

the nucleus. In nuclear matter, the properties of the Y_0^* may be different than when the Y_0^* is formed in the free state through the capture of a K^- on hydrogen. In addition the interaction of the reaction products with the residual nucleus should be taken into account. Nonetheless, our study has shown that the $T=0$ resonance plays a decisive role in the K^- capture and gives a qualitative explanation of the main results. Only after taking the Y_0^* resonance into account can one hope to explore properly the surface properties of nuclei by K^- absorption.

A more detailed account will be published in the near future.

We acknowledge with thanks helpful conversa-

tions with A. K. Kerman.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†Permanent address: Aeronautics Division, Philco-Ford Corporation, Newport Beach, Calif.

¹D. H. Wilkinson, J. Phys. Soc. Japan, Suppl. 24, 469 (1968). This paper reviews the subject of K^- capture and contains many key references.

²D. H. Davis, S. P. Lovell, M. Csejthey-Barth, J. Sacton, G. Schorochoff, and M. O'Reilly, Nucl. Phys. B1, 434 (1967).

³E. H. S. Burhop, Nucl. Phys. B1, 438 (1967).

⁴J. K. Kim, Phys. Rev. Letters 19, 1074 (1967), and private communication.

PHASE CONSPIRACY IN UNOCCUPIED HARTREE-FOCK ORBITALS

W. H. Bassichis and M. R. Strayer

Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 5 May 1969)

Second order energies and the probabilities of two-particle, two-hole components in the ground states of He^4 and O^{16} , calculated using perturbation theory in Hartree-Fock bases, show that excitations to high lying levels are more important than the shell model predicts. The importance of these excitations stems from the relative phases of the Hartree-Fock wave functions for unoccupied states and this phase relation can be explained quite simply.

Hartree-Fock (HF) calculations are presently being performed in bases sufficiently large that they may be considered complete for light nuclei.^{1,2} It is well known, however, that the binding energies thus obtained differ widely from observed binding energies if realistic potentials are used. As Kerman pointed out,³ this lack of agreement should be expected from experimental single-particle energies and their relation to the HF energy. Thus an HF calculation should serve only to define a convenient basis in which to carry out a perturbation calculation. In such a basis, the first-order correction vanishes and only two-particle, two-hole (2p-2h) states contribute in second order, i.e.,

$$\Delta E = \sum_{m < n} \sum_{a, b} \frac{|\langle mn | V_A | ab \rangle|^2}{\epsilon_m + \epsilon_n - \epsilon_a - \epsilon_b} \quad (1)$$

Here m, n are the occupied HF orbitals, a, b the unoccupied orbitals; the ϵ 's are the corresponding HF eigenvalues, and V_A signifies the antisymmetrized matrix element.

An approximate method for evaluating ΔE has

been developed⁴ and quite reasonable agreement with experiment was obtained for O^{16} and Ca^{40} . The method was applied to all the even-even nuclei from helium to calcium with similar success.⁵ The essential approximations were these: (1) replacing the unoccupied orbitals by plane waves and the corresponding ϵ 's by $\hbar^2 k^2 / 2m$, (2) replacing the ϵ 's for the occupied states by a constant equal to their average value, (3) taking the Pauli principle into account approximately. In order to evaluate this procedure we have carried out the evaluation of ΔE exactly in a truncated, but very large, basis.

The HF bases were obtained for He^4 and O^{16} in a space consisting of the $1s_{1/2}$ through the $1i_{13/2}$ harmonic oscillator levels and the two-body potential employed was that of Tabakin.⁶ All of the particles were treated self-consistently. Doubly closed-shell nuclei were treated because the HF orbitals in this case are all eigenstates of l and j and are degenerate with respect to j_z and τ_z , in the absence of the Coulomb force. This allows a certain amount of geometry to be done analytical-

ly so that

$$\Delta E = \sum_{\mu < \nu} \sum_{\alpha \beta} \sum_{J T} \frac{(2J+1)(2T+1)}{\epsilon_{\mu} + \epsilon_{\nu} - \epsilon_{\alpha} - \epsilon_{\beta}} \left| \sum_{i j k l} C_i^{\mu} C_j^{\nu} C_k^{\alpha} C_l^{\beta} \langle i j | V_A | k l \rangle_{J T} \right|^2, \quad (2)$$

where α, β represent the $2(2j+1)$ degenerate unoccupied states of a given l and j ; μ, ν the corresponding occupied states; and the C 's are defined by

$$|\lambda\rangle = \sum_i C_i^{\lambda} |n_i l_{\lambda} j_{\lambda}\rangle. \quad (3)$$

($|n l j\rangle$ are harmonic oscillator states.) The expansion coefficients and epsilons obtained by HF are given in Table I for He⁴.

