sion scale. The peak of the calculated density of states corresponds to the peak in the emission spectrum, which is probably quite reasonable. Burdick's calculated values for the Fermi energy and the energy of some of the high-summetry points are shown in Fig. 3. It would not be reasonable, at the present time, to assign any of the structure to definite critical points. Single-crystal data should hopefully elucidate this information. However, it is of interest to note that on the high-energy side of curve 3, Fig. 3, there is an abrupt increase of intensity at 933.8 eV and a small peak at 933.5 eV. This is the same as Burdick's calculated value of 933.5 eV for the Fermi energy.

The advantages of this technique over those which attempt to determine band structure, such as photoemission spectroscopy,[7] ion neutralization,[8] and optical and piezo-optical measurements,[9,10] are apparent in that the transitions are well defined (one of the states being an atomic level) and the specimen surface is no longer a critical consideration. Once the spectrum of the pure metal is understood, the application of this technique to the study of alloys will be useful in understanding their electronic structure.

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# PHONON FOCUSING IN SOLIDS*

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In phonon-propagation studies using heat pulses it has been found that predominantly transverse phonons are observed for [100] propagation in LiF whereas mainly longitudinal phonons are observed for this direction in KCl. This result and similar effects previously reported on other crystals are explained in terms of a phonon-focusing effect that arises from elastic anisotropy.

The characteristics of phonon propagation in dielectric solids are conveniently studied by means of heat (phonon) pulses.[1-5] These pulses are produced by passing short-duration (typically $10^{-7}$-sec) pulses of current through a resistive metal film (the "source") evaporated onto a selected area of the crystal under study. The phonon pulses are detected, after traveling through the crystal, at the face opposite the source. The detection of the arriving phonons is achieved by measuring the change in the electrical resistance of a superconducting film maintained very near its transition temperature. In the present investigation sources consisting of Constantan films, and detectors composed of an indium-tin alloy (94% In, 6% Sn), were used. The measurements were carried out in the temperature range $2^\circ\mathrm{K} \lesssim T \lesssim 3.6^\circ\mathrm{K}$ using a magnetic field to vary the transition temperature of the detector. The crystals were ultraviolet-grade LiF and optical-grade KCl supplied by the Harshaw Chemical Company.

In these experiments separate pulses are observed due to transverse and longitudinal phonons, these pulses arriving at different times because of the different velocities of the phonons. We have found that the relative amplitudes of these pulses depend very strongly on the propagation direction and that in the [100] direction the predominant mode differs in KCl and LiF. Figure 1 shows a pulse observed for propagation in the [100] direction in LiF. The velocity of the phonons associated with this pulse is that of

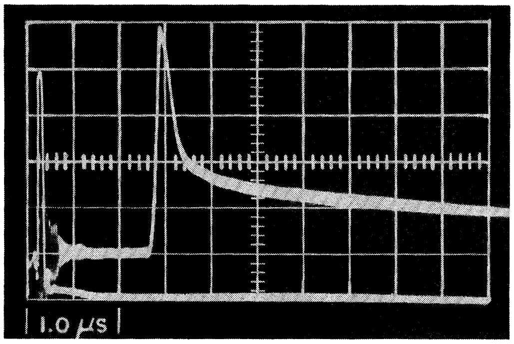

FIG. 1. Oscilloscope trace of heat pulse in the [100] direction in LiF. Lower trace shows input pulse. Upper trace shows detected and amplified heat pulse. Path length, 6 mm. Only the transverse pulse is observed in this trace. Temperature, 3.3°K.

transverse waves in the [100] direction. For the amplification used in obtaining the trace shown in Fig. 1 no pulse due to longitudinal phonons is observable. In contrast to LiF, we observed that longitudinal phonons predominate in pulses propagating in the [100] direction in KCl. In the [110] direction for LiF the fast transverse mode predominates over the longitudinal and the slow transverse by factors of about 10 and 20, respectively. A considerably greater amplitude longitudinal pulse is observed in the [110] direction than in the [100] for LiF.

