Phys. Rev. Letters 19, 447 (1967).
[12]R. C. Morrison, thesis, Yale University, 1965 (unpublished).
[13]H. E. Gove, A. E. Litherland, and R. Batchelor, Nucl. Phys. 26, 480 (1961).
[14]D. E. Frederick, Nucl. Phys. A101, 250 (1967).
[15]The assumption $\eta_{3/2,1}-\eta_{1/2,1}\simeq 0$ is not overly restrictive since a value as large as 75° would result in $\rho_{3/2,1}=0.98$, affecting the calculated ratio $\sigma(E2)/\sigma(E1)$ by less than 30%.
[16]M. Gell-Mann and V. L. Telegdi, Phys. Rev. 91, 169 (1953).
[17]A more complete report of this work will be published elsewhere.

EXPERIMENTAL SEARCH FOR SEMILEPTONIC NEUTRINO NEUTRAL CURRENTS*

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A search for the rare K^+ decay mode $K^+\to\pi^+\nu\bar{\nu}$ has been carried out using stopping K^+ mesons in a heavy-liquid bubble chamber. No events were found, and an upper limit on the branching ratio of this mode of 5×10^{-5} to all K^+ decays is obtained. Comparison is made with various theories which imply the existence of neutral neutrino currents.

A recurring question in the study of weak interactions is the possible existence of neutral leptonic currents. Although no theoretical model specifically predicts the absence of neutral currents, all present expermental data are consistent with the nonexistence of such currents, at least for first-order weak interactions.[1,2] However, up to the present, experimental searches for neutral-current processes have looked for decay modes of K mesons with either e^+e^- or $\mu^+\mu^-$ pairs in the final state, and the limits on these branching ratios are presently in most cases in the range 10^{-6}. No published limits exist for neutral-current modes with two neutrinos in the final state.[3]

Recently several investigations of possible mechanisms of CP nonconservation in weak interactions[4] and renormalizable theories of weak interactions[5-7] have suggested the possible existence of neutral leptonic currents coupled primarily to neutrinos

In this note we present the essential details of a search for the decay modes

$$K^+\to\pi^+\nu_\mu\bar{\nu}_\mu \text{ or } \pi^+\nu_e\bar{\nu}_e. \qquad (1)$$

Examples of Reaction (1) were searched for using film from an exposure of the Argonne National Laboratory–Michigan bubble chamber to a stopping K^+ beam at the zero-gradient synchrotron.[8] The bubble chamber was filled with heavy Freon, and the magnetic field was run at 46 kG. An average of 3-4 K^+ were stopped in an appropriate fiducial volume for each picture. In this note we report on the search for decay modes (1) in a sample of 206 000 K^+ decays.

In order to separate decay mode (1) from all other K^+ decays, we make use of three characteristics of these decays: (a) detection of a stopping π^+ in the final state which is uniquely identified by the observation of a $\pi\to\mu\to e$ decay chain at the stopping point, (b) nonobservation of converted gamma rays coming from the K^+ decay point, and (c) a π^+ momentum measured by range in the bubble chamber that is different from that expected for $K_{\pi2}$ or $K_{\mu2}$ decays. Each of these characteristics is discussed below.

The unique identification of a π^+ as the charged decay product of a stopped K^+ is used to separate Reactions (1) from $K_{\mu3}$, $K_{\mu2}$, or $\mu\nu\gamma$ decays. There are a number of ways by which a stopping μ^+ track can appear to have a $\pi\to\mu\to e$ chain at the stopping point. In order to reduce the probability of such "fake π's," restrictive criteria for identification of the $\pi\to\mu\to e$ chain were used. By studying the $\pi\to\mu\to e$ chains on π^+ from τ decays, it was observed that 48% of stopping π^+ satisfy these criteria. In contrast only $\frac{3}{4}$% of μ's passed the $\pi\to\mu\to e$ chain test. It was also observed that slightly more rigid criteria for $\pi\to\mu\to e$ identification could easily reduce the $\frac{3}{4}$% by one half without changing the $\pi\to\mu\to e$ detection efficiency appreciably.

Excluding $K_{\mu2}$ and K_{e2} decays, all known decays of K^+ mesons with one charged particle in the final state result in one or more photons in the final state. If the final state has a charged π^+, the following modes are allowed: $\pi^+\pi^0$, $\pi^+\pi^0\pi^0$, $\pi^+\pi^0\gamma$, and $\pi^+\gamma\gamma$.[9] The latter two modes have branching ratios of less than 3×10^{-4}[10] and 10^{-4},[11] respectively. Using the mean probability for gamma-ray conversion into an electron-positron pair in this experiment, it is expected that the approximate fractions 2×10^{-2}, 5×10^{-4}, 3×10^{-3}, and 2×10^{-2} of these modes will be observed with no associated gamma rays. With the exception of the $\pi^+\pi^0$ mode, the other modes can therefore be separated from Reaction (1) down to the branching ratio of 10^{-5} through the observation of gamma rays alone. Similarly $K_{\mu 3}$ decays with a fake $\pi\to\mu\to e$ chain are separated at the level of less than 10^{-5} in this way.

