ical calibration curves cannot be replaced by predictions based on DWBA theory. Calculations were made for the reactions on $^{52}$Cr and $^{54}$Fe. They are in good qualitative agreement with vector-analyzing-power measurements. However, the measured and calculated curves show significant quantitative differences, especially where the vector analyzing power is changing rapidly with angle. In addition, the calculations did not reproduce the shape of the observed cross-section angular distributions at back angles. These deviations cause a considerably poorer fit to the $^{53}$Cr data than the fit shown in Fig. 1. The use of empirical calibration curves has the advantage that it allows a determination of $\sigma(\frac{1}{2}^-)$ and $\sigma(\frac{3}{2}^-)$ which is independent of any theory.

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# EVIDENCE FOR $Y_4$ DEFORMATION IN $^{20}$Ne AND OTHER $s$-$d$ SHELL NUCLEI*

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The sign and magnitude of the haxadecapole deformation in $^{20}$Ne is determined from measurements of the inelastic scattering of 24.5-MeV protons. A coupled-channels analysis yields a value for $\beta_4$ of $+0.28$. A similar analysis of other available data in the $s$-$d$ shell suggests a large hexadecapole deformation in $^{28}$Si (+0.25) and $^{32}$S (+0.25); $^{24}$Mg is found to have a $Y_4$ moment close to zero.

Accurate measurements of the large intrinsic quadrupole deformation of the first excited $2^+$ states in $^{20}$Ne, $^{24}$Mg, and $^{28}$Si have recently been performed.[1] Such data are a critical test of the detailed microscopic calculations of nuclear properties which are now being carried out by methods such as deformed Hartree-Fock.[2] Some of these calculations suggest that nuclei of the $2s$-$1d$ shell should also have a ground-state hexadecapole deformation which changes both in size and sign through the shell.[3-5] The size and sign of $Y_4$ moments of rare-earth nuclei have previously been determined by a coupled-channels analysis of the cross sections for excitation of the ground state rotational band by 50-MeV alpha particles.[6] The advantage of this method was that all multiple-excitation paths between these states were treated consistently. A similar analysis of scattering data in the $2s$-$1d$ shell has been performed only for $^{24}$Mg, but no $Y_4$ deformation was observed in the ground-state band.[7] On the other hand, previous inelastic-scattering results analyzed with the distorted-wave Born approximation, Austern-Blair, and other less sophisticated models indicate that large direct transition strengths are needed in order to explain the magnitude of the cross sections for the first $4^+$ states in $^{20}$Ne and $^{28}$Si.[8-10]

We have measured the inelastic scattering of 24.5-MeV protons from $^{20}$Ne. The cross sections for the lowest $0^+$, $2^+$, $4^+$, and $6^+$ states are analyzed with the same coupled-channels method used for the rare-earth nuclei. The same model is then used to analyze the data of Crawley and Garvey[11] for inelastic scattering of 17.5-MeV protons from $^{24}$Mg, $^{28}$Si, and $^{32}$S. Evidence is found for large positive hexadecapole deformation in $^{20}$Ne, $^{28}$Si, and $^{32}$S, while $^{24}$Mg is determined to have a small, possibly negative, hexadecapole moment. These results are relatively independent of the size and sign of $\beta_2$.

The $^{20}$Ne experiment was performed at the Berkely 88-in. cyclotron. A 24.5-MeV proton beam was scattered from a gas target filled to 10- or 20-cm Hg pressure with enriched $^{20}$Ne. A set of four Si(Li) detectors were used to count the scattered particles; the detectors were 4 mm thick and cooled to $-25$°C. The total energy resolution achieved was about 50 keV. The angular distributions are shown in Fig. 1, along with theoretical curves described below.