The significant point to note in Table I is that for the pairs of unoccupied HF states, which by orthogonality must have the form

$$\begin{aligned} |\lambda_1\rangle &= a|n l j\rangle - b|n' l j\rangle, \\ |\lambda_2\rangle &= b|n l j\rangle + a|n' l j\rangle, \end{aligned} \quad (4)$$

the eigenfunction which is the sum of the two components in every case corresponds to a larger eigenvalue than the eigenfunction which is the difference. This feature, as seen below, will determine the character of the 2p-2h state mixing into the ground state. It is also to be noted that although the occupied orbitals are nearly pure harmonic-oscillator functions, the unoccupied orbitals have appreciable mixing, i.e., the coefficient b in (4) is not negligible.

The existence of the phase rule can be understood from simple matrix algebra as follows. The HF basis can be defined by the fact that in the HF representation, the HF Hamiltonian does not connect occupied to unoccupied states. If, in addition to this, one considers a doubly closed-shell nucleus (all j shells filled), then the HF matrix in the harmonic-oscillator basis,

$$\langle \alpha | h | \beta \rangle \equiv \langle \alpha | t | \beta \rangle + \sum_{\lambda} \langle \alpha \lambda | V_A | \beta \lambda \rangle, \quad (5)$$

will not connect α to β unless they both have the same l and j . Since in the basis considered there are at most two unoccupied states of a given l and j , noted symbolically as $|1\rangle$ and $|2\rangle$, there will be a number of two-by-two matrices not connected to any other part of the HF matrix and these can be diagonalized separately. Explicitly such a matrix is

$$\begin{pmatrix} \langle 1 | h | 1 \rangle & \langle 1 | h | 2 \rangle \\ \langle 2 | h | 1 \rangle & \langle 2 | h | 2 \rangle \end{pmatrix}, \quad (6)$$

where

$$\langle i | h | j \rangle \equiv \langle i | t | j \rangle + \sum_{\lambda} \langle i \lambda | V_A | j \lambda \rangle \equiv \mathcal{E}_{ij},$$

and

$$\langle i | h | j \rangle = \langle j | h | i \rangle. \quad (7)$$

Writing the eigenfunctions as

$$\begin{pmatrix} C_1^k |1\rangle \\ C_2^k |2\rangle \end{pmatrix}$$

and the corresponding eigenvalues as ϵ_k we have

$$\mathcal{E}_{11} C_1^k + \mathcal{E}_{12} C_2^k = \epsilon_k C_1^k \quad (8)$$

or

$$\mathcal{E}_{12} C_1^k + \mathcal{E}_{22} C_2^k = \epsilon_k C_2^k \quad (9)$$

and

$$\epsilon_k = \frac{1}{2} \{ \mathcal{E}_{11} + \mathcal{E}_{22} \pm [(\mathcal{E}_{22} - \mathcal{E}_{11})^2 + 4\mathcal{E}_{12}^2]^{1/2} \}. \quad (10)$$

Since $\mathcal{E}_{22} > \mathcal{E}_{11}$ and \mathcal{E}_{12} is real, we can write

$$\begin{aligned} [(\mathcal{E}_{22} - \mathcal{E}_{11})^2 + 4\mathcal{E}_{12}^2]^{1/2} &= \mathcal{E}_{22} - \mathcal{E}_{11} + \Delta \\ &\text{with } \Delta > 0. \end{aligned} \quad (11)$$