In order to separate $K_{\pi 2}$ decays from Reaction (1), the unique π^+ momentum of this mode is used. Only stopping π^+ tracks without a nuclear scatter were used. In order to test the reliability of the momentum determined from range measurements and the reconstruction program,[12] the ranges of a number of stopped π^+ from $K_{\pi 2}$ decays were measured. The π^+ momentum obtained from the range measurements was 207 ± 3 MeV/c to be compared with the expected value of 205 MeV/c.

In order to search for Reactions (1) all one-prong K^+ decays with the K^+ stopping in an appropriate fiducial volume were scanned for. Events with a charged decay track identified as a π^+ and with no associated γ rays converted in the bubble chamber were selected. Each of these events was looked at by a physicist and events with a π^+ nuclear interaction were discarded. For the remaining events the range of the π^+ and the angle ($\theta_{K\pi}$) between the π^+ direction and the direction of the incoming K^+ (prior to stopping) were measured. There were 217 events remaining in the sample. In Fig. 1 a scatter plot of the π^+ momentum versus $\theta_{K\pi}$ is shown. An important feature of this plot is the strong concentration of π^+ momenta in the band 201-212 MeV/c. These events are undoubtedly $K_{\pi 2}$ decays with the decay gamma rays being undetected. Cutting out the obvious $K_{\pi 2}$ events there are 11 remaining candidates for Reaction (1). The remaining source of background comes from $K_{\pi 2}$ decays with the K^+ decaying in flight.

Because of the large increase in dE/dx for low K^+ momentum it is impossible to separate visually at-rest decays from decays with K^+momentum up to 200 MeV/c. Beyond 200 MeV/c the ionization of the K^+ track can be used to identify the in flight decay. For $K_{\pi 2}$ decays in flight, the π^+ momentum is no longer unique but depends on the angle between the K^+ direction and the π^+ direction. Forward going π^+'s will have a π^+ momentum greater than 205 MeV/c whereas backward going π^+'s will have a momentum of less than 205 MeV/c. In order to separate Reaction (1) from $K_{\pi 2}$ decays in flight,[13] a cut was made on the angle $\theta_{K\pi}$. In Fig. 1 the kinematic limits on P_{π^+} as a function of $\theta_{K\pi}$ are shown by the dashed line for K^+ decays in flight with K^+ momenta up to 200 MeV/c. The regions marked "search area" are expected to be free of $K_{\pi 2}$ decays, in flight or at rest, and are, therefore, the regions where events of Reaction (1) would be found. There are no remaining candidates in these regions. Thus the present experiment does not give evidence for the existence of decay modes (1).

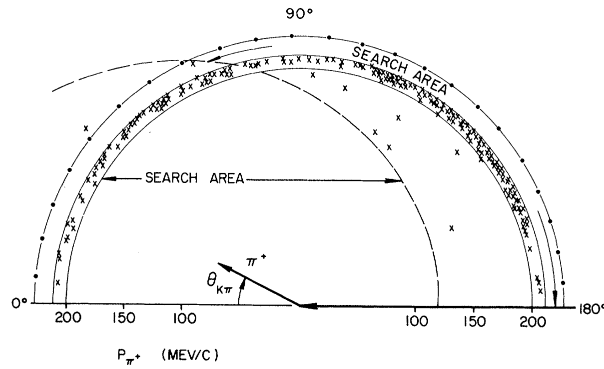

FIG. 1. Scatter plot of the π^+ momentum determined by range and the angle between the outgoing π^+ and the incident K^+ (prior to stopping). $K_{\pi 2}$ events from stopping K^+ decays are expected to fall in the band between 201 and 212 MeV/c. The regions labeled "search area" are expected to be free from $K_{\pi 2}{}^+$ decay-in-flight background. The dash-dotted curve indicates the kinematic limits for the $\pi^+\nu\bar{\nu}$ decay.

