the mean free paths for heavy particles in hydrogen. With the present view that cosmic-ray particles, on the average, have passed through about 3 g/cm^2 of hydrogen, the effect on the heaviest particles is clearly large, and a far more extensive knowledge of the fragmentation parameters is needed, in addition to better experimental statistics.

Our intention, in this Letter, has been to draw attention to a powerful new technique for cosmic-ray studies and its first results. The various figures which have been given, even in the absence of reliable values of fragmentation parameters, permit us to estimate definite lower limits to the fluxes of the VVH particles, and provide, for the first time, a flux estimate for the charge region $33 \leq Z \leq 40$.

We wish to thank Professor P. H. Fowler of the University of Bristol for his willingness to receive one of us (W.C.W.) into his laboratory to perform the photometric measurements on the Barndoor-II tracks. We also wish to acknowledge the splendid support of the National Center for Atmospheric Research Scientific Balloon Flight Station, Palestine, Texas.

*Work supported in part by the National Aeronautics and Space Administration, the U. S. Army Research Office (Durham), and the McDonnell-Douglas Company.

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POLE-EXTRAPOLATION RESULTS FROM $pp \to \Delta^{++}n$ AT 6.6 GeV/c *†

Z. Ming Ma, Gerald A. Smith, and R. J. Sprafka
Physics Department, Michigan State University, East Lansing, Michigan 48823

and

Eugene Colton and Peter E. Schlein
Physics Department, University of California, Los Angeles, California 90024

(Received 25 June 1969)

We present an experimental study of the low–momentum-transfer $\Delta^{++}n$ component of 6424 $pp \to p\pi^+n$ events at 6.6 GeV/c. The π^+p elastic cross sections in the Δ^{++} region are measured by means of several different pole-extrapolation procedures. We find that the conventional Chew-Low extrapolation procedure yields results not in satisfactory agreement with the known on-shell cross sections. We suggest a modified extrapolation procedure which in our case yields results in good agreement with the on-shell values.

The proper extraction of $\pi\pi$ and $K\pi$ scattering cross sections from πp and Kp experimental data is a subject of increasing importance in many high-energy experiments. Lack of sufficient statistics in single-momentum experiments and ambiguities in pole-extrapolation procedure provide compelling reasons to study pp reactions from which the already known πp elastic cross sections can be obtained. These pp studies would allow, for example, a determination of the minimum statistics necessary to get reliable results and a comparison with the results using several different extrapolation procedures. In short, until it can be shown that πp cross sections can be reliably extracted from pp experiments, the credibility of $\pi\pi$ and πK results will be in doubt. We present in this Letter results of a study of the $(p\pi^+)$ effective-mass and momentum-transfer dependences of the reaction

$$pp \to p\pi^+n \qquad (1)$$

at 6.6 GeV/c from which the elastic π^+p on-shell cross sections are successfully obtained in the Δ^{++} region.[1]

6424 examples of Reaction (1) were obtained in an exposure of the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber to the external proton beam from the Beavatron. We find a cross section for Reaction (1) of 5.73 ± 0.35 mb. In the analysis which follows, we confine the discussion to those 1750 events[2] which simultaneously satisfy the selection criteria

$$t < 0.3 \text{ GeV}^2, \quad 1.14 < M < 1.42 \text{ GeV}, \tag{2}$$

where M is the $p\pi^+$ effective mass and t is the lower of the two possible values of momentum transfer to the outgoing $p\pi^+$ system. (We take t to be positive in the physical region.) The cross section for the $p\pi^+n$ events satisfying the cut (2) is found to be 1.50 ± 0.12 mb. The experimental differential cross sections of $d\sigma/dM$ and $d\sigma/dt$ for these events are shown in Figs. 1(a) and 1(b). The curves appearing in Fig. 1 are described below.

