object. While this conclusion is consistent with the inelasticity we observe for the $I=0$ D wave, we note that it depends strongly on successfully eliminating the $K^*(890)$ contribution.

[15]G. Veneziano, Nuovo Cimento 57A, 190 (1968); C. Lovelace, Phys. Letters 28B, 265 (1968).

[16]See, G. Ascoli, H. B. Crawley, D. W. Mortara, and A. Shapiro, Phys. Rev. Letters 21, 1712 (1968), for the 4π decay of the f^0, and Ref. 3.

$\pi^0\pi^0$ MASS SPECTRUM AND $\delta_0{}^0$ BELOW 1 GeV *

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We have measured the $\delta_0{}^0$ phase shifts from $\pi^+\pi^-$ elastic scattering and compared the four possible solutions with our $\pi^0\pi^0$ mass spectrum. Only one solution, characterized by a very gradual increase of the phase shifts to 90°, compares favorably.

In this paper we present a unique set of $I=0$, $J=0$ π-π scattering phase shifts ($\delta_0{}^0$) based on an analysis of the reactions

$$\pi^+n \rightarrow p\pi^+\pi^- \tag{1}$$

and

$$\pi^+n \rightarrow p\pi^0\pi^0 \tag{2}$$

near 2-GeV/c incident-pion momentum. This paper differs from an earlier report[1] in essentially two respects: (a) We now include in the analysis a new sample of 4044 low-momentum-transfer events from Reaction (1), thus providing a prediction for the $\pi^0\pi^0$ mass spectrum within the framework of our own experiment, and (b) we have redone background subtractions on the $\pi^+n \rightarrow p+$"neutrals" events based on considerably more reliable information for the $p+3\pi^0$ reaction. We find that only one set of phase shifts, the so-called up-down solution, satisfies both the $\pi^+\pi^-$ and the $\pi^0\pi^0$ data. This solution is characterized by a gradual increase of the phase shifts to 90° near 750 MeV, thereafter remaining constant to 1000 MeV.

The method described by Schlein[2] and applied to π-π scattering by Malamud and Schlein[3] has been adopted for the phase-shift analysis. Since the $I=2$, $J=0$ phase shifts are not available to us in this experiment as a consequence of the fact that a deuterium target has been used, we have assumed values of $\delta_0{}^2$ based on averaging values reported by several authors.[4] Since the method of analysis has been discussed elsewhere, only the relevant points will be reviewed here. We have used the coordinate definitions and expressions given by Schlein for the moments $N\langle Y_l{}^m\rangle$ and cross sections of the π-π scattering angular distributions in order to fit our experimental data. The quantities $|\vec{s}|^2$, $|\vec{p}_1-\vec{p}_{-1}|^2$, $|\vec{p}_0|^2$, and $(|\vec{p}_1|^2+|\vec{p}_{-1}|^2)$ are helicity-amplitude quantities, and $\theta(\vec{p}_0, \vec{s})$ and $\theta(\vec{s},\vec{p}_1-\vec{p}_{-1})$ are angles between the helicity amplitudes. All off-mass-shell and absorption corrections to the simple one-pion-exchange (OPE) Born helicity amplitudes are included in the definition of the above helicity-amplitude quantitites. Expressions for the S- and P-wave scattering amplitudes involving explicitly the $\delta_0{}^0$, $\delta_1{}^1$, and $\delta_0{}^2$ phase shifts may be found in Ref. 3.[5,6]

A crucial assumption of this method of analysis is that the entire π-π mass dependence of the moment equations resides in the π-π scattering amplitudes which are assumed to be independent of t and E^* (overall c.m. energy). Likewise, the helicity-amplitude quantities are assumed to depend only on t and E^*. A recent article by Bander, Shaw, and Fulco[7] has shown that below a π-π mass of 600 MeV this assumption may not be valid. Since in our analysis we have examined regions of the Chew-Low plot down to 500 MeV, it is necessary to check this assumption. We have evaluated the moments of the π-π scattering angular distributions as a function of π-π mass for several limits on t. We have found that the inclusion of data between 500 and 600 MeV does not significantly raise or lower the confidence level for either the $l=1$ or 2 moment tests. Therefore, we feel that the initial assumption on the constancy of the helicity-amplitude quantities as a function of π-π mass is valid down to 500

MeV.

The data sample used in the phase-shift analysis of Reaction (1) consists of 4044 events with a c.m. production-angle cosine of the π-π system greater than 0.80 and c.m. energy $2071 < E^* < 2285$ MeV. In order that there be no significant difference between the Chew-Low plot boundary and that for the corresponding charge-symmetric free-nucleon reaction $\pi^- p \rightarrow n\pi^+\pi^-$, we have chosen this sample such that it contains only events with spectator momentum less than 120 MeV/c. This insures that the difference between the boundaries (which is due to the Fermi motion of the neutron target) remain less than 0.005 GeV^2/c^2 for all values of π-π mass less than 1000 MeV at these values of c.m. energy. The median value of E^* for this sample is 2218 MeV, corresponding to an effective "laboratory" beam momentum of 2.14 GeV/c.

