NEW STATE OF FERROMAGNETISM IN DEGENERATE ELECTRON GAS AND MAGNETIC FIELDS IN COLLAPSED BODIES

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A new state of "ferro"magnetism in a degenerate electron gas is found and shown to be stable. This magnetism is the sum of all microscopic magnetic moments associated with electrons in their respective Landau levels while the Landau levels of the system are in turn maintained by this macroscopic magnetization. The maximum field in the Landau orbital ferromagnetism state is 10^7 G for white-dwarf densities and 10^{12} G for neutron-star densities.

Recently there has been considerable interest in problems of intense magnetic fields in gravitationally collapsed bodies. Although the physical properties of an electron gas in a magnetic field are now well understood,[1,2] the origin of the field is still uncertain.[3] Up to now, magnetic fields in astrophysics are believed to be produced by a current in the form of drifting charges, but such a current is subject to resistive dissipation.

In this Letter we present a new mechanism as a possible source for a strong magnetic field in gravitationally collapsed bodies. This is a new state of self-consistent magnetization of a degenerate electron gas. This self-consistent macroscopic magnetization is the sum of microscopic magnetic moments associated with all electrons in their respective Landau levels while the Landau levels of the system are in turn maintained by the macroscopic magnetization of the system, which will be referred to as LOFER (Landau orbital ferromagnetism).

The magnetic induction B of a system is given by the well-known relation $B=H+4\pi M$, where H is the field due to a true current. In our case the magnetization M is a function of B and since we are considering the current-free case with $H=0$, the value of M in a LOFER state is then given by the solutions of the self-consistent relation $B=4\pi M(B)$, or alternatively, by the relation $M=M(4\pi M)$, where $M(B)$ or $M(4\pi M)$ is derived by using the interaction term of the Hamiltonian, $-\vec{J}\cdot\vec{A}$, where the induced current $\vec{J}$ and the vector potential $\vec{A}$ are due to (Landau) orbital motions of all electrons. This LOFER state is quasistable, in the sense that the probability of making a macroscopic transition into another state of different magnetization is extremely small.

This LOFER state exists for a degenerate electron gas of all densities, but here we shall consider only the nonrelativistic case which can be discussed analytically. Since the discreteness of the Landau levels is vital to the existence of the LOFER state, the level broadenings due to impurities and other effects are assumed to be small and these conditions are fully fulfilled in condensed celestial bodies such as white dwarfs or neutron stars.[4] Also we will ignore the spin part of the Hamiltonian since this would not qualitatively affect our conclusions. As we shall see, the most interesting regime is that in which there are many Landau levels below the Fermi level, namely $\omega_c<\mu$ ($\hbar=1$), where μ is the chemical potential (in the degenerate case μ is also the Fermi energy) and $\omega_c=eB/mc$ is the cyclotron frequency, and other symbols have their usual meanings. In this case, the magnetization M per unit volume is given by $M^{(\mathrm{n.o.})}+M^{(\mathrm{osc})}$, where $M^{(\mathrm{n.o.})}$ is a smooth nonoscillatory function associated with the semiclassical diamagnetism of an electron gas and $M^{(\mathrm{osc})}$ is an oscillatory function associated with the discreteness of the Landau levels,[5] and

$$M^{(\mathrm{n.o.})}=-[ep_\mathrm{F}/3(2\pi)^2\beta c]\beta\omega_c[1+O(\omega_c/\mu)+O^2(1/\beta\mu)],$$

$$M^{(\mathrm{osc})}=-\left[\frac{ep_\mathrm{F}}{2\pi\beta c}\right]\left(\frac{2\mu}{\omega_c}\right)^{1/2}\sum_{l=1}^{\infty}\frac{(-)^l}{l^{1/2}}\frac{\sin[2\pi l(\mu/\omega_c)-\frac{1}{4}\pi]}{\sinh(2\pi^2 l/\beta\omega_c}\left[1+O\left(\frac{\omega_c}{\mu}\right)+O\left(\frac{1}{\beta\mu}\right)\right], \tag{1}$$

where p_F is the Fermi momentum and $\beta^{-1}=kT$. As was suggested by Shoenberg[6] and subsequently discussed by many others,[7] the field B appearing in previous expressions through the cyclotron frequency ω_c is the magnetic induction B. This dependence of ω_c on M gives rise to the equation $M=M(H+4\pi M)$. In the following we shall show that there are nonzero solutions for M even when H vanishes. These nonzero solutions are then the self-consistent macroscopic magnetization associated with the LOFER state.

