GeV^2). This ratio is consistent with the decrease expected for baryon exchange.

Barger and Cline[2] have made predictions for $\bar{p}+p\rightarrow\pi^{\mp}+\pi^{\pm}$ based on fits of a Regge-pole model with nucleon and Δ_δ exchange to the recent data on $\pi^{\pm}$-p backward elastic scattering of Orear et al.[10] For $\bar{p}+p\rightarrow\pi^-+\pi^+$ they find two solutions depending on the relative sign of the two exchange amplitudes. Predictions of the model are given in Table I and Fig. 2. While the overall average of the data lies higher then either prediction by two standard deviations, the agreement is better with solution 2 in the small-$|t|$ region. There is good agreement with their prediction for $\bar{p}+p\rightarrow\pi^++\pi^-$.

We wish to express our appreciation for the helpful discussions we have had with G. Renninger, K. V. L. Sarma, and V. Barger concerning the theoretical interpretation of these data, and to S. Singh for his help in the data analysis.

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FINE STRUCTURE IN THE MESON SPECTRUM

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The predictions of an algebraic model are compared with the experimental meson mass spectrum.

One of the most interesting results in meson spectroscopy is the fine structure in the mass spectrum.[1-7] It appears that the spectrum for the charged (probably $I=1$ and $Y=0$) mesons below 2400 MeV falls into four bands, the ρ band, the A_2 band, the R band, and the STU band, and that the fine-structure splitting inside each band increases from the lower to the higher band.[8]

The mass splitting between the different bands—or, more precisely, between the highest members of each band—has been described recently by an algebraic structure A_1,[9] which is essentially the Majorana representation of a relativistic symmetry $\mathfrak{S}=\mathcal{P}\dashv SO(3,2)$[10] broken by a non-Lie algebraic relation (generalized infinite-component wave equation). The mass spectrum which was derived in Ref. 9 is the rotator spectrum

$$M^2=\lambda^2\alpha^2-(9/4)\lambda^2+\lambda^2s(s+1), \tag{1}$$

where the symmetry-breaking constant λ has the value $\lambda^2 = 0.29$ BeV^2 and α^2 is a parameter that describes the representation of A_1, i.e., characterizes the physical system of which ρ, A_2^H, R_4, and U are different states.

The problem we want to consider here is whether the splitting inside each band can be described in an analogous manner, i.e., whether there is an algebraic structure that describes the whole spectrum including the fine structure. To do this we certainly need a new quantum number that distinguishes the various levels in the same band. There are two ways to accomodate such a new quantum number: (1) to choose a larger algebra[11] and (2) to choose a more complicated representation. We shall try here the second way for which there is already an analogy in atomic physics: The states of the nonrelativistic hydrogen atom are described by the degenerate series representation of SO(4, 1), and to obtain also the fine structure one has to use a more complicated representation of the same SO(4, 1).[12]

As the space-time part remains the same we will have to look for a more general representation of the spectrum-generating group SO(3, 2) of $\mathfrak{S}$.

Two kinds of representations of the relativistic symmetry $\mathfrak{S} = \mathcal{P}_{L_{\mu\nu}, P_\mu} \vdash \mathrm{SO}(3,2)_{\Gamma_\mu S_{\mu\nu}}$ are well known, one of which is the finite-dimensional Dirac representation (in which Γ_μ is represented by $\frac{1}{2}\gamma^\mu$ and $S_{\mu\nu}$ by $\frac{1}{2}\sigma_{\mu\nu}$) and the other the four Majorana representations,[13] the most degenerate unitary representations. It is useful to characterize the representations of $\mathfrak{S}$ by the multiplicity pattern of Ehrman[14] that describes which eigenvalue of the spin operator

$$M^{-2}\mathcal{W}_\mu \mathcal{W}^\mu = s(s+1)$$

$$(\mathcal{W}^\mu = \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}P_\nu L_{\rho\sigma}, \quad M^2 = P_\mu P^\mu) \tag{2}$$

and which of the operator

$$M^{-1}P_\mu \Gamma^\mu = n \tag{3}$$

appear in an irreducible representation. E.g., the multiplicity pattern for the Dirac representation $\mathfrak{S}^{(\mathrm{Dirac})}$ is given in Fig. 1(a), and those for the Majorana representations $\mathfrak{S}^{(\mathrm{Majorana})}$ are given in Figs. 1(b) and 1(c). Here the first number in each box gives the value for n and the second number gives the value for s; the line between the boxes indicates that there are nonzero matrix elements of elements of the Lie algebra (Γ_i) between these states. The letter in the boxes gives the particle symbol which has been assigned to these states (cf. Ref. 9 for the Majorana representation). In the Majorana representation n is not an independent quantum number but $n = s + \frac{1}{2}$. In the Dirac representation the sign of n distinguishes between particle and antiparticle.