We have divided the sample into 63 bins in π-π mass and t (average of 64 events/bin) and evaluated the π-π scattering angular distribution moments and their errors. For each of the 63 bins we have fitted the data by the moment expressions given by Schlein.[2] Free parameters used to minimize χ^2 were as follows: (a) Each of the four quantities $|\vec{s}|^2$, $|\vec{p}_0|^2$, $(|\vec{p}_1|^2+|\vec{p}_{-1}|^2)$, and $|\vec{p}_1-\vec{p}_{-1}|^2$ was parametrized to functions of the form $Ae^{-bt}[1-\frac{1}{3}H(t)]$ (two parameters each)[8]; (b) the quantities $\theta(\vec{s},\vec{p}_0)$ and $\theta(\vec{s},\vec{p}_1-\vec{p}_{-1})$ were each parametrized to quadratic expressions in t of the form $A+Bt+Ct^2$ (three parameters each); (c) m_ρ and Γ_ρ were assumed unknown for each bin (making a total of 16 free parameters for δ_0^0). In the above scheme we have a total of 32 free parameters and 378 data points. The minimization procedure utilized the program MINFUN. We have attempted other parametrizations, such as the helicity-amplitude squares fit with higher exponential forms, $Ae^{-bt}(1-ce^{-dt})[1-\frac{1}{3}H(t)]$; however, the fitted values of the parameters c were consistent with zero and χ^2 for the fits was not significantly lowered. We have attempted cubic parametrizations for the helicity-amplitude angles of the form $A+Bt+Ct^2+Dt^3$; however, this also failed to lower significantly the values of χ^2 for the fits.

In general, since the moments $\langle Y_l^m \rangle$ depend quadratically on $\sin\delta_0^0$, there should be two distinct branch solutions found for this quantity. However, the two solutions approach so closely near a π-π mass of 725 MeV that they are equal at this mass within statistical errors. Therefore, there can exist two other "cross-over" solutions.[3] Since it is impossible to tell which are the two "true" solutions and which are the two "cross-over" branches, all four must be considered when attempting to find a preferred solution. The results of these fits for the δ_0^0 phase shifts as a function of π-π mass are shown in Fig. 1. The confidence levels for these four fits (shown in Fig. 1) are sufficiently similar in magnitude to rule out at this point a selection of a preferred set of phase shifts.

It should be emphasized that this analysis assumes that OPE is the only production mechanism. On a Dalitz plot of $m^2(p\pi^+)$ vs $m^2(\pi^+\pi^-)$ the $N^{*++}(1238)$ band, defined as $1.35 \leq m^2(p\pi^+) \leq 1.69$ GeV^2, overlaps the region corresponding to very backward π-π scattering (as measured with respect to the incident pion in the final-state π-π c.m.). We have found that, as witnessed on the $m^2(p\pi^+)$ projection of the same Dalitz plot, there is a detectable $N^{*++}(1238)$ signal above OPE background of magnitude $4.0 \pm 0.8\,\%$ of the total sample of 4044 events. Also, we have been unable to detect evidence for interference effects in the N^*-ρ overlap region. We feel it is very unlikely that our phase-shift solutions have been seriously altered by $N^{*++}(1238)$ production. The excellent agreement of our solutions shown in Fig. 1 with those of Malamud and Schlein at higher momenta adds further support to this conclusion. In addition, we feel that the alternative procedure of removing all events in the $N^{*++}(1238)$ region would present serious difficulties in the

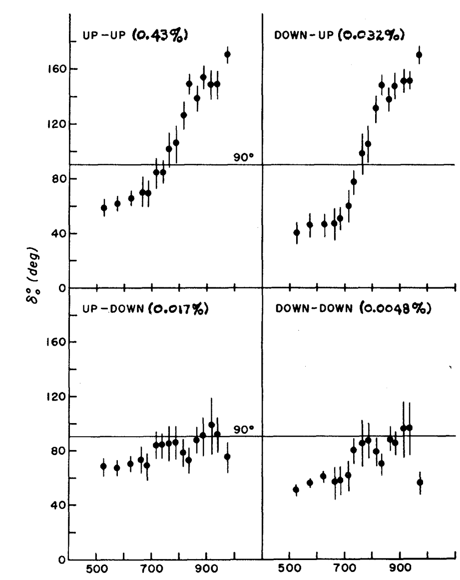

FIG. 1. $I=0$, $J=0$ π-π phase shifts based on an analysis of $\pi^+ n \rightarrow p\pi^+\pi^-$ discussed in the text.

moment analysis of the π-π scattering and is completely unwarranted, considering the very small $N^{*++}(1238)$ contamination present.

