mates of 3Γ, which must be considerably smaller than for CaF_2.

The zero-phonon transition from the orbital singlet ground state of Eu^{2+} to the A level is expected to be forbidden in absence of strain due to vanishing vibrational overlap with the singlet ground state.[5] However, this tunneling level has been observed optically by Kaplyanskii and Przevuskii[9] who measured the stress splitting of the E level of Eu^{2+} in CaF_2 and SrF_2 and found a weak transition, with strain-dependent intensity, to an unidentified level at 15.3 and 6.5 cm^{-1}, respectively, above the E level. In SrF_2 this level is close enough to anticross with the split components of E and borrow transition moment from them when stress is applied along [001] and [110]. The magnitude of the stress coupling of this level to E and the diagonal matrix elements of the stress within the E level can be related by the wave functions in (1). The result is, for $\gamma \ll 1$, $\langle A_1|e_\theta|E_\theta\rangle/\langle E_\theta|e_\theta|E_\theta\rangle = \sqrt{2}$. This ratio was used in Eq. (3) to obtain 3Γ from the ESR data. In the opposite limit where the second-order coupling is negligibly small so that the levels are almost purely rotational, this ratio can be shown to be unity. The measured strain data[9] give a ratio of 1.49 for which we estimate a possible error of up to 10% from the spread in the published data. It thus appears that the optical data for $SrF_2:Eu^{2+}$ are better fitted by the tunneling model of Eq. (1). On the basis of the evidence we propose that the work of Ref. 9 is the first conclusive optical observation of such a tunneling level.

Selective broadening similar to that of Fig. 1 is also found in the resonances of Cu^{2+} in MgO and CaO[10] and possibly in Sc^{2+} in CaF_2[2] and can be used to estimate the position and symmetry of the tunneling level in these cases.

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DIRECT OBSERVATION OF THE LORENTZIAN LINE SHAPE AS LIMITED BY QUANTUM PHASE NOISE IN A LASER ABOVE THRESHOLD*

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The quantum-phase-noise–limited Lorentzian power spectral densities of $Pb_{0.88}Sn_{0.12}Te$ diode lasers were directly measured. Linewidths as narrow as 54 kHz were observed and quantitatively analyzed. The predicted inverse dependence of linewidth on laser power was also demonstrated. These results were obtained from the beat-note spectra produced by heterodyning the diode laser, operating cw at 10.6 μm, with a stable, single-frequency CO_2 gas laser.

This paper describes measurements of the output power spectra of $Pb_{0.88}Sn_{0.12}Te$ diode lasers above the threshold of oscillation.[1] The observed spectral widths as narrow as 54 kHz were dominated by fluctuations of the oscillation phase due to spontaneous emission (quantum phase noise), rather than extraneous modulation caused by environmental disturbances.

Theoretical results[2] for amplitude and phase noise in lasers have been generally derived from solutions of equations for a van der Pol oscillator with appropriate noise sources. Thus it was predicted that the output power spectrum of a single-frequency laser may be approximated by a Lorentzian profile whose width is given by

$$\Delta\nu = A\alpha\pi h\nu_0(\Delta\nu_c)^2/P, \qquad (1)$$

where h, ν_0, $\Delta\nu$, and $\Delta\nu_c$ denote Planck's constant, the laser center frequency, and the full widths between half-power points of the laser output and the "cold" cavity, respectively. P is the laser power external to the cavity, and includes the (often negligible) power lost by absorption and scattering in the reflecting surfaces. $\alpha = N_2[N_2-(g_2/g_1)N_1]^{-1}$ represents the degree of population inversion, with N_2, g_2, N_1, and g_1 denoting the populations and degeneracies of the upper and lower levels, respectively. A is a parameter whose value is unity well above threshold and gradually increases to 2 well below the threshold of oscillation. Equation (1) with $A=2$ yields the linewidth originally predicted by Schawlow and Townes.[3] The "cold"-cavity bandwidth is defined[4] by

$$\Delta\nu_c \equiv \nu_0/Q_c = (\beta/\pi)\Delta\nu_{\text{axial}}, \qquad (2)$$

where β represents the total single-pass transmission, absorption, diffraction, and scattering losses of the cavity without any gain, and $\Delta\nu_{\text{axial}}$ the frequency difference between adjacent cavity modes.