It can be easily shown that, if $M=M^{(\mathrm{n.o.})}$, then only the trivial solution $M=0$ is allowed. Thus, in order to have nonzero solutions it is necessary that $M^{(\mathrm{osc})}\gg M^{(\mathrm{n.o.})}$. As we shall see, this condition can be satisfied when $\mu/\omega_c\gg 1$.

For mathematical simplicity we will consider the regime where $x\equiv 2\pi^2/\beta\omega_c\sim 1$. Other regimes including the relativistic case can be analyzed accordingly, but we are yet unable to carry out the analysis in an analytical form. Numerical solutions have been found and will be published separately.[8] Thus we can write $M^{(\mathrm{osc})}$ as

$$M^{(\mathrm{osc})}\cong(ep_F/2\pi\beta c)(2\mu/\omega_c)^{1/2}\sin[(\beta\mu/\pi)x-\tfrac{1}{4}\pi](\sinh x)^{-1}. \tag{2}$$

It is then easily seen that when the condition

$$(2\mu/\omega_c)^{1/2}|\sin[(\beta\mu/\pi)x-\tfrac{1}{4}\pi]|\gg x^{-1}\sinh x \tag{3}$$

is satisfied, then $M^{(\mathrm{n.o.})}\ll M^{(\mathrm{osc})}$ and we can write $M\cong M^{(\mathrm{osc})}$. Noting that $\omega_c=4\pi(eM/mc)$, the relation $M=M(4\pi M)$ is satisfied with nonzero solutions if

$$\alpha\pi^{-3}(\beta\mu)^{1/2}(v_F/c)>x^{-3/2}\sinh x, \tag{4}$$

where v_F is the Fermi velocity of the electron gas and $\alpha=e^2/\hbar c$ is the fine-structure constant. As we restrict ourselves to $x\simeq 1$, the inequality (4) becomes

$$\beta\mu>1.31(\pi^3c/\alpha v_F)^2\sim 10^7(c/v_F)^2 \tag{5}$$

which is the sufficient condition for the existence of the LOFER state for the regime of temperature and density under consideration. When Eq. (5) is satisfied, a set of discrete solutions for the relation $M=M(4\pi M)$ exist. However, because of the sine function on the right-hand side of Eq. (2) and of the large factor $(\beta\mu/\pi)$ appearing in its argument, these nonzero solutions appear in one cluster and the separation of M between two neighboring solutions is proportional to $(\omega_c/\mu)M$. Since the existence of the LOFER state will be more favorable in the regime $x\equiv 2\pi^2/\beta\omega_c\ll 1$ than for the regime we are considering, our analysis gives a value of M lower than the maximum value of nonzero solution allowable to a given temperature and density.

As the temperature is increased $M^{(\mathrm{n.o.})}$ will dominate and therefore a temperature exists above which LOFER state will disappear, and the electron gas will become diamagnetic. This gives the transition temperature T_c between LOFER and diamagnetic states. For the present case T_c is given by Eq. (5) with the equality sign. [Note that Eq. (5) is inapplicable to neutron stars or white dwarfs because of the prevalence of relativistic condition.] One should also note that the geometry of the Fermi surface can strongly affect this transition temperature. In this case the transition temperature T_c is given by

$$\frac{KT_c}{\mu^*}=\cos^2\left(\frac{\pi m^*}{m}\right)\left(\frac{\alpha}{\pi^3}\right)^2\left(\frac{2\mu^*}{m^*c^2}\right)(x^*)^{3/2}\times[\sinh x^*]^{-1}2\pi\left\{\left|\frac{\partial^2S(\mu,p_z)}{\partial p_z^{\,2}}\right|_e\right\}^{-1}. \tag{6}$$

Here $S(\mu,p_z)$ is the cross-sectional area of the Fermi surface at p_z (M is assumed to be in the z direction) and the subscript e indicates that the expression is evaluated at the extreme orbitals.[5] μ^* and m^* are given by $2\pi m^*=[\partial S(\mu,p_z)/\partial\mu]_e$ and $2\pi m^*\mu^*=[S(\mu,p_z)]_e$. x^* is x evaluated with starred quantities. Thus the transition temperature can be strongly enhanced for a system of electrons with a Fermi surface having a small cyclotron mass and a large radius of curvature in the direction perpendicular to the direction of M at the extremal orbits. In certain metals the transition temperature may be sufficiently high for the LOFER state to be observable in the laboratory.