For each of our four solutions for $\delta_0^{\ 0}$ we have calculated the predicted $\pi^0\pi^0$ mass spectrum. In order to best compare with out previously reported data, the solutions have been translated down to a beam momentum of 2.01 GeV/c, which is the median value of beam momentum for our sample of π^0−π^0 data. This involves multiplying $|\vec{s}|^2$ by a correction factor of $(2.14/2.01)^2 = 1.13$, since the cross sections are known to follow the p_{lab}^{-2} dependence of OPE.

The techniques used to arrive at our final $\pi^0\pi^0$ sample have been discussed in detail in Ref. 1. Beyond that, we have made several revisions to the spectrum shown in Fig. 3 of our previous work. First, for purposes of making the subtraction of $N^*\pi$ events reflected into the π-π system, we have assumed an $N^*\pi$ spectral shape according to the Stodolsky-Sakurai production mechanism. The distribution of subtracted events has been found to be identical to that based on our previous statistical model assumption. Second, we have established a new overall track-length normalization factor. This revised track-length factor has been based on a normalization to $\pi^+n \to p\pi^0$ (charge symmetric to $\pi^-p \to n\pi^0$), the cross sections for which have been measured in direct hydrogen experiments in our energy range.[9] As a further check of this method, we find our cross sections for $\pi^+n \to p\pi^+\pi^-$, arrived at by a similar normalization, are in good agreement with that reported for the charge-symmetric reaction. Third, we have renormalized our $3\pi^0$ subtraction to a recent cross-section measurement by the Brown–Harvard–Padova–Weizmann Institute–Massachusetts Institute of Technology group.[10] The $3\pi^0$ production has otherwise been assumed statistical in nature. Fourth, the $3\pi^0$ subtraction has been further revised based on the most recent value of the ratio of $\eta^0 \to 3\pi^0$ to $\eta^0 \to$ "all neutrals" of 0.42.[11] This number is valuable in that it enables us to avoid subtracting $\eta^0 \to 3\pi^0$ events twice. The methods of the $3\pi^0$ (and η^0, ω^0) subtractions have been discussed in detail in our previous paper.

The aforementioned revisions to our previously reported $\pi^0\pi^0$ spectrum have resulted in two basic changes in the data: (1) the overall cross section has been reduced by a factor of 1.17 and (2) the shape of the spectrum above 800 MeV falls off less rapidly. This latter effect results directly from the modified $3\pi^0$ subtraction and the fact that the $3\pi^0$ phase space peaks in this region.

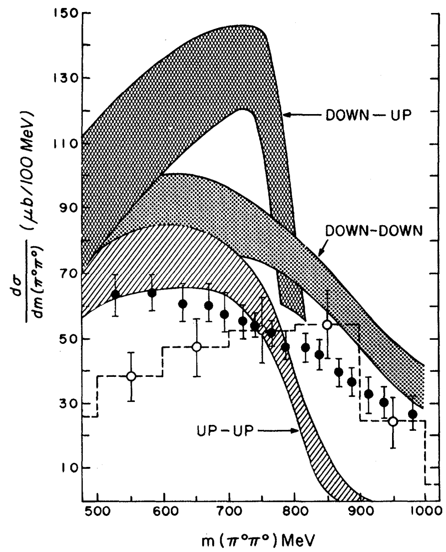

FIG. 2. Experimental $\pi^0\pi^0$ mass spectrum (open circles) compared with the four phase-shift solutions. The down-up, down-down, and up-up solutions are shown shaded, where the half-width of the shaded region corresponds to 1 standard error. The preferred up-down prediction is shown by the solid circles.

The $\pi^0\pi^0$ mass spectrum (open circles) and the four phase-shift solutions are compared in Fig. 2. The data have been restricted to those events with a c.m. production-angle cosine of the π-π system greater than 0.80, identical to the cut used for the phase-shift sample. It is clear that only the up-down solution (closed circles) adequately describes the $\pi^0\pi^0$ data. A straightforward χ^2 comparison shows that the up-down solution fits the data at the 10% confidence level, whereas the next best solution, up-up, fits with only a <0.01% confidence level. We note that the disagreement with the up-up solution, which rises rapidly through 90° near 750 MeV as seen in Fig. 1 and is preferred by Malamud and Schlein,[3] is most extreme in the region above 800 MeV. Furthermore, our preferred up-down solution, which increases gradually to 90° near 750 MeV and then remains essentially constant, is in quite good agreement with the solution of Walker et al.[4] In conclusion, we wish to note that we are aware of three additional experimiments which, by various means, have attempted to study the $\pi^0\pi^0$ system near 2-GeV/c pion momentum.[12-14] Without exception, these authors are in agreement with ourselves on the very broad nature of the $\pi^0\pi^0$ mass spectrum.