Measurements of amplitude noise[5] in gas lasers agree remarkably well with theory; however, experiments designed to measure phase noise and to verify Eq. (1) have always been hampered by environmental disturbances which mask the spontaneous emission noise by introducing additional random-noise-like modulations of the laser output. The difficulty arises from the inherently narrow linewidths of gas lasers (usually considerably less than 10 Hz) imposed by the spontaneous emission. To our knowledge, all previous attempts to measure phase noise in gas lasers involved observations of fluctuations in the beat note obtained by heterodyning either two similar single-mode lasers or two modes of a single (multimode) laser. Neglecting amplitude fluctuations, the observed beat note varies as $\cos[2\pi\nu_b t + \varphi(t)]$, where ν_b is the difference between the two optical frequencies, and $\varphi(t)$ represents fluctuations in the phase. Measurements of the spectral density of the beat note even under the quietest environmental conditions[6,7] resulted in linewidths which were more than two orders of magnitude greater than the predictions of Eq. (1). In more recent experiments,[8-10] attempts were made to separate the spontaneous-emission phase noise from extraneous modulation by determining the power spectral density of the frequency fluctuations $\dot{\varphi}(t)$ and attributing the high-frequency portion to quantum phase noise.

In a typical semiconductor laser the combination of lower output power and an inherently smaller cavity Q_c produces a linewidth which is orders of magnitude greater than that of even a low-power gas laser. Nevertheless, attempts to measure the linewidth of GaAs diode-laser emission[11-13] have resulted only in upper limits defined by the measuring equipment.

By heterodyning the coherent emission from a liquid-helium-cooled $Pb_{0.88}Sn_{0.12}Te$ diode laser, operating cw near 10.6 μm, with that from a stable, single-frequency CO_2 gas laser, we have made direct observations of the Lorentzian distribution of the power spectral density of the diode laser and verified the inverse power dependence of the linewidth given in Eq. (1). These heterodyne experiments were performed by combining the diode- and gas-laser radiation in a liquid-helium-cooled Ge:Cu detector. The resulting beat note is displayed directly on a spectrum analyzer; the frequency of the beat note can be varied continuously by current-tuning the wavelength of the diode-laser emission.[14] As previously shown,[15] the linewidth of a 1-W CO_2 laser is sufficiently narrow that its power spectral density may be treated as a delta function in interpreting the spectrum-analyzer display.

Figure 1 shows the beat note between a 240-μW beam from a single-mode $Pb_{0.88}Sn_{0.12}Te$ diode

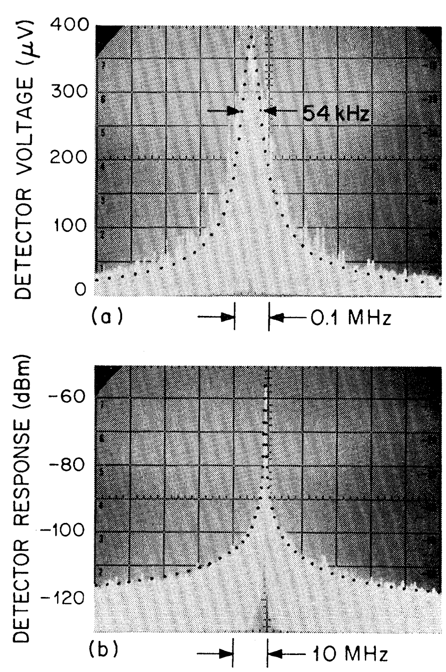

FIG. 1. Spectrum-analyzer display of beat note between a 240-μW single-frequency $Pb_{0.88}Sn_{0.12}Te$ diode laser (well above threshold) and the $P(14)$ transition of the CO_2 gas laser. I.f. bandwidth is 10 kHz. Sweep rates and exposure times are (a) 0.2 sec/cm and 2 sec and (b) 0.002 sec/cm and 0.5 sec.

laser, well above threshold, and the $P(14)$ transition of the CO_2 laser. Figures 1(a) and 1(b) are different representations of the same beat note, with the ordinate linearly proportional to the detector voltage in 1(a), but proportional to the logarithm of detector voltage in 1(b). The dotted lines correspond to Lorentzian fits to the power spectral density:

$$S(\nu)=\frac{1}{1+[2(\nu-\nu_0)/\Delta\nu]^2}$$

with $\Delta\nu=54$ kHz. In Fig. 1(b) the Lorentzian profile holds over the entire 60-dB power range displayed, as it should for a laser well above the threshold of oscillation.[2]