In order to determine an upper limit on the branching ratio for Reactions (1) the detection efficiency was estimated. This estimate included corrections for loss of possible $\pi^+\nu\bar{\nu}$ events due to π^+ nuclear interactions, for the detection efficiency of the $\pi\to\mu\to e$ chain, for a steepness cutoff imposed on the π^+ track, and for the scanners' efficiency. In addition the loss of events due to $\theta_{K\pi}$ and π^+ momentum cuts was included.[14] The latter correction requires an estimate of the π^+ momentum spectrum for Reactions (1). It should be emphasized that our experiment is very insensitive to the assumed momentum spectrum since only the 201- to 212-MeV/c part of the spectrum was removed. However, a reasonable guess at the expected π^+ spectrum for (1) would

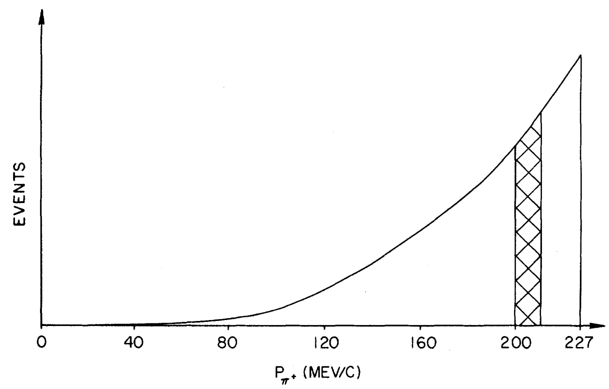

FIG. 2. Assumed π^+ momentum spectrum for $\pi^+\nu\nu$ decay. The shaded region is cut out to remove $K_{\pi 2}$ decay background.

equate it to the π^0 spectrum for K_{e3} decay. Figure 2 shows the resulting assumed spectrum with the $K_{\pi 2}$ cut indicated. Folding in all corrections, the effective sample of K^+ decays that were searched for decay modes (1) contains 21 800 K^+ decays. Although no events were found, had one event been observed the branching ratio would be $4.6\times10^{-5}/(K^+$ decay). This is also the 63% confidence level upper limit on the branching ratio for decay modes (1). The 90% confidence level for these decays is $1.0\times10^{-4}/(K^+$ decay).

This is also the upper limit on the branching ratio for previously undetected K^+ decays of the form

$$K^+\rightarrow\pi^+X^0,$$

where X^0 decays neutrally into nonphotonic final states. An example of such a nonphotonic final state would be a decay into previously undetected neutral massive leptons.

This experiment conclusively shows that semileptonic neutral currents involving two neutrinos are strongly suppressed relative to charged semileptonic currents. This evidence when combined with the negative searches for semileptonic neutral currents involving two electrons or muons leads to the conclusion that the suppression of these currents is independent of the nature of the leptons involved in these currents.[1]

Our present experimental limit on decay modes (1) is still a factor of 2 larger than the branching ratio suggested by the theory of Oakes[4]; thus the predictions of this theory are consistent with experiment.

The cleanliness of the present results and estimates of background rates lead us to believe that the present experimental technique can be used to search for decay modes (1) to the level of ~5 $\times10^{-6}$ relative to all K^+ decays. The broad π^+ spectrum range covered by this experimental technique makes the search for this mode relatively independent of the details of the matrix element for Reactions (1). It is our intention to continue the search for these modes to the lowest possible level in the near future.

We wish to thank R. Klem, D. Nordby, and V. Sevcik for their cooperation in the smooth running of this experiment at the zero-gradient synchrotron. We are grateful to Professor C. T. Murphy, Professor R. March, and Mr. B. Lagerroos for their help in making the construction program SHAPE work. Finally we wish to thank J. Kolonko and the Wisconsin scanning and measuring staff for their diligent work.

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†Alfred P. Sloan Fellow (1968-1970).

[1]For a recent review see D. Cline, in Methods of Subnuclear Physics, edited by M. Nikolic (Gordon and Breach Publishers, Inc., New York, 1969), Vol. III, p. 355.

[2]A. Pais and S. Treiman, Phys. Rev. 176, 1974 (1968).

[3]A crude limit on the $\pi^+\nu\bar{\nu}$ mode was reported by D. Cline, thesis, University of Wisconsin, 1965 (unpublished).

[4]R. Oakes, Phys. Rev. Letters 20, 1539 (1968), and private communication. The combined branching ratio for (1) is estimated to be $1.8\times10^{-5}/(K^+$ decay) in this theory.

[5]G. Segrè, "A Model of Weak Interactions with a 300 BeV Cutoff," (to be published).

[6]N. Christ, Phys. Rev. 176, 2086 (1968).

[7]Yu. Galaktionov and E. P. Shabalin, Zh. Eksperim. i Teor. Fiz.—Pis'ma Redakt. 8, 600 (1968) [translation: JETP Letters 8, 369 (1968)].

[8]M. Derrick and G. Keyes, in Proceedings of International Conference on Instrumentation for High Energy Physics, Stanford, 1966 (International Union of Pure and Applied Physics and U. S. Atomic Energy Commission, Washington, D.C., 1966), p. 618.