In order to display the momentum-transfer dependence at low t in the absence of a kinematic cutoff, we show in Fig. 1(c) $d\sigma/dt'$, where $t' = t - t_{\text{min}}$. The dashed curve is the result of a least-squares fit to the t' distribution for $0.03 < t' < 0.30$ GeV^2 of the assumed form $d\sigma/dt' = Ae^{Bt' + ct'^2}$. The best-fit parameters are $A = 21.3 \pm 2.3$ mb/GeV^2, $B = -16.1 \pm 1.9$ GeV^{-2}, and $C = 23.0 \pm 6.2$ GeV^{-4}. We note that the first bin point ($t' < 0.01$ GeV^2) is 4.3σ below this curve indicating that the low-t' data are compatiable with a drop-off in the forward direction.

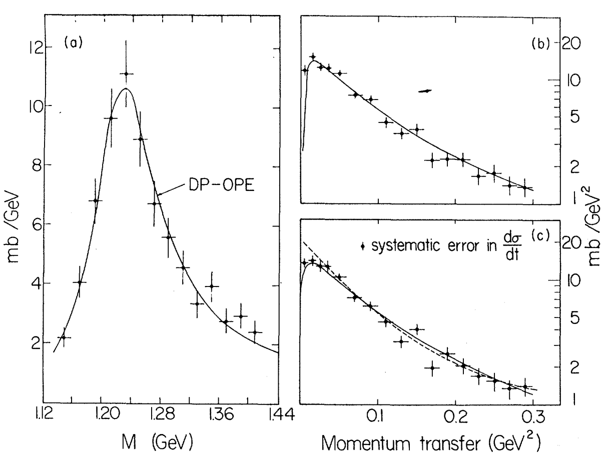

FIG. 1. (a) $p\pi^+$ effective-mass distribution $d\sigma/dM$ for $pp \rightarrow p\pi^+n$ at 6.6 GeV/c for momentum transfer to the neutron $t < 0.3$ GeV^2. The solid curve is calculated from DP-OPE, using Wolf's values for R_n, R_Δ, and c. (b) $d\sigma/dt$ for $1.14 < M < 1.42$ GeV. The curve is the t projection corresponding to the curve in (a). (c) $d\sigma/dt'$, where $t' = t - t_{\text{min}}$, for the same events plotted in (b). The dashed curve is the result of a fit to the data with $0.03 < t < 0.30$ GeV^2 of $ae^{bt' + ct'^2}$. The solid curve is the prediction of DP-OPE. The errors on the individual points in (b) and (c) are only the statistical $\sqrt{N}$ errors. The systematic cross-section error is also shown.

We have divided the data shown in Fig. 1 into nine bins of π^+p mass and extrapolated the relevant t-dependent quantities to the pion-exchange pole in Reaction (1) to obtain measurements of the on-shell cross section of the reaction

$$\pi^+p \rightarrow \pi^+p. \tag{3}$$

For Reaction (1), the pole equation or Chew-Low formula[3] is obtained from the differential cross section in M and t as $t \rightarrow -\mu^2$:

$$\lim_{t \rightarrow -\mu^2} \frac{d^2\sigma}{dtdM} = \frac{2}{4\pi m_p^2 P_{\text{lab}}^2} \frac{G^2}{4\pi} \frac{t}{(t+\mu^2)^2} \times M^2 Q \sigma(M). \tag{4}$$

P_{lab} is the laboratory beam momentum, m_p is the proton rest mass, μ is the pion rest mass, $G^2/4\pi \simeq 2 \times 14.6$, Q is the momentum in the Δ^{++} rest frame, and $\sigma(M)$ is the on-shell π^+p elastic-scattering cross section.

The conventional pole-extrapolation procedure to obtain the elastic cross section $\sigma(x\pi^+ \rightarrow x\pi^+)$ from a reaction of the type $xp \rightarrow x\pi^+n$ is to fit the ratio

$$\text{“}t\sigma\text{”} = \frac{(d\sigma/dt)_{\text{expt}}}{(1/t)(d\sigma/dt)_{\text{pole eq.}}} \tag{5}$$