With Eqs. (1) and (2) one can, in principle, predict the laser linewidth from the population inversion, output power, axial-mode spacing, and total single-pass cavity loss. Since the diode laser is well above threshold for Fig. 1 (923 mA versus threshold current of approximately 400 mA), $A\simeq 1$. Because of the low operating temperature, the population inversion parameter α is also expected to be near unity.[16] The scattering and diffraction losses are small compared with the total loss and will be neglected in this analysis. The single-pass transmission loss is $1-R=0.4$. The reflectivity R was calculated from the effective refractive index ($n\simeq 8$) obtained from the measured axial-mode spacing of $\Delta\nu_{\rm axial}=c/2nL=29.4$ GHz, where the cavity length L is 0.0635 cm. Accurate determination of the cavity absorption loss is difficult to make, although an approximate value may be obtained by measuring the absorption coefficient of material having the same chemical composition as that used in the diode laser, but with a low carrier concentration to simulate the active region. The absorption constant of a thin slab of $Pb_{0.88}Sn_{0.12}Te$ was found to be a strong function of temperature and photon energy near the bandgap, yielding values of 10-30 cm^{-1} in the laser-action region. The theoretical linewidth may be brought into exact correspondence with the experimentally measured value of 54 kHz if we assume a single-pass absorption loss $\beta_a/L=18.6$ cm^{-1}. It should be noted that the effective absorption constants for other abrupt-junction diode lasers[17] are also in the neighborhood of 20 cm^{-1}.

The diode-laser output power for Fig. 1 was in a single TEM_{30} mode, with the radiation linearly polarized in the junction plane. The far-field pattern was describable by Hermite-Gaussian functions, which are eigenmodes of confocal resonators with rectangular mirrors.[4] (Similar mode patterns have been reported for GaAs diode lasers.[18]) Individual lobes of the TEM_{30} pattern produced identical beat frequencies (at the same diode current) when heterodyned separately with the single-spot TEM_{00} beam from the CO_2 laser. It is improbable that filamentary diode emission could have yielded such a result. Moreover, from the far-field measurements we determined that the diode-laser emission emanated from an approximately 0.004×0.022 cm^2 portion of the cleaved end face, the latter dimension in agreement with microscopic examination of the width of this face. The total cavity power used in Eq. (1) for the above calculation is, therefore, 240 μW.

In order to demonstrate the inverse power variation of linewidth, the diode laser was operated at different power levels, with the results shown in Fig. 2. Although the current was reduced from 865 mA in Fig. 2(a) to 845 mA in 2(b), the beat frequency and junction temperature were held constant by a simultaneous increase (less than 1°K) in heat-sink temperature. The dotted lines of Fig. 2 represent theoretical Lorentzian enve-

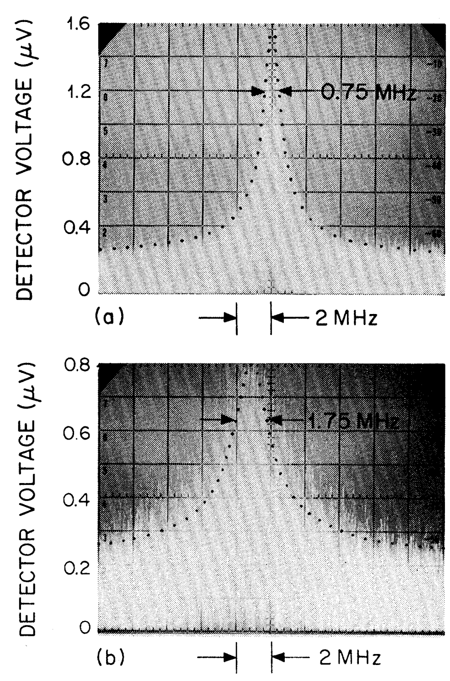

FIG. 2. Spectrum-analyzer display of beat note between low-power diode laser mode and $P(16)$ transition of the gas laser, corresponding to a diode-laser current of (a) 865 mA and (b) 845 mA. Center frequency is 92 MHz (maintained by adjustment of heat-sink temperature), and i.f. bandwidth is 30 kHz. Sweep rate is 0.01 sec/cm, and exposure time, 1 sec.

