distribution from the spectral data presented. Indeed the most difficult problem here is that of appropriate unfolding of the line profile. However if we assume somewhat arbitrarily that the energetic excited atoms result mainly from resonant charge exchange on excited atomic neutrals, we can use the observed spectrum to estimate a distribution of fast ions within the plasma. The resulting average ion energy we find to be in rough agreement with that inferred from velocity analysis of escaping neutrals atoms (1-2 keV) as expected.

We wish to express our appreciation to B. S. Turner and J. G. Harris for their inventiveness and technical skill in the development of the experiment.

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OSMOTIC PRESSURE OF DEGENERATE He^3-He^4 MIXTURES*

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The osmotic pressure of He^3-He^4 mixtures has been measured down to 0.027°K. The results are used (a) to check the validity of the He^3-He^3 effective interaction deduced from transport data using recent exact solutions of the Boltzmann transport equation, (b) to obtain the limiting solubility of He^3 in He^4 as a function of pressure, and (c) to demonstrate the metastability of supersaturated mixtures of He^3 in He^4 in the absence of a free surface.

Recently, exact solutions for the Boltzmann equation of a degenerate Fermi liquid have been worked out by Brooker and Sykes[1] and by Højgard Jensen, Smith, and Wilkins.[2] The exact solutions have been employed by Ebner[3] to redetermine the Bardeen, Baym, and Pines[4] (BBP) He^3-He^3 effective interaction from experimental data on the transport properties of helium mixtures. The static or thermodynamic properties of mixtures also depend on the interaction, and one can make an independent test of the theory by comparing thermodynamic data with the predictions of BBP calculated from transport data. Previous tests of this sort[5-7] in the degenerate region have been only semiquantitative or have needed extrapolation of the Fourier transform $V(k)$ of the interaction to higher values of k than can be deduced from the transport data.

The osmotic pressure π, which can be thought of as the pressure of the He^3 quasiparticle gas, has a fairly large contribution from the effective interaction at 0°K and this is one reason why the present relatively precise measurements of π were undertaken. The experiments were performed at pressures P up to 20 atm so that, in addition, we could determine how the solubility of He^3 in He^4 varied with pressure. This result, as well as the metastability of supersaturated mixtures observed in the experiments, has some relevance to the possibility of a pairing condensation at low temperatures[4,8] and will be useful for the design and analysis of other experiments on mixtures under pressure.

The experimental cell was maintained at temperatures from 0.027 to 0.65°K by a dilution refrigerator, and contained two chambers each with a capillary, one for adding He^3 and one for pressurizing the system with He^4. The mixture chamber contained a cerium magnesium nitrate thermometer and communicated with the pure He^4 chamber through a superleak.[9] The differential pressure across the superleak, the osmotic pressure π, was measured with a specially designed diaphragm pressure gauge,[10] calibrated in situ against the earlier, absolute-osmotic-pressure measurements of Wilson, Edwards, and Tough[11] at 0.65°K.

Measurements obtained with the present apparatus at 0.32°K and He^3 mole fraction $X = 1.5\,\%$ agree very well with the data of Wilson, Edwards, and Tough. The conclusion of these authors, that there is a serious disagreement with Ebner's extension of the BBP theory to high temperatures,[12,13] remains unchanged.

(A) Osmotic pressure at 0°K and $P=0$.—Measurements at 0.26 atm were extrapolated with negligible error to 0°K and $P=0$ to give $\pi_0(P=0)$. The results, which are consistent with the less accurate values of π_0 deduced by Seligmann et al.[6] from heat-of-mixing measurements, are as follows:

X (%)	1.45	2.73	4.06	4.47	6.28	7.14
π_0 (Torr)	1.73	4.64	8.43	9.7	15.7	18.9

The data should be compared with the equation from the BBP theory[13]:

$$\pi_0 v_4^0 = \tfrac{2}{5}XRT_{\mathrm{F}0}[1+\tfrac{5}{8}X(1-\tfrac{3}{5}\alpha)]+\pi_{\mathrm{int}}v_4^0, \tag{1}$$

where

$$\pi_{\mathrm{int}}=\tfrac{1}{2}n_3^2V_0-\frac{3n_3^2}{(2k_{\mathrm{F}})^4}\int_0^{2k_{\mathrm{F}}}\left(1-\frac{k^2}{4k_{\mathrm{F}}^2}\right)V(k)k^3dk. \tag{2}$$