In order to determine whether differences in attenuation of various modes were responsible for these results, measurements were performed on crystals of different thickness; it was found that the predominance of the modes discussed above was not significantly affected by changes in path length. We conclude, therefore, that these results are not due to differences in attenuation.

Other experimental results showing large differences in the amplitudes of the various modes have been previously reported. Pomerantz and von Gutfeld[6] reported propagation of heat pulses in silicon and germanium in the [100] direction in which the transverse-mode amplitude is very much greater than the longitudinal, and Rogers and Rollefson[7] observed a longitudinal amplitude of about one-quarter of the transverse amplitude in NaF.

We are proposing here that elastic anisotropy accounts for the observed differences in the propagation behavior of phonons in these solids, as discussed in detail below. It should be noted, however, that Pomerantz and von Gutfeld[6] have reported changes in the ratio of transverse to longitudinal pulse amplitude in Ge and Si as a function of impurity content. The present results, obtained on alkali halides, correlate well with the proposed phonon-focusing effect; the same effect also accounts for the observations on Ge and Si with low impurity content. It is concluded, therefore, that the impurity effect which leads to differences in attenuation for different modes is quite distinct from the one discussed here.

In an elastically anisotropic solid the direction of energy flow associated with a plane wave does not, in general, coincide with the direction of the wave vector. It follows that even when waves are excited in a given region of a crystal with a uniform angular distribution of wave vectors, the energy flow will be enhanced in some directions and decreased in others with respect to the average. This enhancement or decrease can be calculated for any direction in a crystal from the relation between the directions of the wave vector $\vec{k}$, or the phase velocity $\vec{V}_p$, and the corresponding directions of energy flow or group velocity $\vec{V}_g$ which Musgrave and Miller[8,9] have derived. By using their expressions, a computer calculation was carried out of the directions of the group velocity associated with a large number of wave vectors. These computations covered wave-vector directions spanning, and extending at least 30° in every direction beyond, the irreducible symmetry triangle for cubic crystals. Conventional spherical coordinates were used with $25° < \theta < 120°$ and $-30° < \varphi < 75°$, and wave vectors were taken at 1.25° intervals in $\theta$ and $\varphi$. The number of group-velocity vectors of each of the three modes lying within solid angles defined by increments of 2° in $\theta$ and $\varphi$, and hence the degree of enhancement or focusing of each mode in that direction, was determined. Corrections were applied for changes in the density of wave vectors and the sizes of the solid angles as the angle $\theta$ departs from 90°. Group velocities of the same mode lying within the same solid angle but differing in magnitude, i.e., corresponding to different portions of the group-velocity surface in the vicinity of a cusp, were not distinguished. In many cases the differences in magnitudes resulted in transit time differences which were below the resolution of the experimental technique. The result of the computation was obtained for each mode as a ratio of phonon intensity for various crystal directions to the intensity expected in an isotropic solid.

It was found that for LiF the longitudinal phonons were enhanced in the [111] and [110] directions by factors of 2.5 and 1.5, respectively, and

Table I. Summary of calculated and experimental results.

| Solid | Propagation direction | Amplitude ratio, longitudinal:fast-transverse:slow-transverse Calculated | Observed |
|---|---|---|---|
| LiF | [100] | 1:40[a] | ~1:100 |
| LiF | [110] | 6:60:1 | 2:20:1[b] |
| KCl | [100] | 60:1[a] | ~7:1 |

[a] Transverse modes degenerate.