In the coupled-channels calculations the states explicitly coupled are assumed to be the lowest members of a pure $K=0$ rotational band. The intrinsic deformation of these states is parametrized according to the following definition of the nuclear radius:

$$R(\theta)=R_0[1+\beta_2Y_{20}(\theta)+\beta_4Y_{40}(\theta)].$$

The interaction potential arises from the deformation of both the real and imaginary central terms of the optical potential and is calculated correctly to all orders. Thus, all possible multiple excitation paths between the coupled states are explicitly included. Coulomb excitation and deformed spin-orbit terms are included in many of the calculations but are found to have no significant effect. The coupled-channels code of A. D. Hill, which includes a spin-orbit term in the optical potential, is used for most of the calculations. The predictions for the $6^+$ state in $^{20}$Ne are made with the program of N. K. Glendenning, which, however, does not include a spin-orbit term. The $2^+$ and $4^+$ curves are insensitive to the spin-orbit potential.

Starting optical-model parameters were obtained by fitting only the elastic cross sections using the search code MERCY. Very good fits to the elastic cross sections were obtained with several distinct sets of optical-model parameters. These parameters were obtained either by searching only on the well depths using the geometrical parameters recently found for nuclei in the $1f_{7/2}$ shell[12] or by searching on all parameters. These parameters were then adjusted to preserve the fits to the elastic scattering in the coupled-channel calculations. Usually it was sufficient to adjust only $W_D$, $a_I$, and $V_0$. The parameters used for the curves shown in Figs. 1 and 2 are given in Table I.

The experimental cross sections and theoretical curves for the $0^+$, $2^+$, $4^+$, and $6^+$ states in $^{20}$Ne are shown in Fig. 1. With values of +0.47 for $\beta_2$ and +0.28 for $\beta_4$, good fits are obtained to the shape and magnitude of the $0^+$, $2^+$, and $4^+$ cross sections; the $6^+$ cross section is underestimated by a factor of about 2. The sensitivity of the predictions for the $4^+$ and $6^+$ states to the value of $\beta_4$ is also illustrated in this figure. When $\beta_4$ is omitted, the predicted $4^+$ and $6^+$ cross sections are too small by one or two orders of magnitude. If $\beta_4$ is negative, the predicted shape of the $4^+$ angular distribution does not match the experimental curve. In fact, changing $\beta_4$ by +0.05 or −0.05 is sufficient to destroy the good agreement with the $4^+$ cross sec-

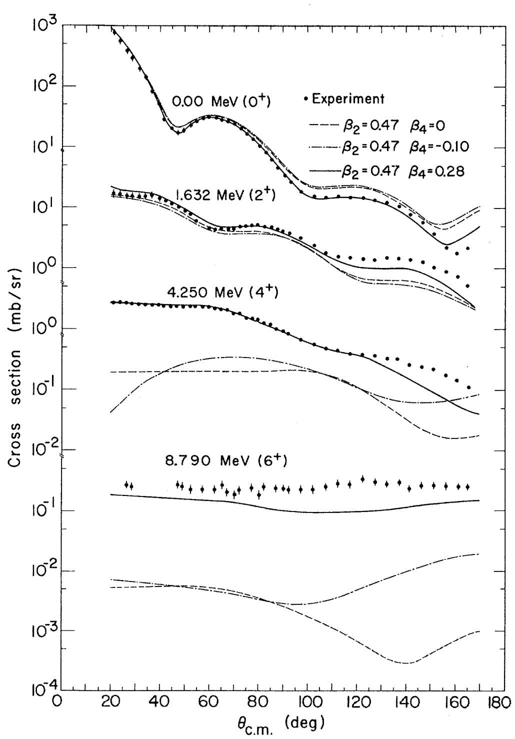


FIG. 1. Measured cross sections and theoretical predictions for the inelastic scattering of 24.5-MeV protons from $^{20}$Ne. The curves were calculated with a coupled-channels program; a rotational model was used with the values of $\beta_2$ and $\beta_4$ indicated.