Here X is the mole fraction of He^3, v_4^0 is the molar volume of He^4 at 0°K, and α is the BBP parameter defined by the empirical expression for the He^3 number density n_3:

$$n_3=N_{\mathrm{A}}X/(1+\alpha X)v_4^0, \tag{3}$$

where N_{A} is Avagadro's number. The first term on the right-hand side of Eq. (1) represents the "kinetic pressure" of the He^3, in which the Fermi temperature is given by $k_{\mathrm{B}}T_{\mathrm{F}0}=\hbar^2k_{\mathrm{F}}^2/2m_0^* = (\hbar^2/2m_0^*)(3\pi^2n_3)^{2/3}$ using the effective mass at $X=0$, $m_0^*=(2.34\pm0.04)m_3$ (Ref. 4). This term has been subtracted from the data to give empirical values of the interaction term, $\pi_{\mathrm{int}}v_4^0$, which are compared with the theoretical curve in Fig. 1. The theoretical expression [Eq. (2)] has been evaluated using Ebner's new version[3] of the effective interaction $V(k)$. Strictly, Ebner's $V(k)$ is valid only up to $X=5\%$.

The error bars on the empirical values arise mainly from the uncertainty in the effective mass m_0^*. The discrepancy between theory and experiment which is equivalent to about 10-20% in $V(k)$ is larger than the experimental uncertainty and is a factor 2 worse than the fit obtained to the transport data. On the other hand, it is strikingly better than the serious disagreement which exists in the nondegenerate region[11] at 0.32°K. As Ebner[3] has remarked, the theory is probably oversimplified in treating the quasiparticle scattering amplitude as being concentration and velocity independent.

(B) The limiting solubility $X_0(P)$.—When the He^3-He^4 mixture had separated into two phases, our experimental method yielded the osmotic pressure of the lower, superfluid phase $\pi^l(P,T)$. This saturated osmotic pressure is, of course, independent of the nominal concentration of the mixture. Extrapolation to $T=0$°K gives $\pi_0^l(P)$ from which, using graphs of our single-phase data, we obtained the concentration $X_0(P)$ in the saturated, lower phase at 0°K. The results are shown in Fig. 2, with error bars drawn only for the random errors. There is an additional estimated ±0.06-mole% systematic error in $X_0(P)$ due to the determination of the sample-mixture concentration.

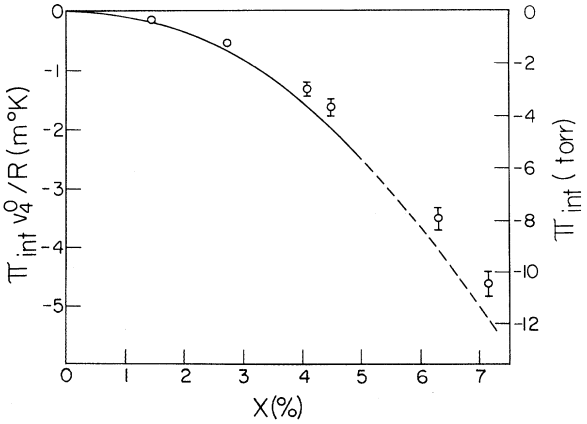

FIG. 1. Contribution to the osmotic pressure at zero temperature and pressure $\pi_0(P=0)$ due to the He^3-He^3 effective interaction. The points are empirical values defined by Eq. (1), while the curve is the theoretical result from Eq. (2).