Then $\epsilon_+ = \mathcal{E}_{22} + \frac{1}{2}\Delta$ and $\epsilon_- = \mathcal{E}_{11} - \frac{1}{2}\Delta$ and Eqs. (8) and (9) become

$$\begin{aligned} C_2^- &= \frac{\epsilon_- - \mathcal{E}_{11}}{\mathcal{E}_{12}} C_1^- = -\frac{\Delta}{2\mathcal{E}_{12}} C_1^-, \\ C_2^+ &= \frac{\epsilon_+ - \mathcal{E}_{22}}{\mathcal{E}_{12}} C_1^+ = +\frac{\Delta}{2\mathcal{E}_{12}} C_1^+. \end{aligned} \quad (12)$$

Table I. The Hartree-Fock eigenvalues and expansion coefficients as defined in Eq. (3), for He⁴. The phase rule is seen by considering pairs of states with the same l and j .

	ϵ	c_1	c_2	c_3
s _{1/2}	-12.5746	0.9944	0.0096	0.1054
p _{3/2}	5.0273	0.9006	-0.3728	0.2236
p _{1/2}	6.8956	0.8442	-0.4769	0.2446
s _{1/2}	12.6788	0.0339	0.9149	-0.4023
s _{1/2}	45.6644	0.1003	-0.4036	-0.9094
p _{3/2}	22.9647	0.4346	0.7810	-0.4484
p _{3/2}	57.8694	0.0075	-0.5010	-0.8654
p _{3/2}	26.2201	-0.5312	0.6832	0.5011
p _{1/2}	65.4235	0.0718	0.5530	0.8301
d _{5/2}	16.4701	-0.9061	0.4231	0.0
d _{5/2}	44.2181	0.4231	0.9061	0.0
d _{3/2}	18.0403	-0.8777	0.4792	0.0
d _{3/2}	51.9933	0.4792	0.8777	0.0
f _{7/2}	23.9227	-0.8690	0.4949	0.0
f _{7/2}	56.5564	0.4949	0.8690	0.0
f _{5/2}	25.0396	-0.8514	0.5246	0.0
f _{5/2}	63.3625	0.5246	0.8514	0.0
g _{9/2}	41.3433	1.0000	0.0	0.0
g _{7/2}	44.0878	1.0000	0.0	0.0
h _{1/2}	50.2311	1.0000	0.0	0.0
h _{9/2}	52.1566	1.0000	0.0	0.0
i _{13/2}	58.7923	1.0000	0.0	0.0

Since \mathcal{E}_{12} contains a large, positive kinetic-energy term, it is generally positive for the interaction used here and will probably be positive for any potential of reasonable strength. Thus the phase rule: C_1^+ and C_2^+ , corresponding to the larger eigenvalue, ϵ_+ , are of the same sign and C_1^- and C_2^- , corresponding to the smaller eigenvalue, ϵ_- , are of opposite signs.

To illustrate the effect of this phase rule, consider the two unoccupied $d_{5/2}$ states in He^4 :

$$\begin{aligned} |d_{5/2}\rangle_1 &= -0.9061|1d_{5/2}\rangle + 0.4231|2d_{5/2}\rangle \equiv |1\rangle, \\ |d_{5/2}\rangle_2 &= +0.4231|1d_{5/2}\rangle + 0.9061|2d_{5/2}\rangle \equiv |2\rangle. \end{aligned} \quad (13)$$

Since the corresponding eigenvalues are $\epsilon_1 = 16.47$ MeV and $\epsilon_2 = 44.22$ MeV and since the eigenvalue of the occupied orbital is -12.57 MeV, one would expect the state with two particles in $|1\rangle$ to contribute twice as much to ΔE as the state with two

particles in $|2\rangle$. Furthermore, since the probability of finding such 2p-2h states in the true ground states is given by

$$P_{ab}^{mn} \sim \frac{|\langle mn|V_A|ab\rangle|^2}{(\epsilon_m + \epsilon_n - \epsilon_a - \epsilon_b)^2} \quad (\text{unnormalized}), \quad (14)$$

one might expect the probability of finding the lower state to be four times that of the upper state. In fact, the contribution to ΔE of the upper state is five times larger than that of the lower state (-0.53 MeV compared with -0.10 MeV) and the probability of finding the upper state is two and a half times larger than that of the lower state (0.36% compared with 0.14%)! That this result is directly attributable to the phase relations is shown as follows. Ignoring the small admixtures in the occupied orbital, we have