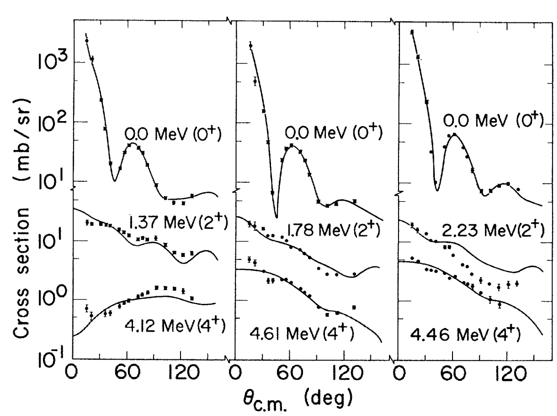


FIG. 2. Experimental cross sections of Crawley and Garvey (Ref. 11) for the inelastic scattering of 17.5-MeV protons from $^{24}$Mg, $^{28}$Si, and $^{32}$S. The curves are coupled-channels predictions with the values of $\beta_2$ and $\beta_4$ indicated in Table I.

Table I. Values of parameters used in the coupled-channels calculation.

| | $V_0$ (MeV) | $r_0$ (F) | $a_0$ (F) | $W_D$ (MeV) | $r_I$ (F) | $a_I$ (F) | $V_S$ (MeV) | $r_S$ (F) | $a_S$ (F) | $\beta_2$ (expt) | $\beta_4$ (expt) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $^{20}$Ne | 54.4 | 1.05 | 0.73 | 6.30 | 1.26 | 0.55 | 3.57 | 0.95 | 0.33 | +0.47 | +0.28 |
| $^{24}$Mg | 46.0 | 1.22 | 0.60 | 3.60 | 1.27 | 0.64 | 7.26 | 1.22 | 0.60 | +0.47 | −0.05 |
| $^{28}$Si | 46.0 | 1.24 | 0.62 | 8.0 | 1.19 | 0.40 | 6.0 | 1.24 | 0.62 | −0.34 | +0.25 |
| $^{32}$S | 47.0 | 1.21 | 0.62 | 9.5 | 1.26 | 0.28 | 6.0 | 1.21 | 0.62 | −0.30 | +0.25 |

tion. Only a large positive $Y_4$ deformation could reproduce the shape as well as the magnitude of the $4^+$ cross section. Changing $\beta_2$ has a smaller effect on the predicted cross sections. If it is omitted (as it is, e.g., in a calculation employing the distorted-wave Born approximation), the value of $\beta_4$ must be increased to about 0.36 and the fit deteriorates somewhat, especially at back angles.

The coupled-channels predictions for the $0^+$, $2^+$, and $4^+$ states in $^{24}$Mg, $^{28}$Si, and $^{32}$S are shown in Fig. 2; no $6^+$ data were available. The values of $\beta_2$ and $\beta_4$ are given in Table I. The signs of the $\beta_2$ deformations in $^{24}$Mg and $^{28}$Si were chosen to agree with the results of Ref. 1. Hartree-Fock calculations[5] predict an oblate deformation for $^{32}$S, but this has not yet been verified experimentally and is not determined by the present analysis. The sign and magnitude of $\beta_4$ for $^{32}$S are not very sensitive to the sign of $\beta_2$. The fits to the elastic scattering are good for all three nuclei; the striking difference in the shape of the $4^+$ angular distribution for $^{24}$Mg from those for $^{28}$Si and $^{32}$S is also qualitatively explained. However, the general quality of the fits shown in Fig. 2 is inferior to that obtained for the $0^+$, $2^+$, and $4^+$ states in $^{20}$Ne. A conservative error of $\pm 0.08$ is thus assigned to the value of $\beta_4$ determined for these nuclei because of ambiguities in the optical parameters and imperfections of the fit. The corresponding error for $^{20}$Ne is $\pm 0.05$.

