BREAKING DUALITY*

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A qualitative understanding of the observed pattern of Regge trajectories is obtained from approximate duality.

By examining the assumptions underlying duality,[1] we formulate a concept of broken duality which allows an understanding of the observed patterns of Regge trajectories.

Exact duality[2] implies exchange degeneracies between meson trajectories, baryon trajectories,[3,4] and the existence of exotic meson trajectories,[5] which should be approximately degenerate with ordinary meson trajectories. While a few of these exchange degeneracies are extremely well satisfied in nature, others are badly violated. In fact, many of the trajectories predicted seem to be entirely absent. These difficulties arise through the assumption of high-energy resonance saturation and low-energy Regge-pole dominance, but are removed by specifying the ranges of validity of these assumptions.

Implicit in exact duality is the decomposition[2] of scattering amplitudes into diffractive and nondiffractive parts. The nondiffractive pieces are assumed to be represented simultaneously either by resonances in one channel or Regge poles in another. From these assumptions, in particular the absence of an imaginary part in nonresonating channels, certain exact exchange degeneracies are predicted.[3] Exchange-degenerate nonets are required in meson-meson scattering, patterns such as $\underline{1}\oplus\underline{8}$ exchange degenerate with $\underline{8}\oplus\underline{10}$ are necessary in meson-baryon scattering, and exotic $(\underline{27},\underline{10},\underline{10}^*)$ mesons approximately exchange degenerate with ordinary mesons are needed in baryon-baryon scattering. While mesons do occur in signature-doubled nonets, the π and the X_0 are far from degenerate. For baryons, at least a $\underline{70}$ $L=0$ and $\underline{70}$ $L=1$ are required to be exchange degenerate with the $\underline{56}$ $L=0$. The $\underline{70}$ $L=1$ is observed, but the $\underline{70}$ $L=0$ is not. In particular, the $\frac{3}{2}^+$ decuplet should be accompanied by at least $\frac{5}{2}^-$ and $\frac{3}{2}^+$ octets. The $\frac{5}{2}^-$ is observed where expected, but there is no trace of the $\frac{3}{2}^+$ octet which should have been degenerate with the $\frac{3}{2}^+$ decuplet. The exotic mesons, which should have accompanied all ordinary mesons, seem absent.

The crucial assumption leading to exchange degeneracies is that the nondiffractive piece of a scattering amplitude may be expressed in Regge form with its imaginary part vanishing in exotic channels. This is expressed as[3]

$$\sum_{b,i}(X_{st})^{ab}\beta_i{}^b(t)s^{\alpha_i{}^b(t)}=0,$$
$$\sum_{b,i}(X_{ut})^{ab}\tau_i{}^b(t)s^{\alpha_i{}^b(t)}=0, \tag{1}$$

where X_{ts} is the crossing matrix from the s to the t channel and $\tau_i{}^b$ is the signature. At high energies, only the resonance saturation assumption, the right-hand side, could fail at positive t, while at low energies the Regge parametrization, the left-hand side, is expected to break down. The presence of Regge cuts, which at extremely high energies dominates the leading poles for $t<0$, provides an additional breakdown of the left-hand side of the duality equations, indicating that <u>the validity of their consequences for both $\alpha(t)$ and $\beta(t)$ should deteriorate with increasing $-t$.</u> We confine the succeeding discussion to the region of positive t.

The extent to which the assumptions of resonance saturation and Regge behavior fail allows us to estimate the deviations from exchange degeneracy. There should be a region of s and t inside of which our equations will be accurate to a precision ϵ:

$$\frac{|\sum_{b,i}(X_{st})^{ab}\beta_i{}^b(t)s^{\alpha_i{}^b(t)}|}{\sum_{b,i}|(X_{st})^{ab}\beta_i{}^b(t)s^{\alpha_i{}^b(t)}|}\leqslant\epsilon,$$
$$s_{\min}\leqslant s\leqslant s_{\max}. \tag{2}$$