$$\Delta E(1, 1) = \sum_{JT} \frac{(2J+1)(2T+1)}{-58.08} |0.82V_{JT}^{11} + 0.18V_{JT}^{22} - 0.77V_{JT}^{12}|^2$$

and

$$\Delta E(2, 2) = \sum_{JT} \frac{(2J+1)(2T+1)}{-113.58} |0.18V_{JT}^{11} + 0.82V_{JT}^{22} + 0.77V_{JT}^{12}|^2, \quad (15)$$

where

$$V_{JT}^{11} \equiv \langle (1s_{1/2})^2 | V_A | (1d_{5/2})^2 \rangle_{JT}, \quad V_{JT}^{22} \equiv \langle (1s_{1/2})^2 | V_A | (2d_{5/2})^2 \rangle_{JT},$$

and

$$V_{JT}^{12} \equiv \langle (1s_{1/2})^2 | V_A | 1d_{5/2}2d_{5/2} \rangle_{JT}. \quad (16)$$

For the Tabakin interaction, the values of the nonzero matrix elements are

$$\begin{aligned} V_{01}^{11} &= -3.0787, & V_{10}^{11} &= -1.8386, \\ V_{01}^{22} &= -2.2493, & V_{10}^{22} &= -1.4209, \\ V_{01}^{12} &= -2.2733, & V_{10}^{12} &= -1.4005; \end{aligned} \quad (17)$$

and (15) becomes

$$\Delta E(1, 1) = -\frac{3}{58.08} \{ |-2.53 - 0.49 + 1.73|^2 + |-1.51 - 0.26 + 1.08|^2 \} = -0.11 \text{ MeV}, \quad (18)$$

and $\Delta E(2, 2)$, which differs mainly in the sign of the third terms in the absolute value brackets and, of course, the denominator, is given by

$$\Delta E(2, 2) = -\frac{3}{113.58} \{ |-0.55 - 1.84 - 1.73|^2 + |-0.32 - 1.06 - 1.08|^2 \} = 0.61 \text{ MeV}. \quad (19)$$

This illustrative example overestimates the ΔE 's because the admixture in the $s_{1/2}$ state was ignored but still serves to demonstrate the dominant role played by the relative phases.

The overall effect of the relative phases can be seen by comparing the results of this calculation with one in which all the orbitals are assumed to be pure harmonic oscillators. The results for He^4 and O^{16} are shown in Table II. The column labeled "Denominator" represents the average of $\epsilon_m + \epsilon_n - \epsilon_a - \epsilon_b$ for states which would correspond to the same number of $\hbar\omega$ in excitation in the usual shell model. HF labels the Hartree-Fock results, SM(1) the corresponding results when pure oscillator

Table II. The contributions to ΔE and P , the probability of 2p-2h components in the ground state, as a function of the energy denominator. The column labeled HF is obtained using HF wave functions and epsilons, that labeled SM(1) uses instead pure harmonic-oscillator functions, and SM(2) uses, in addition, multiples of $\hbar\omega$ for the epsilons.

Denominator	% ΔE			Probability (%)		
	HF	SM(1)	SM(2)	HF	SM(1)	SM(2)
He^4						
- 37.07	13.97	39.33	46.28	6.14	21.16	27.13
- 56.34	20.78	32.60	28.59	6.13	11.79	8.38
- 84.65	24.12	17.75	15.63	4.71	4.29	3.05
-112.66	26.79	8.19	7.50	3.69	1.42	1.10
-140.31	14.34	2.13	1.99	1.56	0.29	0.23
Total $\Delta E =$	-20.46	-32.15	31.20	P = 22.23%	38.95%	39.89%
O^{16}						
44.11	20.81	29.63	32.04	19.93	27.30	34.56
77.84	23.08	36.51	36.08	10.77	17.36	19.45
112.13	30.02	24.59	23.23	9.54	7.98	8.35
146.14	23.89	8.59	7.98	5.74	2.12	2.15
182.17	2.20	0.68	0.67	0.39	0.13	0.15
Total $\Delta E =$	-64.27	-78.96	-97.63	P = 46.37%	54.89%	64.66%