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[1]G. A. Smith and R. J. Manning, Phys. Rev. 171, 1399 (1968).

[2]P. E. Schlein, Phys. Rev. Letters 19, 1052 (1967).

[3]E. Malamud and P. E. Schlein, Phys. Rev. Letters 19, 1056 (1967).

[4]W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters 18, 630 (1967); E. Malamud and P. E. Schlein, Bull. Am. Phys. Soc. 13, 31 (1968); J. P. Baton, G. Laurens, and J. Reignier, Nucl. Phys. B3, 349 (1967); E. P. Tyron, Phys. Rev. Letters 20, 769 (1968).

[5]No statistically significant $l>2$ moments for the π-π scattering angular distribution were found for π-π mass less than 1000 MeV. Because of kinematic limitations on the boundary of the Chew-Low plot at our energies, we cannot perform a meaningful phase-shift analysis for values of π-π mass greater than 1000 MeV. Therefore, our data can be accounted for by including only S- and P-wave π-π scattering.

[6]As pointed out in Ref. 2, the absence of events of the type $\pi^-p\rightarrow n2\pi^+2\pi^-$ with the mass of the four final-state pions less than 1000 MeV [see S. U. Chung, O. I. Dahl, J. Kirz, and D. H. Miller, Phys. Rev. 165, 1491 (1968)] indicates a very small π-π inelasticity at a mass less than 1000 MeV.

[7]M. Bander, G. L. Shaw, and J. R. Fulco, Phys. Rev. 168, 1679 (1968).

[8]The deuteron form factor, $H(t)$, has been taken into account for this analysis. The specific form follows the treatment of Benson as applied to π^+d interactions at 3.65 GeV/c [see G. C. Benson, University of Michigan Technical Report No. C00-1112-4, 1966 (unpublished)].

[9]L. Guerriero, Proc. Roy. Soc. (London) 289, 471 (1966); A. E. Pifer, High Energy Physics Group, Brown University, Internal Report No. 109, 1968 (unpublished).

[10]H. R. Crouch, Jr., et al., Phys. Rev. Letters 21, 845 (1968).

[11]N. Barash-Schmidt et al., Rev. Mod. Phys. 41, 109 (1969).

[12]K. J. Braun, D. Cline, and V. Scherer, Phys. Rev. Letters 21, 1275 (1968).

[13]I. F. Corbett et al., Phys. Rev. 156, 1451 (1967).

[14]It is our understanding that the preliminary results of the Walker-Erwin group using the Princeton-Pennsylvania Accelerator bubble chamber with plates also favor a broad $\pi^0\pi^0$ spectrum. See W. D. Walker, in Proceedings of a Conference on the $\pi\pi$ and $K\pi$ Interactions, Argonne National Laboratory, 14-16 May 1969 (unpublished).

OBSERVATION OF TRANS-IRON NUCLEI IN THE PRIMARY COSMIC RADIATION*

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Interleaved layers of nuclear photographic emulsion and plastic detectors, covering a total area of 21 m^2, were exposed to the primary cosmic radiation on high-altitude balloon flights. Flux values, in particles/m^2 sr sec, have been estimated to be

$J(Z\geq 33)\geq 2.6\times10^{-5}$,
$J(33\leq Z\leq 40)\geq 1.9\times10^{-5}$,
$J(Z\geq 70)\gtrsim 1\times10^{-6}$.

These values refer to the top of the atmosphere, after extrapolation through 1.5 g/cm^2 of detector and 3.5 g/cm^2 of atmosphere, for particles with magnetic rigidities above 5 GeV.

It is only recently that extremely heavy particles with $Z\geq 30$ have been identified unambiguously in the cosmic radiation. First measurements of the flux of these particles, referred to as VVH, were derived from stored nuclear tracks in crystals found in meteorites[1] and represent, therefore, a flux value that was averaged over the exposure time of the meteorites, typically 10-100 million years.[2] The presence of VVH particles in the present-day cosmic radiation was first shown by Fowler et al.,[3] using nuclear emulsions flown on a balloon at high altitudes. Where previously typical emulsion areas had been of the order of 200 cm^2, Fowler et al. used 4.5 m^2 and found nine nuclei with $Z>40$, including two inferred as having charges in the vicinity of 92. Nuclei with $26<Z<40$ were not readily identifiable because of their possible confusion with the very much larger number of slowing-down iron nuclei.