to a polynomial in t. [When this ratio is properly extrapolated to the pion pole, it is equal to $t \times$ (on-shell $x\pi^+$ cross section).] Here $(d\sigma/dt)_{\text{expt}}$ is the experimental $d\sigma/dt$ for the mass bin ΔM in question, and $(d\sigma/dt)_{\text{pole eq.}}$ is the right-hand side of Eq. (4) evaluated assuming that $\sigma = 1$ mb and integrated over ΔM. A polynomial form $a + bt + ct^2 + \cdots$ is then fit to the experimental "$t\sigma$" points, statistics usually prohibiting the use of higher powers than quadratic. Because of the low statistics and relatively low beam momenta used thus far for extrapolation analyses, the data are not sensitive to the presence of a small nonzero constant term a. Thus a is usually constrained to be zero.[4]

We have followed this procedure[5] and fit our "$t\sigma$" points to the polynomial $bt + ct^2$ to obtain $\sigma(\pi^+p \rightarrow \pi^+p)$ for nine π^+p mass bins. These results are shown in column 4 of Table I. Whereas the "$t\sigma$" points are well fit by the $bt + ct^2$ expansion (for the nine fits we find a total χ^2 of 23 for 36 degrees of freedom—confidence level 95%), the extrapolated π^+p cross sections are

Table I. Extrapolated values for π^+p elastic cross section (in mb).

$\overline{M}$ (MeV)	KNOWN ON-SHELL VALUES OF σ_{π^+p} (mbarn)	EXTRAPOLATED CROSS SECTIONS σ_{π^+p}: DP-OPE "$t\sigma$" bt fit	Conventional "$t\sigma$" bt + ct^2 fit	Conventional "σ" b + ct fit
1164	63	62 ± 6	87 ± 13	88 ± 13
1190	127	130 ± 12	155 ± 21	158 ± 21
1210	188	179 ± 14	217 ± 22	219 ± 22
1230	197	193 ± 15	234 ± 22	240 ± 22
1249	167	150 ± 12	151 ± 17	152 ± 17
1269	123	110 ± 10	91 ± 13	93 ± 13
1298	82	75 ± 6	67 ± 7	69 ± 7
1340	49	47 ± 4	40 ± 5	41 ± 5
1390	29	29 ± 3	20 ± 4	21 ± 3
FWHM (MeV)	105	∿ 105	∿ 80	∿ 80
Fit to data	χ^2/DF (Prob.)	42/45 (60%)	23/36 (95%)	26/36 (88%)
Agreement with $\sigma_{\text{on-shell}}$	χ^2/DF (Prob.)	6/9 (76%)	30/9 (<0.1%)	31/9 (<0.1%)

not in satisfactory agreement with the on-shell values shown (the χ^2 for this equality is 30 for nine degrees of freedom—confidence level $\lesssim 0.1\%$). The difficulty is that the low-mass extrapolated cross sections are too large, while those at high mass are too small. Almost exactly the same results are found (shown in column 5 of Table I) using the approach of Marateck et al.[6] in which a linear fit $b+ct$ is made to the quantity

$$``\sigma" = \frac{(d\sigma/dt)_{\text{expt}}}{(d\sigma/dt)_{\text{pole eq.}}}. \tag{6}$$

As expected, $bt+ct^2$ fits to "$t\sigma$" are equivalent to $b+ct$ fits to "σ."

The essential difficulty in the conventional pole extrapolation described above is that considerably more data are needed to determine the higher order coefficients and/or constant term which are evidently required for a perfect extrapolation. This necessity for a more complex extrapolating function arises directly from the fact that the normalizing function used in the denominator of "$t\sigma$" has a t dependence quite different from that of $(d\sigma/dt)_{\text{expt}}$. If one could choose a normalizing function which has very nearly the same t dependence as $(d\sigma/dt)_{\text{expt}}$ (and which reduces to the pole equation as $t \to -\mu^2$), then a less complex function of t would be required to fit the "$t\sigma$" points. This point is most simply illustrated if one imagined that the normalizing function has exactly the t dependence of $(d\sigma/dt)_{\text{expt}}$; in this case "$t\sigma$" would be linear in t and have a slope equal to the on-shell cross section. In view of this point, it seems evident that use of the pole equation as a normalizing function unnecessarily increases the complexity of the required extrapolation function.