$s_{\max}$ is the point where the error made by assuming resonance saturation is ϵ, and is expected to be related to the importance of inelastic channels. $s_{\min}$, the point where an error ϵ is made by assuming the Regge parametrization, should be related to the position of threshold. If only two trajectories couple to a reaction, the difference between their trajectory functions, $\Delta\alpha$, is given by

$$(s_{\max}/s_{\min})^{|\Delta\alpha|}=[(1+\epsilon)/(1-\epsilon)]^2. \tag{3}$$

We can use these ideas in the limit of exact SU(3) symmetry to extract the qualitative character of the breaking of exchange degeneracy. However, because the point at which the Regge parametrization fails depends strongly on the masses of the external particles in a reaction, more precise conclusions would be obtained with the actual values of their masses.

For mesonic reactions for which the quantum numbers are the same (so that s_{max} is effectively the same), Eq. (3) shows that the larger the external particle masses (and therefore the larger the s_{min}), the larger $\Delta\alpha$ will be. Exchange degeneracies which come from the consideration of reactions with smaller external masses should be better than those which come from higher mass reactions. The most reliable mesonic exchange degeneracy comes from consideration of pseudoscalar-pseudoscalar (P-P) scattering and is the degeneracy of the octet of vector-meson trajectories with the nonet of tensors. Next in reliability should be the predictions following from pseudoscalar-vector (P-V) scattering. Here we find that the ninth vector meson should be degenerate with the octet, that the octet of pseudoscalar trajectories should be degenerate with the nonet of axial vector $\mathcal{C}=-1$ trajectories [including $B(1220)$], and that the octet of axial vector $\mathcal{C}=+1$ trajectories [including $A_1(1070)$] should be degenerate with a nonet of pseudotensor $\mathcal{C}=-1$ trajectories.

Only two new trajectories are predicted by including the least reliable meson reaction, V-V scattering. They are the ninth pseudoscalar and axial-vector $\mathcal{C}=+1$ trajectories which are predicted to be the least degenerate with their respective octets. This hierarchy of degeneracy breaking is confirmed experimentally. Specifically, the φ and f' are close to their respective octets, while the X_0 is far removed from the pseudoscalar octet. Our expectation that the ninth axial-vector $\mathcal{C}=+1$ state is widely split from the octet leads us to the conclusion that the Gell-Mann–Okubo mass formula (without mixing) should hold. Therefore, the $D(1285)$ completes the 1^{++} octet along with the A_1 and $K^*(1240)$.

From the point of view of broken duality, baryon-antibaryon (B-$\overline{B}$) channels are highly unreliable; s_{min} could, in fact, exceed s_{max}, totally invalidating the degeneracy equations (1). Therefore, the exchange degeneracies coming from the baryon cross-channel in meson-baryon scattering should be far more accurate than those coming from the meson channel. The simplest patterns derived from the baryon channel are either a decuplet exchange degenerate with an octet whose F/D is $-\frac{1}{3}$, or two exchange-degenerate octets, although there are many more complicated solutions. The observed quark representations $\underline{56}$ $L=0$ and $\underline{70}$ $L=1$ satisfy the baryon-channel exchange-degeneracy constraints.

In particular, the $\frac{3}{2}^+$ decuplet is required to be exchange degenerate with the $\frac{5}{2}^-$ octet, whose F/D should be $-\frac{1}{3}$. Its measured F/D is approximately -0.2.[6] Had we, in addition, used the equations coming from the meson channel, an unobserved octet of $\frac{3}{2}^+$ baryons degenerate with the $\frac{3}{2}^+$ decuplet would have been required.[7]