lopes (including 0.2-μV flat amplifier noise) where the linewidth $\Delta\nu$ is 0.75 MHz for Fig. 2(a), and 1.75 MHz for 2(b). Relative values for the mode power producing each beat note can be obtained by integration under the Lorentzian power spectral density envelopes, and the resulting power ratio of $(1.6-0.2)^2(0.75)/(0.8-0.2)^2(1.75)$ is identical with the reciprocal linewidth ratio of 1.75/0.75, in agreement with Eq. (1). (The predicted inverse power dependence of linewidth has been observed for other $Pb_{0.88}Sn_{0.12}Te$ diode lasers as well.) Since the low-power diode-laser mode of Fig. 2 oscillated together with a much stronger mode (beyond the beat frequency range of the detector), no direct power measurements were made. Furthermore, since these curves were produced with the $P(16)$ CO_2 transition, whereas those of Fig. 1 involved the $P(14)$ transition, direct comparisons of linewidth versus power between Figs. 1 and 2 are not meaningful.

We believe that the data presented here verify the general validity of Eq. (1) in describing the quantum noise limit imposed upon the spectral width of a laser far above the threshold of oscillation. However, further experiments are planned to determine more precisely the cavity losses and inversion parameter. By additional measurements of both phase and amplitude noise near threshold, we hope not only to determine these values more exactly, but also to show the change in the parameter A from 2 to 1 as the laser output is increased from below to above the threshold of oscillation.

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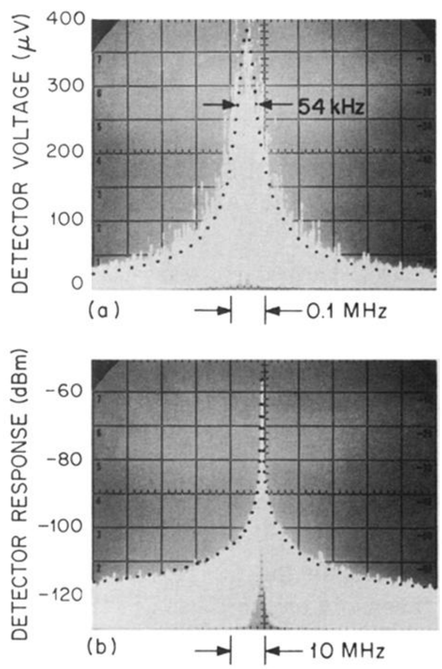

FIG. 1. Spectrum-analyzer display of beat note between a 240-μW single-frequency $Pb_{0.88}Sn_{0.12}Te$ diode laser (well above threshold) and the $P(14)$ transition of the CO_2 gas laser. I.f. bandwidth is 10 kHz. Sweep rates and exposure times are (a) 0.2 sec/cm and 2 sec and (b) 0.002 sec/cm and 0.5 sec.

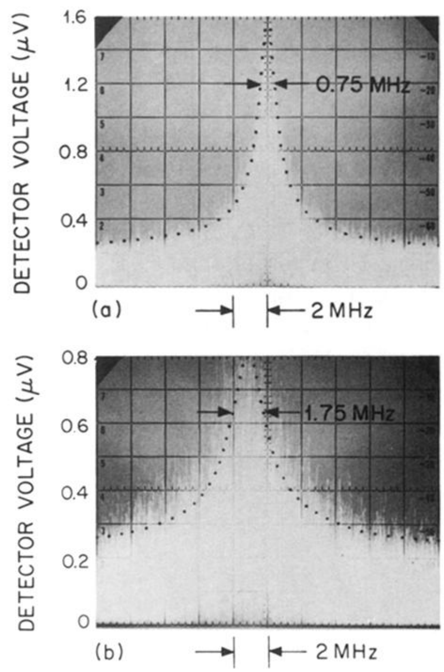

FIG. 2. Spectrum-analyzer display of beat note between low-power diode laser mode and $P(16)$ transition of the gas laser, corresponding to a diode-laser current of (a) 865 mA and (b) 845 mA. Center frequency is 92 MHz (maintained by adjustment of heat-sink temperature), and i.f. bandwidth is 30 kHz. Sweep rate is 0.01 sec/cm, and exposure time, 1 sec.