Recent successes[7-9] in fitting the Chew-Low distributions of a large class of reactions using the Dürr-Pilkuhn[10] modified pole equation (DP-OPE) prompt us to suggest that, in fact, DP-OPE would be a far superior choice of normalizing function than the pole equation itself. To illustrate the rather good agreement of DP-OPE with the data for Reaction (1), we show in Fig. 1 curves calculated using Eq. (4) modified for use in the physical region of t by the following DP vertex correction factors:

$$t \to t\left[\frac{1+R_n{}^2q^2}{1+R_n{}^2q_t{}^2}\right], \qquad (7)$$

$$Q\sigma(M) \to Q\left[\frac{(M+m_p)^2+t}{(M+m_p)^2-\mu^2}\right] \times \left\{\sigma_{s\,1/2}+\left(\frac{Q_t}{Q}\right)^2\left[\frac{1+R_\Delta{}^2Q^2}{1+R_\Delta{}^2Q_t{}^2}\right]\sigma_{p\,3/2}\right\}. \qquad (8)$$

$\sigma_{l,J}$ are the on-shell partial cross sections in π^+p elastic scattering ($\sigma_{p\,1/2}$ is very small[11] in the Δ^{++} region and can be ignored). Q_t (Q) are the incident (outgoing) proton momenta in the Δ^{++} c.m. system. Similarly, q_t is the momentum of the incident proton evaluated in the neutron rest frame and q is this quantity taken on-shell. In addition to these DP vertex factors which, it should be pointed out, are both mass and t dependent, we also use the "universal" weakly t-dependent form factor[12] $G(t)^2=[(2.3-\mu^2)/(2.3+t)]^2$ which Wolf found was necessary in addition to the DP factors in order to obtain good fits to the experimental distributions. R_n and R_Δ were set at Wolf's values, namely, 2.66 and 4.0 GeV^{-1}, respectively.[13] The DP-OPE curves are seen to reproduce the t and M dependence of the data quite well in both shape and normalization. In addition to reproducing the full width at half-maximum (~116 MeV) of the Δ^{++} in the physical region of Reaction (1), the model also is in approximate agreement with the drop-off in $d\sigma/dt'$ at small t'.

The near correctness of DP-OPE in describing $d\sigma/dt$ distributions in the physical region implies that its use as the normalizing function in pole extrapolation has great vitrue. To illustrate this point we show in Fig. 2 the quantity "$t\sigma$" $=(d\sigma/dt)_{\mathrm{expt}}[(d\sigma/dt)_{\mathrm{DP\text{-}OPE}}t^{-1}]^{-1}$ plotted versus t for the nine indicated π^+p mass bins. In calculating $(d\sigma/dt)_{\mathrm{DP\text{-}OPE}}$ for the extrapolation, we ignore the small $\sigma_{s\,1/2}$ term in Eq. (8) and assume that the entire π^+p cross section (unknown and set equal to 1 mb for the extrapolation) is multiplied by the factor shown for $\sigma_{p\,3/2}$.

The results of fitting the data points of Fig. 2 to the expression bt are shown in column 3 of Table I. The χ^2 for this linear fit to the "$t\sigma$" points is seen to be 42 for 45 degrees of freedom, giving a confidence level of ~60%. (There are a total of 54 data points and nine free b parameters in these fits.) Furthermore, the nine extrapolated π^+p cross sections give a χ^2 of 5.8 (confidence level ~76%) for the hypothesis that they