Nondirect processes may be responsible for some of the discrepancies, especially for the $6^+$ state in $^{20}$Ne and for the $4^+$ state in $^{24}$Mg. However, there is evidence that such processes are not important at forward angles for the $4^+$ states with larger cross sections. Prior to this experiment, an excitation function for the $0^+$, $2^+$, and $4^+$ states in $^{20}$Ne was measured at proton energies between 23 and 26 MeV in 500-keV steps. The excitation functions of these states are smooth between 23 and 26 MeV. In addition the $4^+$ cross section decreases by more than an order of magnitude between forward and backward angles. The cross section for the $4^+$ state in $^{28}$Si measured at 21.2 MeV[10] is similar to the 17.5-MeV cross section.[11]

An additional source of ambiguity is the known imperfection in the rotational description of these nuclei. The energy levels, particularly in $^{32}$S, already show considerable deviation from the strict rotational-model pattern. The values of $B(E2)$ for the intraband transitions measured in various laboratories are not always consistent.[13] The ratio $[B(E2)4^+ \rightarrow 2^+]/[B(E2)2^+ \rightarrow 0^+]$ seems to be generally slightly less than 1.0, whereas the rotational model prediction is 1.43. For $^{32}$S, however, a recent measurement[14] gives $2.6 \pm 0.7$. This means that the multiple excitation contributions to the $4^+$ cross sections are somewhat overestimated except in $^{32}$S. The $[B(E2)6^+ \rightarrow 4^+]$ in $^{20}$Ne seems to be considerably larger than the measured $[B(E2)2^+ \rightarrow 0^+]$ in this nucleus, and is larger than expected from the rotational model. This may be another reason why the predicted $6^+$ cross section in $^{20}$Ne is too small.

In terms of the rotational model, nonzero values of $\beta_4$ imply a hexadecapole moment in the ground state and in all the states of the rotational band built upon it. However, the inelastic scattering data alone might be equally well described by a vibrational model, with some modifications of the values of $\beta_2$ and $\beta_4$. Thus, the interpretation of $\beta_4$ as describing the static $Y_4$ deformation of the ground-state band relies upon measurements of a nonzero quadrupole moment; such a measurement has not yet been made for $^{32}$S. The quadrupole moments of the $2^+$ states of $^{20}$Ne and $^{24}$Mg are about 30% larger than expected[1] on the basis of the rotational model from the electromagnetic values of $[B(E2)2^+ \rightarrow 0^+]$. Since the analysis of the inelastic scattering depends upon the evaluation of matrix elements between the ground state and excited states, the present results should be interpreted in terms of a transition probability instead of a static moment when the two are not consistent.

Benson and Flowers[4] have predicted a value of about 0.17 for $\beta_4$ in $^{20}$Ne; Hartree-Fock calculations[5] predict a large $Y_4$ moment for $^{28}$Si but a small $Y_4$ moment for $^{32}$S. We have made an estimate of the value of $\beta_4$ expected in the Nilsson model, according to the simple method of Harada[15] which accurately predicted the relative values for rare-earth muclei. The predictions follow the general trend of the experimental values except for $^{32}$S which is again underestimated.

To summarize, the coupled-channels analysis of the present $^{20}$Ne data shows clearly the existence of a large hexadecapole deformation. A similar analysis of available data suggests a large hexadecapole deformation also in $^{28}$Si and $^{32}$S while $^{24}$Mg is found to have a very small $Y_4$ deformation.

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# COULOMB EXCITATION OF $^{28}$Si PROJECTILES

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The static quadrupole moments of the first excited $J^\pi = 2^+$ states in $^{28}$Si and $^{62}$Ni were measured to be $Q = 0.17 \pm 0.05$ b and $Q = 0.08 \pm 0.12$ b, respectively, using the reorientation effect in Coulomb excitation with 70-MeV $^{28}$Si projectiles incident on a $^{62}$Ni target.

Measurement of the static quadrupole moments of the first excited $J^\pi = 2^+$ states in light even-even nuclei by means of the reorientation effect in Coulomb excitation has recently become feasible. In principle, accurate values for the static quadrupole moments of these nuclei can be obtained, since the influence of other low-lying states and of the states of the giant dipole resonance[1] on the population of the first excited $2^+$ state is small. The results so far obtained for $^{24}$Mg,[2-4] $^{20}$Ne, and $^{22}$Ne[5] show the expected prolate deformation of the intrinsic state, although the