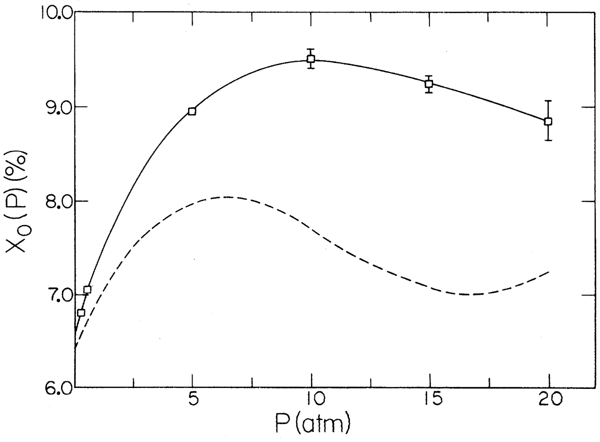

FIG. 2. Saturation He^3 concentration at 0°K as a function of pressure. The error bars include only random errors and are shown when larger than the data symbols. The dashed curve is a calculation of $X_0(P)$ from Ref. 7.

The low-pressure data extrapolate to $X_0(P=0) = (6.60 \pm 0.15)\%$, in reasonable agreement with previous determinations.[14-16] The value of $\pi_0{}^l\,(P=0) = (16.8 \pm 0.4)$ Torr agrees with the value (17.2 ± 0.5) Torr reported by London, Phillips, and Thomas.[17]

Although $X_0(P)$ has not been previously measured, the data can be compared with a thermodynamic calculation[7] of $X_0(P)$ based upon the empirical pressure dependence of α,[18] which is shown by the dashed line in Fig. 2. It is likely that the discrepancy is due to the measured values of α being too large, especially at the lower pressures.

(C) Supersaturated solutions.—Seligmann *et al.*[6] have suggested that mixtures with concentrations up to at least 16% should be metastable with respect to phase separation unless a nucleus of very concentrated He^3 can be formed. The fact that supercooling has not been observed in experiments at the saturated vapor pressure indicates that the liquid-vapor surface, where Andreev[19] has demonstrated the existence of a He^3-enriched film, acts as such a nucleus. Supercooling has been observed in the present experiments, and in similar ones at Cornell,[20] where the capillary to the sample mixture was completely filled to a temperature outside the two-phase region, eliminating the free surface.

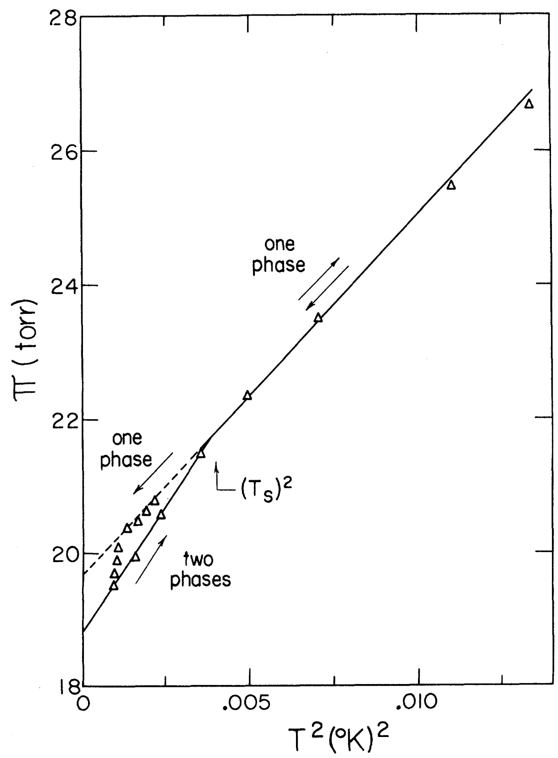

FIG. 3. Osmotic pressure of a 7.28% solution at 0.53-atm pressure showing supercooling below the phase-separation temperature T_S. The dashed line is a continuation of the single-phase osmotic pressure. Data on this line indicate the mixture is supercooled. The transition from the metastable, supersaturated, single-phase state to the stable two-phase state was spontaneous.

Figure 3 shows the supercooling of a 7.28% mixture at a pressure $P = 0.53$ atm. In this particular example, the He^3-rich phase appeared spontaneously, and the time required to attain the equilibrium two-phase value after nucleation was about 2 h, indicating that the nucleus was in the capillary. In other examples, there was no spontaneous nucleation for a period of up to 3 h, and the transition was induced by lowering the pressure on the system. In all cases, the subsequent behavior of the osmotic pressure indicated that the He^3-rich phase was in the capillary. We did not attempt to observe supercooling at pressures greater than 0.53 atm, or at concentrations more than about 0.3 mole% greater than X_0.

