (S.J.B. and A.J.D.) wish to thank Dr. S. D. Drell for his suggestions and encouragement.

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