The approximate exchange-degeneracy pattern for the leading quark doublet trajectories (the $\frac{1}{2}^+$ octet, and the $\frac{3}{2}^-$ singlet, octet, and decuplet) is 8 exchange degenerate with $1\oplus8\oplus10$. There are many coupling patterns compatible with this solution to the broken duality equations; however, the observation that the $\frac{3}{2}^-$ decuplet couples weakly to P-B even though it is very broad, and so presumably strongly coupled to P-Δ, leads to unique F/D values for both octet trajectories. Note that the F/D value is constant along each of the three leading octet trajectories, $(\frac{1}{2}^+,\frac{5}{2}^+,\cdots)$, $(\frac{3}{2}^-,\frac{7}{2}^-,\cdots)$, and $(\frac{5}{2}^-,\cdots)$. The F/D value for the $\frac{1}{2}^+$ and $\frac{3}{2}^-$ octet trajectories should be 1, as compared with the measured values $F/D\approx1.0$ for the $\frac{1}{2}^+$ octet (determined[8] from $g_{\Sigma NK}^2/g_{\Lambda NK}^2=0.02\pm0.03$, which is predicted to be 0, and $g_{\Lambda NK}^2/g_{NN\pi}^2=1.1\pm0.2$, which is predicted to be $\frac{4}{3}$), and $F/D\approx1.2$ for both the $\frac{3}{2}^-$ and $\frac{5}{2}^+$ octets.[6]

We expect that the exchange degeneracies coming from P-B scattering should be better than those coming from P-Δ scattering. P-B gives degeneracy between the octet $\frac{1}{2}^+$ trajectory and the octet and singlet $\frac{3}{2}^-$ trajectories, while P-Δ scattering gives degeneracy between the octet $\frac{1}{2}^+$ trajectory and the octet and decuplet $\frac{3}{2}^-$ trajectories. The large violations of these exchange degeneracies may be related to the fact that three trajectories here contribute to each degeneracy equation, while in the previously discussed cases only two trajectories contributed to each equation.

Broken duality resolves the problem of B-$\overline{B}$ scattering; exact duality predicted exotic mesons approximately degenerate with the ordinary mesons, but this prediction came from consideration of the B-$\overline{B}$ channel. These exotic mesons, and presumably also the $\underline{70}$ $L=0$ of baryons, if present at all, should be high in mass.

The SU(3)-symmetric results are summarized in Table I. More detailed predictions follow from consideration of specific reactions. Mesonic re-

Table I. Summary of SU(3)-symmetric results.

A. Degeneracy Patterns[a]

Mesons: Exchange degenerate nonets of opposite P and C.

Baryons: (1) Negative natural parity -- 10 exchange degenerate with 8.

(2) Positive natural parity -- 8 exchange degenerate with $1 \oplus 8 \oplus 10$.

B. Reactions Giving Specific Degeneracies

Trajectories Related	Elastic Reactions
$8(1^{--})$ - $9(2^{++})$	PP
$9(1^{--})$ - $8(2^{++})$ $8(1^{++})$ - $9(2^{--})$ $8(0^{-+})$ - $9(1^{+-})$	PV
$9(1^{++})$ - $8(2^{--})$ $9(0^{-+})$ - $8(1^{+-})$	VV
$10(3/2^{+})$ - $8(5/2^{-})$ $8(1/2^{+})$ - $(8 \oplus 1)$ $(3/2^{-})$	PB
$8(1/2^{+})$ - $(8 \oplus 10)$ $(3/2^{-})$	PΔ

C. Meson Mixing

The ninth $(1^{++}, 3^{++}, \ldots)$ and $(0^{-+}, 2^{-+}, \ldots)$ trajectories are split far from their respective octets, so that these octets are essentially unmixed.

D. F/D Ratios for Coupling to PB

(F/D is constant along each trajectory)

Octet	F/D Predicted	F/D Experimental
$1/2^{+}$, $5/2^{+}$	1	1.0, 1.2
$3/2^{-}$	1	1.2
$5/2^{-}$	-1/3	-0.2

[a]The Toller M quantum numbers of trajectories related through exchange degeneracy are the same.

Table II. Approximate exchange degeneracies and the reactions giving them.