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BROKEN SYMMETRY AND DECAY OF ORDER IN RESTRICTED DIMENSIONALITY

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The ordering of one- and two-dimensional systems with a continuous symmetry is considered in the *absence* of a symmetry-breaking field. It is shown rigorously that no spontaneous ordering can occur; bounds on the order-order correlation function integrated over a subdomain indicate how the short-range order decays with distance.

It has been appreciated heuristically for some time that in a one- or two-dimensional system, i.e., a system of finite cross section or thickness, which has a continuous symmetry (such as the gauge invariance of a Bose fluid or rotational isotropy in a ferromagnet), the fluctuations in the order parameter are so large as to destroy any ordered state with spontaneously broken symmetry even though such can arise in the fully three-dimensional system. Hohenberg[1] has demonstrated that Bogoliubov's inequality,

$$\tfrac{1}{2}\langle\{A, A^\dagger\}\rangle \geq k_B T|\langle[C, A]\rangle|^2/\langle[[C, \mathcal{H}_\Omega], C^\dagger]\rangle, \quad (1)$$

in which $\mathcal{H}_\Omega$ is the Hamiltonian for the system confined to a domain Ω, can be used to substantiate this idea, and Mermin and Wagner[2] have proven that if the dimensionality of Ω is less than three, an isotropic Heisenberg ferromagnet can exhibit no spontaneous magnetization, i.e.,[3]

$$M_0(T) = \lim_{H\to 0^+} M(T, H) = 0 \quad (T > 0). \quad (2)$$

As indicated by (2) [see also Chester, Fisher, and Mermin,[4]] the existing proofs[5] first introduce a *symmetry-breaking field* η (the magnetic field H for a ferromagnet), *then* proceed to the thermodynamic limit [volume $V(\Omega) \to \infty$], and finally, show that the induced order parameter $\Psi(\eta)$ vanishes as the field η is removed ($|\eta| \to 0$). For a magnet $\Psi \sim M$, while for a Bose fluid one considers

$$\Psi(T, \eta) = \lim_{V(\Omega)\to\infty} [V(\Omega)]^{-1} \int_\Omega \langle\psi(\vec{r})\rangle_\Omega d\vec{r}. \quad (3)$$

These results are satisfying, but they leave open some more fundamental questions, namely:

(A) How does the order-order correlation function $\sigma(\vec{r}, \vec{r}')$ behave as $|\vec{r}-\vec{r}'| \to \infty$? For a magnet with localized spin variables $\vec{S}(\vec{r})$ we may take

$$\sigma(\vec{r}, \vec{r}') = \langle S_z(\vec{r})S_z(\vec{r}')\rangle \text{ or } \langle S_+(\vec{r})S_-(\vec{r}')\rangle. \quad (4)$$

One would like to say something about the rate of decay and to prove that $\sigma \to 0$ as $|\vec{r}-\vec{r}'| \to \infty$, so as to demonstrate the absence of long-range order [$\sigma(\infty) \equiv 0$]; but as a matter of fact, even when (2) holds one cannot be sure that $\sigma(\infty) = 0$.[6] For a Bose fluid one is interested in the off-diagonal order or one-body density matrix

$$\sigma(\vec{r}, \vec{r}') = \langle\psi^\dagger(\vec{r}')\psi(\vec{r})\rangle. \quad (5)$$

A second question is the following:

(B) Can one dispense with the symmetry-breaking field in proving the absence of ordering? (This question is especially pertinent for a Bose fluid,[7] where the relevant "off-diagonal" field cannot be realized physically.) An answer might be provided by considering (with $\eta \equiv 0$) the rms order parameter Ψ_σ defined by

$$(\Psi_\sigma)^2 = \lim_{V(\Omega)\to\infty} [V(\Omega)]^{-2} \times \int_\Omega d\vec{r} \int_\Omega d\vec{r}'\, \sigma_\Omega(\vec{r}, \vec{r}'), \quad (6)$$