After this introduction we can now present the model that describes the fine structure. We shall only state the results here and compare them with experimental data. The derivation of the representation in analogy to Refs. 9 and 13 for the Majorana representation and a theoretical discussion of the symmetry-breaking relation (mass formula) has to be deferred to a more detailed mathematical paper.

We call $\mathfrak{S}^{(R)}$ the representation of $\mathfrak{S}$ which is characterized by the following eigenvalues of the Casimir operators:

$$-\Gamma_\mu \Gamma^\mu - \tfrac{1}{2}S_{\mu\nu}S^{\mu\nu} = R, \tag{4}$$

$$Q_2 = -\omega_a \omega^a = \tfrac{1}{4}R(R-2) \tag{5}$$

$$(\omega^a = \tfrac{1}{8}\epsilon^{abcde}S_{bc}S_{de}, \quad a = 5, 0, 1, 2, 3, \quad S_{5\mu} = \Gamma_\mu),$$

where R is a real constant with $R > 2$; and whose multiplicity pattern is given in Fig. 2. From the multiplicity pattern we see that this representation space of $\mathfrak{S}$ contains all integer spins $s = 0, 1, 2, \cdots$, and the new quantum number n has for a given s the spectrum $n = s, s-1, \cdots, -s$.

Before we can assign the meson resonances to each box of the multiplicity pattern we have to make some assumptions about the symmetry

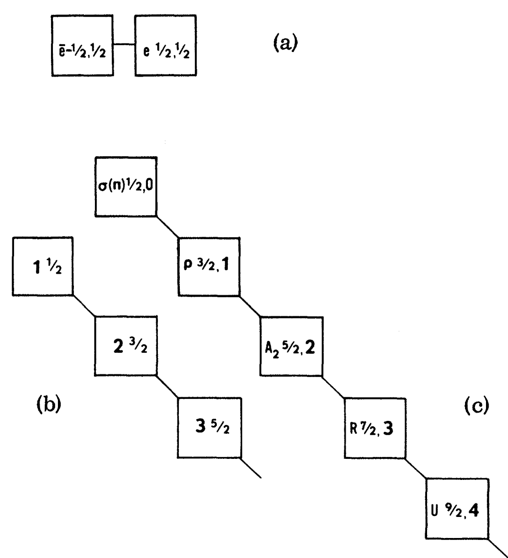

FIG. 1. Multiplicity pattern for the following representations of $\mathfrak{S}$ or $\mathrm{SO}(3,2)_{S_{\mu\nu}\Gamma_\mu}$: (a) Dirac representation. (b) Majorana representation with half-integer spin ($k_0 = \frac{1}{2}, c = 0$). (c) Majorana representation with integer spin ($k_0 = 0$, $c = \frac{1}{2}$). The letters in (a) and (c) give a possible particle assignment.

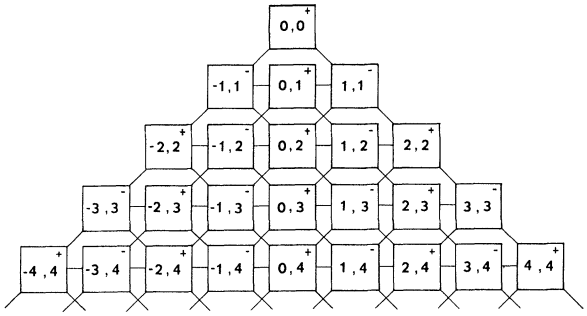

FIG. 2. Multiplicity pattern of the integer-spin singleton representation $\mathfrak{S}^{(R)}$ of $\mathfrak{S}$ or $SO(3,2)_{S_{\mu\nu}\Gamma_\mu}$.