[9]Other rare modes like $\pi^+\pi^0\gamma\gamma$ and $\pi^+\pi^0\pi^0\gamma$ are neglected because of the expected small branching ratios.

[10]D. Cline and W. Fry, Phys. Rev. Letters 13, 101 (1964).

[11]M. Chen, D. Cutts, P. Kijewski, R. Stiening, C. W. Wiegand, and M. Deutsch, Phys. Rev. Letters 20, 73 (1968); Cline, Ref. 3.

[12]The reconstruction program SHAPE which was written at the University of Michigan was used in this experiment.

[13]Other decays in flight resulting in π^+ secondaries

(like τ') are estimated to be completely negligible at the statistical level of this experiment.

[14]One can also calculate the detection effeciency for $\pi^+\nu\bar{\nu}$ events from knowing that 217 $K_{\pi 2}$ events with no gammas pointing were detected. Assuming a reasonable gamma-ray detection efficiency one gets very good agreement with the conclusion that we would detect 10.6 % of a real sample of $\pi^+\nu\bar{\nu}$ events.

SYSTEMATIC CONSTRUCTION OF CORRELATED MANY-PARTICLE INTEGRAL-TRANSFORM TRIAL FUNCTIONS AND MULTICENTER MOLECULAR ORBITALS

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A systematic approach to the construction of new types of correlated many-particle trial functions is outlined. It is based on the formalism of a special type of multidimensional integral transformation, and combines a conceptually simple geometrical interpretation with computational practicability and the possibility of classifying the correlated wave functions in a simple manner. The approach is applicable to the formation of new, multicenter molecular orbitals as well.

Recently I proposed the use of integral-transform trial functions in quantum mechanical calculations.[1] Such functions are extremely efficient[2-5] and give rise to orbitals of near Hartree-Fock accuracy with only a few adjustable parameters. In this work we generalize and extend the idea in a systematic manner to correlated many-particle trial functions. The formalism is equally applicable for the construction of new kinds of molecular orbitals.

Let H be the Hamiltonian of the system whose solution $\varphi(x)$ we seek. Assume that the exact solution $\varphi_0(x)$ of some related model Hamiltonian H_0 is known. Then one constructs the integral-transform trial function $\varphi_1(x)$, an approximate solution to $\varphi(x)$, by formally scaling φ_0, $\varphi_0(x) \rightarrow \varphi_0(tx)$, and forming

$$\varphi_1(x) = \int_D S(t)\varphi_0(tx)dt, \qquad (1)$$

where D is some suitable domain of integration and $S(t)$, called the shape function, is a weight factor to be determined. The computationally most practical approach is to parametrize some trial form of $S(t)$ and optimize the parameters variationally. The following simple argument[2] aids in the selection of an appropriate analytical form for $S(t)$: As the "perturbation" $(H-H_0)$ approaches zero, both $\varphi_0(tx)_{t=1}$ and $\varphi_1(x)$ tend to $\varphi(x)$. This is consistent only if in that limit $S(t) = \delta(t-1)$. Consequently one chooses a trial $S(t)$ which for certain limiting values of its parameters becomes the delta function, i.e., $S(t)$ should be a delta-convergent sequence.[6]

Since $\varphi_0(xt)$ is a function of the product xt, Mellin transform functions are the natural choice, as in the case of the iterated integral-transform-function approach[4] where the kth iteration $\varphi_k(x)$ is obtained by the prescription

$$\varphi_k(x) = \underbrace{\int \cdots \int}_{k\text{-fold}} \varphi_0\Big(x\prod_{i=1}^{k} t_i\Big)\prod_{i=1}^{k} S_i(t_i)dt_i.$$

One straightforward generalization[1] to many-particle systems is to take some relatively simple N-particle, M-parameter trial function of the independent-particle type and integrate it with respect to the parameters, choosing a suitable $S(t_1, t_2, \cdots, t_M)$. This involves an M-dimensional integration. Particle correlation is to be introduced by coupling the t_i in $S(\vec{t})$. One special version is to use the N scaling parameters as the integration variables t_i and again select some appropriate S. The main objection to the above procedure is that at present there is no previous experience to rely upon in selecting the best way to couple the t_i in S and thus introduce particle correlation. Furthermore, an M-fold integration may be completely intractable from the practical point of view. To overcome these difficulties we shall outline a compact, unified, and quite general method for constructing many-particle correlated trial functions. Recent interest in two-electron trial functions as building blocks for many-electron atomic and molecular wave functions provides a special impetus and makes the method especially topical.

The concept of electron correlation in atoms and molecules is often couched in a spatial geometrical language. Although vague and qualita-