[b] See text for discussion of corrections.

decreased in the [100] direction by a factor of 4. The fast transverse phonons were concentrated about the [100]-[110] zones with a maximum in the [110] direction of about 15 times the average. The slow transverse phonons were concentrated mainly about the [100] direction by a factor of approximately 10 and decreased in the [110] direction by a factor of 4. Thus in LiF in the [100] direction the transverse pulse is expected to be larger than the longitudinal one by a factor of about 40, in qualitative agreement with the experimental results. In the [110] direction the ratios of fast transverse to longitudinal and slow transverse are expected to be about 10 and 60, respectively. However, the fast transverse intensity falls off abruptly for deviations greater than 1° from the [110] direction towards the [001] direction, and thus a reduction in the above factors is accounted for in terms of the finite size of the detector. The angle subtended was approximately ±3° about the propagation direction; this leads to a correction which brings the observed and calculated intensities into close agreement.

The calculation showed that for KCl in the [100] direction the longitudinal phonons were enhanced by a factor of 20, and the transverse phonons were decreased by a factor of 3, giving a ratio of 60:1 for these modes, again in qualitative agreement with the experimental observations. A summary of the results is given in Table I and the effect is represented schematically in Fig. 2.

It is emphasized that the above factors refer to focusing effects due to elastic anisotropy only. In comparing the experimentally observed amplitudes of different modes, differences in the rates of emission from the source must also be considered. The modes are emitted roughly in inverse proportion to the squares of their velocities, which leads to a greater overall abundance of the transverse modes. The variations of this effect for different directions within a crystal are, however, small in comparison with the focusing effect.

It may be noted that the similarity between the

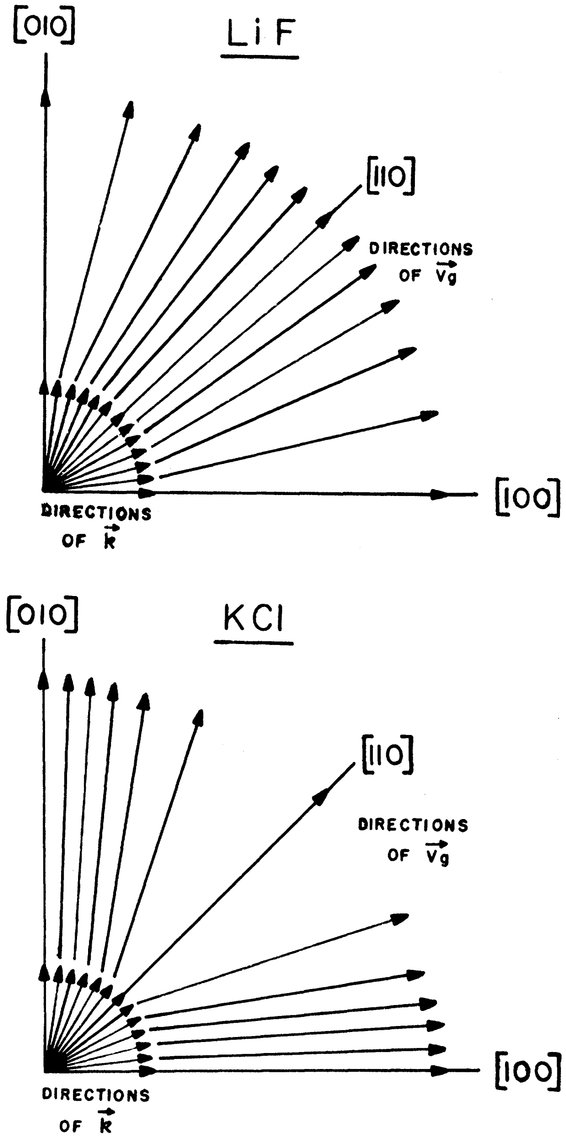

FIG. 2. Diagram showing deviations in directions of longitudinal group velocities $\vec{V}_g$ from their corresponding wave vectors $\vec{k}$ in the [100]-[010] zone for LiF and KCl. Focusing is apparent in the [110] direction in LiF and in the [100] and [010] directions in KCl. In fact the group-velocity vectors emanate from a point. The wave vectors have been included to emphasize the deviations.