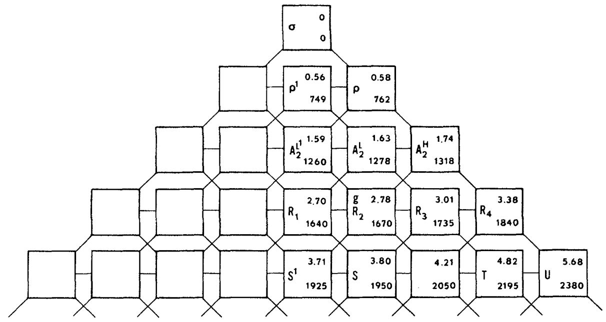

FIG. 3. Multiplicity pattern of $\mathfrak{S}^{(R)}$ with the possible particle assignment and predicted masses. The number in the right upper corner of each box is the mass squared in BeV^2 and the number in the right lower corner is m in MeV.

breaking. The mass formula (1) can be obtained by taking the expectation value of the following operator equation between the particle states:

$$P_\mu P^\mu = m_0^2 + (P_\mu P^\mu)^{-1} \mathfrak{W}_\mu \mathfrak{W}^\mu \lambda^2,$$

where m_0^2 is constant in an irreducible representation. If we replace λ^2 by an operator which has a constant value in the Majorana representation and has a nontrivial spectrum in $\mathfrak{S}^{(R)}$, then we obtain for $P_\mu P^\mu$ a relation which reproduces for the Majorana representation the old result (1) but which gives a mass splitting between states with different value of n but the same s. Such an operator is $\lambda^2 \to \lambda_1^2 - \lambda_2^2 (P_\mu P^\mu)^{-1}(\mathfrak{W}_\mu \mathfrak{W}^\mu - P_\mu \Gamma^\mu P_\rho \Gamma^\rho)$ (where λ_1^2 and λ_2^2 are two constants of dimension BeV^2) because in $\mathfrak{S}^{(\text{Majorana})}$ $(P_\mu P^\mu)^{-1}(\mathfrak{W}_\mu \mathfrak{W}^\mu - P_\mu \Gamma^\mu P_\rho \Gamma^\rho) = -\frac{1}{4}$.[13]

So we postulate as the symmetry-breaking relation for the mass operator

$$P_\mu P^\mu = m_0^2 + M^{-2} \mathfrak{W}_\mu \mathfrak{W}^\mu \times \left[\lambda_1^2 - \lambda_2^2 \left(\frac{\mathfrak{W}^\mu \mathfrak{W}_\mu}{M^2} - P \frac{P_\mu \Gamma^\mu P_\rho \Gamma^\rho}{M^2} \right)\right]. \quad (7)$$

The expectation value of this operator equation in the canonical states $|p_i, n, s, s_3\rangle$ of the irreducible representation space of $\mathfrak{S}^{(R)}$ will then lead to the mass formula

$$m^2 = m_0^2 + \lambda_1^2 s(s+1) - \lambda_2^2 s(s+1)\{s(s+1) - n^2\} \quad (8a)$$

or

$$m^2 = m_0^2 + (\lambda_1^2 - \lambda_2^2 s) s(s+1) - \lambda_2^2 s(s+1)\{s^2 - n^2\}. \quad (8b)$$

Equation (8b) gives for the states with $s^2 = n^2$ the old rotator spectrum, if $\lambda_2^2 \ll \lambda_1^2$ as should be the case for a fine-structure constant. So we shall assign to the boxes with $n = s$ the particle states σ, ρ, A_2^H, R_4, and U, and λ_1^2 is determined from the spectrum of their masses to be

$$\lambda_1^2 = 0.30 \pm 0.01 \text{ BeV}^2. \quad (9)$$

σ corresponds to the one-particle state with the hadron quantum numbers of the vacuum introduced by Böhm and Sudarshan,[15] m_σ^2 is taken to be 0, and $s^P = 0^+$. We remark that the mass difference between the σ state and the π state is of the order of the fine structure (cf. remark below).

The mass fine-structure constant λ_2 is then determined from the mass splitting inside the various bands; a value which gives a good fit to the experimental data is

$$\lambda_2^2 = 0.0061 \pm 0.0002 \text{ BeV}^2. \quad (10)$$

The masses that we predict with the values (9) and (10) of λ_1^2 and λ_2^2 from the mass formula (8) are given in the boxes of the multiplicity pattern in Fig. 3. In each box of the multiplicity pattern of Fig. 3 we have given m^2 in BeV^2 in the right upper corner, m in MeV in the lower corner, and the particle symbol for the possible particle assignment. Comparing these predicted masses with the experimental data we see that the agreement is remarkable. The only apparent discrepancy is in the mass value for R_2, which we have here identified with the g meson of $s^P = 3^-$, and whose predicted mass is 20 MeV below the value one would expect from the experimental data.[1-3,16]

The mass of the state with the quantum numbers $(n, s) = (0, 0)$ has been assumed to be zero, i.e., $m_0^2 = 0$ in (8). An equally good fit of the mass spectrum is obtained if we choose $m_0^2 = m_\pi^2$, so that from this point of view we could equally well have assigned the π meson to the $(n = 0, s = 0)$ state. However, this would lead to difficulties connected with the parity. The parity operator for the singleton representations of