functions are used with the HF epsilons, and SM(2) the results when the denominator is replaced by multiples of $\hbar\omega = 16$ MeV and pure oscillator functions are used. Note that ΔE is overestimated in SM(1) and SM(2) by 50% in He^4 and that the admixture of 2p-2h states in the true ground state is likewise overestimated by a factor of two in He^4 and 1.5 in O^{16} .⁷ The significant difference between the results of the two methods, besides the total ΔE and $\sum P$, is the relative importance of 2p-2h states as a function of their unperturbed energy. In the shell-model approach 40% of the second-order energy shift (for He^4) comes from states of $2\hbar\omega$ and 30% from states of $4\hbar\omega$. Only about 9% of ΔE comes from states of more than $6\hbar\omega$. When the phase relations are considered, i.e., in the exact calculation, the contributions from the various shells increase to a maximum at -112 MeV, corresponding to between 6 and $8\hbar\omega$. In fact, the contribution from states at higher than $6\hbar\omega$ is 41% of the total. Similarly the probability of 2p-2h state admixture in the ground state of He^4 decreases roughly by a factor of 3 from shell to shell in the shell-model calculation while the $2\hbar\omega$ and $4\hbar\omega$ states are equally probable in the exact calculation. In the shell-model calculation only 10% of the total admixture comes from states above $4\hbar\omega$, while in the exact calculation these states constitute 40% of the total admixture. Quite similar statements can be made for the O^{16} results.

There, however, the effect of the phase relation is diminished because the lowest lying unoccupied states ($d_{5/2}$ and $s_{1/2}$) do turn out to be nearly pure harmonic oscillators. The results of Kerman and Pal⁴ indicate that the plane waves which contribute the most have approximately 100 MeV kinetic energy and it thus seems that using plane waves for unoccupied states is a better approximation than using pure oscillator functions.

The result of the plane-wave calculation for O^{16} was $\Delta E = -68$ MeV. The agreement between this and our exact result of -64.27 MeV seems to indicate that this method provides a very quick way to estimate ΔE . This, however, must be verified by considering other nuclei. In applying the plane-wave method to other nuclei the average ϵ for occupied states in O^{16} was used throughout.⁵ Thus only order-of-magnitude estimates were obtained and comparison is not meaningful. Calculations with appropriate average ϵ 's are in progress.

In these calculations no effort has been made to remove the spurious states corresponding to motion of the center of mass. Since the HF state has the center of mass in its ground state² and since the potential is only a function of relative coordinates, this correction can serve only to increase ΔE and the total probability for 2p-2h admixtures in the ground state. Furthermore, it is assumed here that higher lying states, e.g., $3d_{5/2}$ and $4d_{5/2}$, will not mix appreciably with the states included in this calculation. It is unlikely that these states will affect the phase relation since such states will not be connected to the $n=1$ states by the kinetic energy operator and their different radial dependences should render the potential terms small. A small mixing into the lower states is also reasonable from energy denominator considerations but this can only be verified by beginning with an even larger space. Such calculations are in progress.

Finally it should be noted that the use of a pure harmonic-oscillator representation for a perturbation expansion is certainly correct if one includes, also, one-particle, one-hole states. If, rather, one assumes that, since the occupied HF orbitals are nearly pure, a pure oscillator representation will approximate the HF unoccupied states, and carries out a perturbation expansion ignoring 1p-1h contributions, the results will be quite erroneous because of the ignored phase relations.

¹R. M. Tarbuton and K. T. R. Davies, Nucl. Phys.

A120, 1 (1968).

²W. H. Bassichis, B. A. Pohl, and A. K. Kerman, Nucl. Phys. A112, 360 (1968).

³A. K. Kerman, in Proceedings of the Summer School of Theoretical Physics, Cargèse, Corse, 1968 (to be published).

⁴A. K. Kerman and M. K. Pal, Phys. Rev. 162, 970 (1967).

⁵W. H. Bassichis, A. K. Kerman, and J. P. Svenne, Phys. Rev. 160, 746 (1967).

⁶F. Tabakin, Am. Phys. (N.Y.) 30, 51 (1964).