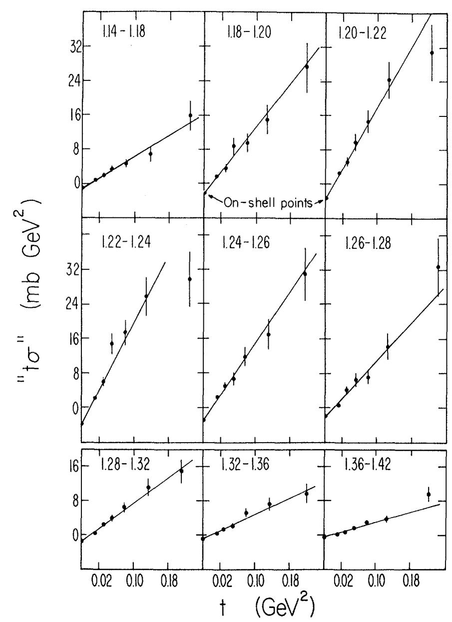

FIG. 2. The experimental quantities "$t\sigma$" defined in Eq. (7) in the text for the nine indicated regions of M. The lines represent the results of least-squares fits to the data using the assumed linear forms bt. Dots on the left side of each plot represent the known on-shell cross sections.

are equal to the known on-shell values given in column 2 of Table I. We find that neither constant nor quadratic terms are necessary to fit our data. We remark that one may not expect this to be always the case, for DP-OPE is surely not the cure-all of all pole-extrapolation work, but rather should be taken as a convenient but not precisely accurate summary of Chew-Low distributions for reactions of the types $xp\to x\pi^-\Delta^{++}$ and $xp\to x\pi^+n$. With higher statistics in our reaction, for example, if one fits "$t\sigma$" $=a+bt+ct^2$, small a and c terms which describe departures of of $(d\sigma/dt)_{\mathrm{DP\text{-}OPE}}$ from $(d\sigma/dt)_{\mathrm{expt}}$ probably will be present.[14]

We conclude from our analysis that the conventional method of using the pole equation (4) as the normalizing function in the extrapolation gives unreliable cross-section results when the "$t\sigma$" $=bt+ct^2$ expansion is used, at least for the reaction considered here. In contrast, however, DP-OPE provides a normalizing function which is so close to the real data that for the statistics presently available to us, no terms are necessary in the expansion to account for departures of DP-OPE from the experimental distributions. The extrapolated cross sections are found to be in excellent agreement with the expected values:

We note in particular that a full width at half-maximum of ~105 MeV results for the Δ, in good agreement with the on-shell value.

*Work supported in part by the U. S. Atomic Energy Commission (University of California, Los Angeles) and a National Science Foundation Science Development Grant (Michigan State University).

†Accepted without review under policy announced in Editorial of 20 July 1964 [Phys. Rev. Letters 13, 79 (1964)].

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$$``t\sigma" = \frac{c}{\int dt\,dM} \sum_{i=1}^{n} \left[\frac{t}{(d^2\sigma/dt\,dM)_{\text{pole eq.}}}\right]_i$$

to evaluate "$t\sigma$" for a given $\Delta t \Delta M$ bin. Here $c = 1500/1750$ μb/event, the sum is over the events in the sample, and the bracketed quantity is evaluated for each event. The integral $\int dt\,dM$ is over that portion of the $\Delta t \Delta M$ bin in question which is included in the physical region of the Chew-Low plot.

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[14]Fits to our data using constant and quadratic terms have also been performed. Although the coefficients of the constant and quadratic terms are consistent with zero in all cases, the constant term carries the same sign in the first five mass bins and the opposite sign in the upper four mass bins, indicating that perhaps we are already observing such an effect.

LEFT-HAND CUTS IN REGGE TRAJECTORIES*

J. S. Ball†
University of Utah, Salt Lake City, Utah

and

F. Zachariasen
California Institute of Technology, Pasadena, California 91109
(Received 9 June 1969)

It is suggested that all Regge trajectories $\alpha(t)$ have a branch point at $t = 0$ and are complex for negative t. A number of consequences and conjectures based on this possibility are discussed.

It is well known that a Regge trajectory $\alpha(t)$ has branch points in t at physical thresholds of all channels to which the trajectory couples. We should like to suggest that every trajectory has a branch point at $t = 0$ as well, and that each trajectory is real only between $t = 0$ and the lowest available threshold.

In potential theory, when two Regge trajectories $\alpha(t)$ and $\alpha_1(t)$ collide [that is, when there is a value t_1 such that $\alpha(t_1) = \alpha_1(t_1)$], then both trajec-