Trajectories Related	Elastic Reaction
ρ f_o	$\pi\pi$
ρ A_2 ω ϕ f_o f_o'	$K\bar{K}$
ω A_2 $A_1(I=0,\ 2^{--})_{1,8}$ π $(I=0,\ 1^{+-})_{1,8}$	$\pi\rho$
π B η $(I=0,\ 1^{+-})_{1,8}$ $A_1(I=1,\ 2^{--})$ D $(I=0,\ 2^{--})_{1,8}$	$\bar{K}K^*$
η B	$\rho\rho$
η X_o $(I=0,\ 1^{+-})_{1,8}$ D $(I=0,\ 1^{++})_1$ $(I=0,\ 2^{--})_{1,8}$	$K^*\ \bar{K}^*$
$K^*(890)$ $K^*(1420)$	πK
K(495) $K_A(1320)$ $(I=1/2,\ 2^{--})$ $K^*(1240)$	πK^*
$\Delta(3/2^{+}, 7/2^{+}, 11/2^{+})$ $N(5/2^{-})$ $N(1/2^{+}, 5/2^{+})$ $N(3/2^{-}, 7/2^{-})$	$\pi\Delta$
$\Sigma(3/2^{+}, 7/2^{+})$ $\Lambda(5/2^{-})$ $\Sigma(1/2^{+}, 5/2^{+})$ $\Lambda(3/2^{-})_8$	$\pi\Sigma$
$\Lambda(1/2^{+}, 5/2^{+})$ $\Lambda(3/2^{-}, 7/2^{-})_1$ $\Sigma(1/2^{+}, 5/2^{+})$ $\Sigma(3/2^{-})$ $\Sigma(3/2^{+}, 7/2^{+})$ $\Sigma(5/2^{-})$	$\bar{K}N$
$\Xi(1/2^{+})$ $\Xi(3/2^{-})$ $\Xi(3/2^{+})$ $\Xi(5/2^{-})$	$\pi\Xi$
no trajectory accompanying $\Omega^{-}(3/2^{+})$	$\bar{K}\Xi$

actions remain elastic to very high energies and therefore the position of $s_{\min}$ will primarily determine the accuracy of the exchange degeneracies. In particular, it is only in the highest mass meson reaction, K^*K^*, that we must introduce the X^0 and the ninth 1^{++}, and so we expect these trajectories to be relatively far from their respective octets.

The thresholds in all reactions from which we deduce baryon exchange degeneracies are low compared with other channels with their quantum numbers, and so in addition to $s_{\min}$, $s_{\max}$, which is determined by the onset of inelasticity, will also control the accuracy of baryon exchange degeneracies. The strangeness-(1) baryon channel, from which we deduce the exchange degeneracies between strangeness-(−1) baryon trajectories, are highly elastic; so these exchange degeneracies should be well satisfied. The strangeness-0 and -(−2), and to a lesser extent -(−1), baryon channels become very inelastic at low energies, and therefore the Λ-Σ degeneracies should be moderately broken, while the exchange degeneracies between the strangeness-0 baryon trajectories and between the strangeness-(−2) baryon trajectories should be badly broken. This hierachy is confirmed experimentally. The specific reactions responsible for each exchange degeneracy are shown in Table II.

Meson and baryon trajectories are approximately linear in mass squared, while the Gell-Mann–Okubo formula is quadratic for mesons but linear for baryons. Thus there is an inconsistency between exact exchange degeneracy and the Gell-Mann–Okubo formula for baryons although none exists for mesons. Mesonic exchange degeneracies are very well satisfied, but the baryonic exchange degeneracies are broken in such a way as to preserve the Gell-Mann–Okubo mass formula.

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Note added in proof.—When three trajectories are coupled in a single exchange-degeneracy equation, the pattern of deviations from exchange degeneracy depends on their relative couplings. If two are coupled more strongly than the third, the two strongly coupled trajectories will be closely exchange degenerate, while the weakly coupled one will be further away. Experimentally, the $\frac{3}{2}^-$ octet is more weakly coupled to *PB* than the $\frac{1}{2}^+$ octet or the $\frac{3}{2}^-$ singlet; so the latter two trajectories will be closer to each other than to the $\frac{3}{2}^-$ octet trajectory.

The assignment of the $D(1285)$ to an unmixed 1^{++} octet implies that the decay $D \rightarrow \delta\pi$ should occur. This decay has been observed by Campbell et al.[9]

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[7]In the quark model this would imply at least 70 $L=0$ degenerate with the 56 $L=0$.

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