To discuss the stability of the LOFER state, we consider the Gibbs free energies $\tilde{G}(H)$ and $G(B)$ per unit volume.

The free-energy density $G(B)$ is given by

$$G(B)=\Omega(B)+B^2/8\pi, \tag{7}$$

where the thermodynamic potential[9] $\Omega(B)$ per unit volume is related to the magnetization M by $\partial\Omega(B)/\partial B=-M$ and therefore it can be written in the form $\Omega(B)=\Omega^{(\mathrm{n.o.})}(B)+\Omega^{(\mathrm{osc})}(B)$. The Gibbs free-energy density $\tilde{G}(H)$ determines the free energy of the system at a given configuration of the true current. At $H=0$,

$$\tilde{G}(H)=G(B). \tag{8}$$

Since $\partial G/\partial B=H/4\pi$, the LOFER state corresponds to the extreme of $G(B)$. Because of the oscillatory behavior of $\Omega^{(\mathrm{osc})}(B)$, there exist successive maxima and minima for $G(B)$. The minima then correspond to the stable configurations of the LOFER state.

Thermodynamically speaking, because of Eq. (8) a minimum with a lower value of $G(B)$ is more stable than that with a larger value of $G(B)$. However, in order for the system to make a macroscopic transition from one stable LOFER state to a neighboring stable LOFER state, an energy barrier $\Delta\Omega_B$ per unit volume must be overcome by the system.

The probability for a subsystem to cross over this barrier is proportional to $\exp(-V\Delta\Omega_B/kT)$. Here V is the volume over which M can fluctuate considerably. The lower bound for this volume cannot be smaller than that of a sphere whose radius is the cyclotron radius $R=v_\mathrm{F}/\omega_c$. Then, it can easily be shown that the condition for stability $\exp(-V\Delta\Omega_B/kT)\ll 1$ is equivalent to the following condition.

$$\beta V\Delta\Omega_B \gg \frac{\alpha^{-1}c}{v_\mathrm{F}} \quad \text{for} \quad \frac{T_c-T}{T_c} > \left(\frac{\omega_c}{\mu}\right)^{1/2} \tag{9}$$

which is automatically satisfied in the regime under discussion. Therefore, the LOFER state is quasistable against a macroscopic transition in the sense explained previously.

Equation (5) suggests that the LOFER state can also occur favorably for a relativistic degenerate electron gas. Analyzing the results given by two of the authors,[2] we have obtained, for example, for an electron gas with $\mu\simeq 0.5$ MeV, a set of LOFER states with a self-consistent magnetic field up to 10^7 G. A list of numerical solutions for M in the LOFER state will be published elsewhere.[8]

As for a possible source for a permanent strong magnetic field in a gravitationally collapsed matter such as a neutron star, let us consider an initial field before collapse in the neighborhood of 10^3 G. This field is due to true currents in the form of drifting charges. As time passes by, the neutron star will cool down and after a period of 10^6 yr the internal temperature may be as low as 10^4 °K. The LOFER state will then dominate, and then, even if the true current is already dissipated as suggested by some researchers, the LOFER state will take over to sustain a field as high as 10^{12} G. The total number of extinct white dwarfs with strong magnetic fields can be estimated from their duration of being observable and from the fractional number of observed white dwarfs. It is estimated that there are almost as many extinct white dwarfs as stars in our galaxy. Analogously, the number of extinct pulsars (neutron stars) is estimated to be 1/100 of the number of stars in our galaxy. We therefore suggest that our galaxy may be populated by as many as 10^{11} extinct white dwarfs of fields up to 10^7 G and by as many as 10^9 extinct pulsars (neutron stars) of fields up to 10^{12} G. The presence of these strongly magnetized bodies should be of vital importance in determining the acceleration of cosmic rays and in affecting the magnetic properties of our galaxy.