⁷P. Ellis and L. Zamick, to be published.

180° ELECTRON SCATTERING FROM ³He AND ⁴He AT 56 MeV

B. T. Chertok* and E. C. Jones†

The American University, Washington, D. C. 20016

and

W. L. Bendel and L. W. Fagg

U. S. Naval Research Laboratory, Washington, D. C. 20390

(Received 23 May 1969)

The nuclear magnetic structure of ³He has been investigated by 180° scattering of 56.6-MeV electrons. The first observation of the *M*1 continuum in ³He from 6 to 20 MeV is reported. The *M*1 component of the breakup from ³He(*e*, *e'*)*d*, *p* together with ³He(*e*, *e'*)*p*, *p*, *n* has been measured and is discussed in light of possible isovector and isoscalar meson-exchange currents in the trinucleon system. The elastic magnetic form factor of ³He at *q* = 0.561 fm⁻¹ is 0.80, giving a rms magnetic radius *a* = 1.94 ± 0.19 fm.

The nuclear magnetic structure of ³He has been investigated by 180° scattering of 56.6-MeV electrons from a gaseous ³He target. For comparison, an identical gas target of ⁴He, which has no known magnetic nuclear structure, was bombarded under the same experimental conditions.

We present here a report of the preliminary results which include measurement of a magnetic dipole continuum up to 20-MeV excitation energy in ³He, determination of the magnetic elastic electron scattering cross section at a momentum transfer of *q* = 0.561 fm⁻¹, and observation of electrons produced by magnetic bremsstrahlung from ³He. This is the first observation of the *M*1 continuum in ³He and will complement the well-studied *E*1 structure of ³He and ³H obtained by photodisintegration¹⁻³ and radiative capture measurements,⁴ and by electrodisintegration studies.^{5,6} The elastic magnetic form factor of ³He has been previously measured over a wide range of momentum transfer, 1.0 ≤ *q*² ≤ 8.0 fm⁻², by Collard *et al.*⁷ Electrons produced by magnetic bremsstrahlung from ¹H have been observed by Goldemberg⁸ and probably by several others.

Any discussion of the nuclear physics of ³He must cite the several experiments on the isospin doublet ³H and ³He which complement one another in a forceful way. At the same time one must draw attention to the rather prodigious theoretical complexities of the trinucleon systems which result from adding just one nucleon to the nucleon-nucleon potential. We shall review briefly the magnetic properties of ³H and ³He.

The ground-state magnetic moments of ³H and ³He are +2.9788 and -2.1274 n.m., respectively. By adding μ_{*M*1}(³H) + μ_{*M*1}(³He) and assuming a small ⁴*D*_{1/2} component together with the predominant ²*S*_{1/2}, the ⁴*D*_{1/2} weight is fixed at 3.8%. There are reasons for excluding the *P*_{1/2} states. When the individual moments are then calculated with 3.8% *D* state, there remains an isovector exchange moment (presumably due to meson currents) of 0.27 n.m.⁹ The Stanford measurements of elastic electron scattering from ³H and ³He provide both charge and magnetic form factors over a large range of momentum transfer. To fit these data, Schiff and Gibson¹⁰⁻¹² calculated *T* = ½ ground-state probabilities and found *P*_{*S*} = 92%, *P*_{*D*} = 6%, and *P*_{*S'*} ≤ 2% using spatial wave functions of the Irving-Gunn form, *e*^{-*aR*}/*R*^{*n*}. Their results were also constrained by other measurements, i.e., D(*n*, γ)³H, ³H *e*⁻ ³He, ³He(*e*, *e'*)*dp*, and ³He(μ⁻, ν)³H, and by variational calculations of binding energy of the trinucleons. The increased percentage of ⁴*D*_{1/2} state and possible very small admixture of *T* = ¾ *S'* component led Gibson to re-examine the isovector exchange moment which is required by the static magnetic moments. He found that both isovector and isoscalar exchange moments are required to fit the magnetic form factors.¹² Thus there is not only a sizable meson-exchange current in the isodoublet but it is presumably of different magnitude in ³He and ³H. Let us now look at some pertinent properties of the continuum of the trinucleon systems.