heat pulses in the [100] direction in pure LiF, Si, and Ge is also qualitatively accounted for by the above theory. For LiF the ratio of the elastic constants $C_{11}$:$C_{12}$:$C_{44}$ at low temperatures is 1:0.34:0.52, whereas for Si and Ge these ratios are 1:0.39:0.48 and 1:0.38:0.52, respectively. The deviation between the group and phase velocity does not depend on the magnitude of the elastic constants, but only on their ratios. Since these ratios are very similar for LiF, Si, and Ge, one expects similar heat-pulse propagation in these solids. In contrast, the ratio for KCl is 1:0.11:0.14. Since for NaF the ratio is 1:0.21:0.27, its heat-pulse propagation properties are expected to be intermediate between those of LiF and KCl, which was indeed observed.[7]

The experimental results are consistent with the above theory; phonon focusing in solids has thus been demonstrated. An investigation of heat-pulse propagation in other directions in LiF and KCl is currently being conducted.

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# CHARGE AND SPIN SUSCEPTIBILITY OF A FERROMAGNETIC ELECTRON GAS

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We have obtained the charge and spin susceptibilities in the ferromagnetic state of an electron gas, including the effects of the long-range nature of the Coulomb interaction from a unified point of view. The susceptibilities are discussed as a function of the magnetization for typical ferromagnetic alloy systems.

In this Letter we present a general formulation for the charge and spin responses of a ferromagnetic electron gas to a charge potential or magnetic field. The long-range effects of the Coulomb interaction are adequately accounted for within the random-phase approximation while insuring charge conservation. Besides obtaining the charge susceptibility to a charge potential, $\chi_{ee}$, and the spin susceptibility to a magnetic field, $\chi_{mm}$, we also derive two additional nondiagonal susceptibilities, i.e., the spin response to a charge potential $\chi_{me}$ and the charge response to a magnetic field $\chi_{em}$. The nondiagonal susceptibilities do not appear in the paramagnetic state and have not been previously treated from a general viewpoint. To interpret the electron spin polarization and magnetic moment produced by impurities in ferromagnets Friedel[1] and others[2] have presented a physical picture for the spin imbalance associated with the screening charge about an impurity in a ferromagnetic metal which in a sense corresponds to $\chi_{me}$. Since the nondiagonal susceptibility we derive can be as important as the usual diagonal susceptibility, both must be included in any analysis of the total charge or spin responses of ferromagnets.

We use the following Hamiltonian to calculate the spin and charge response functions for an interacting electron gas:

$$\mathcal{H}=\sum_{l,\sigma}\epsilon_l c_{l\sigma}{}^\dagger c_{l\sigma}+\tfrac{1}{2}\sum_{\substack{l,l',\kappa\\ \sigma,\sigma'}}{}' V(\kappa)c_{l\sigma}{}^\dagger c_{l'\sigma'}{}^\dagger c_{l'-\kappa,\sigma'}c_{l+\kappa,\sigma}+\mathcal{H}_\alpha{}' \quad (\alpha=m \text{ or } e), \tag{1}$$

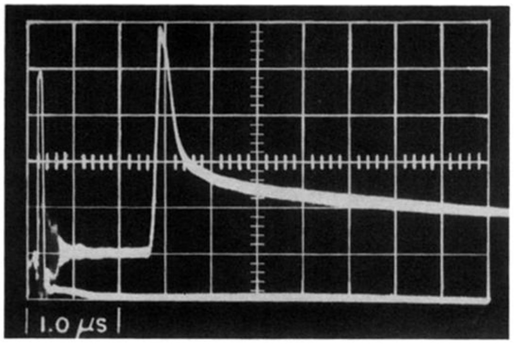

FIG. 1. Oscilloscope trace of heat pulse in the [100] direction in LiF. Lower trace shows input pulse. Upper trace shows detected and amplified heat pulse. Path length, 6 mm. Only the transverse pulse is observed in this trace. Temperature, 3.3°K.