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BRANCHING RATIO AND POSSIBLE CP NONCONSERVATION IN RADIATIVE $K_{\pi 2}$ DECAY

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We have determined with 90% confidence an upper limit of 1.90×10^{-4} for the branching ratio of the decay mode $K^+\to\pi^++\pi^0+\gamma$ in the range of T_{π^+} from 55 to 80 MeV. We discuss the implications of this result for a possible CP nonconservation in radiative $K_{\pi 2}$ decay.

In this Letter we report an upper limit to the branching ratio of the decay mode $K^+\to\pi^++\pi^0+\gamma$. Interest in this mode has been aroused by the possibility[1,2] that a comparison of $K^\pm\to\pi^\pm+\pi^0+\gamma$ might reveal asymmetries which violate CP invariance while still satisfying TCP. The several phenomenological analyses[3,4] have been based on the work of Good.[5] They assume that the decay matrix element contains a direct-emission term which is of the same order as the inner-bremsstrahlung term. The contribution made to the decay by interference between these terms could show a CP-nonconserving asymmetry.

If one assumes further[2-5] that the dominant direct-emission terms are $E1$ and $M1$ and sums over the photon polarization, then interference occurs only between the inner-bremsstrahlung and $E1$ terms. The branching ratio may then be written as

$$R^\pm=B^2[1+\gamma C_1\cos(\delta_{11}-\delta_{20}\pm\varphi)+C_2(\gamma^2+\beta^2)],\quad(1)$$

where B^2 is the branching ratio for pure inner bremsstrahlung and the second and third terms represent the contributions to the branching ratio from the direct-emission terms.[3] γ and β are real positive numbers which give, respectively, the amplitudes for $E1$ and $M1$ direct emission relative to inner bremsstrahlung. In the terminology of Good,[5] $\gamma=gA\mu^4/GM^4$ and $\beta=2gB\mu^4/M^4$. The δ_{IJ} are $\pi\pi$ phase shifts taken at the appropriate energies and φ is a CP-nonconserving phase. B^2, C_1, and C_2 are functions of the kinematic limits for the decay. For T_{π^+} between 55 and 80 MeV and all allowed π^0 energies, $B^2=1.36\times10^{-4}$, $C_1=1.80$, and $C_2=2.00$.[6]

For a comparison of the K^+ and K^- decay modes, the branching-ratio asymmetry is given by

$$A=\frac{R^+-R^-}{R^++R^-}=\frac{\gamma C_1\sin(\delta_{11}-\delta_{20})\sin\varphi}{1+\gamma C_1\cos(\delta_{11}-\delta_{20})\cos\varphi+C_2(\gamma^2+\beta^2)}.\quad(2)$$

Clearly, in order to produce a CP-nonconserving effect it is necessary that γ be nonzero, and it is of considerable interest to know the limits which can be placed on it.

The present experiment is part of a series of stopping-K^+ experiments performed at Nimrod. The apparatus (Fig. 1) has been described elsewhere.[7] Kaons were brought to rest in the beryllium-plate spark chamber. The momentum of the outgoing π^+ was measured by a magnetic spectrometer. Pions were separated from muons by their range-momentum correlation. Electrons were identified by the Cherenkov counter. The three outgoing gamma rays were detected in the spark chambers $B1$-$B4$, which formed four sides of a cube centered at the beryllium chamber. Each chamber contained 35 brass plates 0.12 radiation lengths thick.

614 π^+ were identified with a spectrometer momentum between 105 and 170 MeV/c. After double scanning 41 were found to have three or four gamma rays. These events were measured and the gamma-ray energies estimated from spark counting. The events were least-squares fitted. No events satisfied the hypothesis of $K^+\to\pi^++\pi^0+\gamma$ at the 1% probability level with T_{π^+} between 55 and 80 MeV.[8] The rate for three- or four-gamma-ray events could be entirely accounted for by assuming that they were due to background gamma rays in the spark chambers accompanying a $K_{\pi 2}$ decay in which the π^+ interacted in the beryllium chamber before entering the spectrom-