SO(3, 2) is given by[17]

$$\Pi = e^{i\pi\Gamma_0}; \quad \eta = +1 \text{ or } -1. \tag{11}$$

The resulting parities for our representation with $\eta = +1$ are indicated by a plus or minus sign in the boxes of the multiplicity pattern in Fig. 2. For $\eta = -1$ we would just have to reverse all the signs in Fig. 2 and could then have assigned π to the $(n = 0, s = 0)$ state. However, the parity value of A_2 decided for the choice of $\eta = +1$. With this choice for η, the g meson with $s^P = 3^-$ had to be assigned to the $(n = 1, s = 3)$ state.

To summarize the predictions of our model:

(1) S, T, and U have spin 4 and alternating parity, and there is an additional $s^P = 4^+$ resonance at approximately 2050 MeV.

(2) The S bump is split. Latest experimental data seem to confirm this.

(3) The R mesons have $s = 3$. If R_2 is not identical with the g meson, then the R_1 bump is split and g is the upper part of the R_1 bump. [For the latter possibility cf. Figs. 2(D) and 3(B) of Ref. 1 and Fig. 1, and also Fig. 2 of Ref. 3.]

(4) A_2^L is split into a 2^+ and 2^- state [cf. Fig. 5(B) Ref. 1 with Fig. 3(A) of Ref. 2]. The present best experimental resolution in the A_2 region is 10 MeV, which is just not enough to resolve the predicted splitting of the A_2^L.

(5) The ρ bump contains two ρ states with opposite parity. The experimental situation for the A_2 and R bands suggests that all states in the same band have equal G parity. Then $G = +1$ for $\rho'(1^+)$ and ρ' can at best decay into 4π. There is no experimental evidence against the existence of a narrow 4π resonance in the ρ region.

There remain some serious theoretical problems connected with our model besides the one associated with the interpretation of the σ state. One of the problems is the interpretation of the states with sign $n = -1$. For baryons these states would naturally be interpreted as the antiparticle states in analogy to their interpretation in the Dirac representation.[17] But what about the meson resonances; are these antimesons?

In the above-presented way the fine structure in the meson spectrum could be described at the cost of the introduction of a second constant besides the mass-splitting constant λ. It would be, of course, much more appealing if there were only one universal constant of the dimension of MeV. [We note that the same constant λ also describes the SU(3) mass splitting for mesons; cf. Ref. 15 with references thereof.] However, it is apparent that mass differences of the order of the A_2 splitting or, also, of the π-meson mass cannot simply be described by a constant which is one order of magnitude larger than these quantities, as is the case for $\lambda_1^2 = 0.285$ BeV^2.

The difference from zero of m_π^2 is certainly also a fine-structure effect. In fact, if we assume that the fine-structure effect for the intrinsic quantum numbers is proportional to the second-order Casimir operator $\mathcal{C}$ of SU(3),[9] then the mass difference between the σ state of Ref. 15 and the π state is given by $\lambda_2^2[\mathcal{C}(\pi) - \mathcal{C}(\sigma)] = \lambda_2^2(\nu_\pi^2 + 2\nu_\pi) = 0.0061 \times 3$ BeV$^2 \sim m_\pi^2$.

But perhaps this and even all the number combinations above are just accidental.

It was the enthusiastic encouragement of B. C. Maglić that initiated this investigation. He also contributed valuable information about the experimental situation.

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[16] We want to remark that the predictions are determined from differences of experimental values with large errors, so that the values for the masses given here are not to be taken too seriously; what is, however, to be taken seriously are the mass differences in one band, and in this respect our predictions disagree with the experimental missing-mass spectrometer data (Refs. 1-3).

[17]W. H. Marlow, private communication. It appears that this is the most general form for π. In the Dirac representation, $\Gamma_0 \to \frac{1}{2}\gamma_0$, (11) reproduces the well-known result $\pi^{\mathrm{Dirac}} = \eta e^{i\pi\gamma_0/2} = \eta\gamma_0$.

[18]We remark that there is a representation of $\mathfrak{S}$ very similar to the present representation (with the same eigenvalues of the Casimir operators) but containing only half-integer spins. This representation offers itself for the description of baryons, in particular since the lowest Poincaré group representations that it contains are the same as are contained in $\mathfrak{S}^{(\mathrm{Dirac})}$. This would then be able to describe fine structure in the baryon spectrum as, e.g., given by the $\frac{5}{2}^+$ $N(1688)$ and $\frac{5}{2}^-$ $N(1680)$.

DUALITY, ABSENCE OF EXOTIC RESONANCES, AND THE $\Delta I = \frac{1}{2}$ RULE IN NONLEPTONIC DECAYS

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Based on the current-current theory of weak interactions the $\Delta I = \frac{1}{2}$ rule for nonleptonic decays is derived from a dynamical mechanism that satisfies duality and absence of resonances in exotic channels.

It has been argued,[1] based on universal current-current theory of weak interactions, that the $\Delta I = \frac{1}{2}$ rule in nonleptonic decays may be accounted for either by adding an extra current-current term so as to cancel the $\underline{27}$ portion of the interactions or by a dynamical mechanism that selectively enhances the octet component. The usual mechanism for the octet enhancement, often referred to in terms of "tadpoles,"[2] has been discussed in a variety of ways.[3] None of them, however, appears convincing enough.

In this note we wish to point out that the $\Delta I = \frac{1}{2}$ rule or more precisely, the absence of $\Delta I = \frac{3}{2}$ transitions may be understood as a dynamical mechanism that satisfies duality[4] and the hypothesis of no exotic resonances.[5]

The nonleptonic decays are described as a quasihadronic scattering process in which the weak interaction acts like a scalar and a pseudoscalar spurion carrying zero energy and momentum. We assume, as in the case of ordinary hadron scattering, that they are governed by a dynamics that satisfies duality and the hypothesis of absence of exotic resonances.[6]

Let us first consider octet baryon decays in the SU(3)-symmetric limit. It is then easy to show, under the foregoing assumptions, that

$$S(\underline{27}) + B(\underline{8}) \to M(\underline{8}) + B(\underline{8}) \qquad (1)$$

is forbidden. Here $S(\underline{27})$ stands for scalar or pseudoscalar 27-plet spurion with $I = \frac{3}{2}$ and $|Y| = 1$. The argument that leads to the above conclusion is quite simple: There are six independent amplitudes, corresponding to the SU(3) representations of $\underline{8}$, $\underline{8}'$, $\underline{10}$, $\underline{10}^*$, $\underline{27}$, and $\underline{27}'$ for each parity-nonconserving (pv) and parity-conserving (pc) decay. The same representations arise in all s, t, and u channels. Absence of exotic resonances implies that there be no $\underline{10}^*$, $\underline{27}$, and $\underline{27}'$ in both s and u channels and no $\underline{10}$, $\underline{10}^*$, $\underline{27}$, and $\underline{27}'$ in the t channel. For each pv and pc amplitude, $s(u)$- and t-channel duality then requires seven linearly independent SU(3) amplitudes to vanish and therefore requires that they all vanish. Similarly, s- and u-channel duality gives six independent conditions on six amplitudes, which again requires that all amplitudes must vanish. Thus $\Delta I = \frac{3}{2}$ transitions for baryon decays are forbidden.[7]

It is straightforward to apply the present arguments to $K \to 2\pi$ (3π)[8] and $\Omega^- \to \Xi\pi$ decays. One then obtains the result that $\Delta I = \frac{3}{2}$ transitions for the above processes are again forbidden and therefore these decays should obey the $\Delta I = \frac{1}{2}$ rule.

We have so far assumed, besides our basic assumptions of duality and absence of exotic resonances, the exact SU(3) symmetry. Let us next consider the case of broken SU(3).[9] In that case, it is not difficult to show that similar arguments can still be applied to $\Omega^- \to \Xi\pi$, $\Sigma^- \to n\pi^-$, and $K \to 2\pi$ (3π) decays. As a result, the $\Delta I = \frac{1}{2}$ rule is still valid for these decays. An interesting fact is that absence of a $\Delta I = \frac{3}{2}$ transition for $\Sigma^- \to n\pi^-$ implies the well-known triangle relation $\sqrt{2}\Sigma_0^+ + \Sigma_+^+ = \Sigma_-^-$ and thus effectively the $\Delta I = \frac{1}{2}$ rule for Σ decays.[10]

At this point, let us recall[11] that for pv decays, soft-pion calculations based on $SU(2) \otimes SU(2)$ demand the $\Delta I = \frac{1}{2}$ rule for Λ and Ξ decays but not for Σ decays. It is then an interesting observation that if we combine our results with current-algebra predictions, we are led to the conclusion that all pv decay amplitudes, including Ω^- and K decays, satisfy the $\Delta I = \frac{1}{2}$ rule in broken SU(3), with the additional restrictions $S(\Sigma_+^+) = 0$ and