PION EXCHANGE, DUALITY, AND THE FORWARD STRUCTURE OF SCATTERING AMPLITUDES*

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Forward dips and peaks in π-exchange differential cross sections are analyzed from the t- and s-channel points of view. Simple criteria for the presence of forward structures are given. They lead, among other things, to the prediction that $d\sigma(\pi^-p\rightarrow\rho_{\mathrm{tr}}{}^0n)/dt$, where "tr" denotes transverse polarization, does not have a forward peak with a width of $m_\pi{}^2$, contrary to expectations based on vector dominance.

Pion-exchange amplitudes have attracted new attention during the last few years, mainly because of the intriguing features of the pion Regge trajectory. Several processes (such as $\gamma p\rightarrow\pi^+n$, $pn\rightarrow np$, and $\gamma p\rightarrow\pi^-\Delta^{++}$) exhibit forward structures[1] (dips or peaks) with a characteristic width of μ^2, in the variable t (μ is the pion mass). These structures cannot be explained in terms of a single pion Regge pole, and various conspiracy schemes, Regge cuts, or absorption effects must be invoked in order to account for the observed phenomena.

The proposed duality property of strong-interaction amplitudes[2] has led to attempts at "building" pion-exchange contributions from s-channel terms. This idea was, so far, applied only to π^+ photoproduction, where a finite-energy sum rule (FESR) analysis has indicated[3] that the existence of the narrow forward peak[1] in $d\sigma(\gamma p\rightarrow\pi^+n)/dt$ is consistent with, and may be "explained" by, s-channel contributions (in this case by the s-channel Born term).

In this paper we try to answer the following questions on the nature of π-exchange amplitudes: Can we use "s-channel language" in order to predict the presence or absence of narrow forward peaks or dips in π-exchange reactions other than $\gamma N\rightarrow\pi^\pm N$? What is the "$s$-channel translation" of the various effects of the t-channel pion pole contribution? Can we develop a simple criterion, consistent with both our t-channel and s-channel points of view, which will tell us in which cases we should expect a dramatic variation of a differential cross section in the region $|t|\leq\mu^2$?

A test of our success in answering these questions will be the experimental measurements of various π-exchange amplitudes in the $|t|\leq\mu^2$ region. A particularly interesting case which has actually motivated our investigation is the process $\pi^-p\rightarrow\rho^0n$ with transversely polarized ρ mesons.[4] This is related through the vector-dominance model (VDM) to $\gamma p\rightarrow\pi^+n$. A naive VDM picture would predict a narrow forward peak in $\pi^-p\rightarrow\rho_{\mathrm{tr}}{}^0n$. Our analysis predicts that such a peak will not be found[4] and that $d\sigma(\pi^-p\rightarrow\rho_{\mathrm{tr}}{}^0n)/dt$ does not exhibit any prominent forward structure[5] in the region $|t|\leq\mu^2$.

The general criterion that we find for the existence of a forward structure[5] in a π-exchange amplitude is extremely simple. Consider a t-channel helicity amplitude $A_{\lambda_b\lambda_d,\lambda_a\lambda_c}$ to which the t-channel exchange of a pion is allowed to contribute and define $\Delta h=\max\{|\lambda_a-\lambda_c|,|\lambda_b-\lambda_d|\}$. We then find the following: (a) For $\Delta h=0$ a forward structure is expected. (b) For $\Delta h\geq1$ no forward structure is expected. (c) $\Delta h=1$ amplitudes of processes of the form $\gamma x\rightarrow\pi^\pm y$ are an exception to case (b) and they do exhibit forward structure.

We expect these rules to apply to an elementary one-pion-exchange (OPE) model, with or without absorption, as well as to an evasive or conspiring Reggeized pion with or without cuts or absorption. The processes $pn\rightarrow np$, $\pi^-p\rightarrow\rho_{\mathrm{tr}}{}^0n$, and $\gamma p\rightarrow\pi^+n$ are examples of cases (a), (b), and (c), respectively.

That the "old-fashioned" naive elementary OPE in the t channel satisfies our three rules is, of course, well known. In that model $\Delta h=0$ amplitudes possess a $t=\mu^2$ pole, leading to a strong variation between $t=0$ and $t=-\mu^2$. For $\Delta h\geq1$ elementary OPE does not contribute. Finally, for $\Delta h=1$ in $\pi^\pm$-photoproduction processes the gauge-invariance condition reinstates the OPE contribution in conjunction with s- and u-channel Born terms, and the forward structure is maintained. It is interesting that the various sophisticated versions of the old OPE model (absorption, Reggeization, conspiracies, and cuts) do not change the above simple criteria. We show this both from the t-channel point of view and from an s-channel description, and we shall see that it leads, among other things, to our somewhat surprising conclusion concerning the absence of a forward peak in $\pi^-p\rightarrow\rho_{\mathrm{tr}}{}^0n$.

Which mechanisms could produce a strong variation in $d\sigma/dt$ in the $|t|\leq\mu^2$ region? From a t-channel point of view, we assume that this can be caused only by a $(t-\mu^2)^{-1}$ term in the amplitude, reflecting the pion pole. In all Reggeized versions of OPE this factor is included in the $\sin\pi\alpha$ denominator of the Regge amplitude which, at small t and α, is proportional to $t-\mu^2$.

From an s-channel point of view, we might use ordinary dispersion relations or FESR to study π-exchange amplitudes. We may consider three different approaches: (i) We may write a FESR for the real part of the amplitude. In that case the low-energy part of the FESR cannot be easily discussed in terms of a few s-channel resonances,[6] and the real part of the π-exchange amplitude will be "built" by many resonance tails and background terms, including low-energy contributions of the π exchange itself. (ii) We may write a FESR for the imaginary part of the amplitude. The low-energy resonances will presumably dominate the integral in the FESR, and they will produce the imaginary part of the π-exchange term. The latter is, however, extremely small for $|t| \leq \mu^2$ in all the versions of the π-exchange mechanism, and it does not contain the pion pole. The FESR for the imaginary part is therefore not particularly illuminating for most π-exchange processes.[7,8] (iii) The most interesting s-channel point of view is that of a fixed-t dispersion relation in ν. In this case the real part of the π-exchange amplitude which possesses the forward structure is expressed in terms of the imaginary part of the amplitude, which may be dominated by resonances and/or by high-energy terms. In that case we shall assume that a strong t dependence at $|t| \leq \mu^2$ can come from two possible sources in the dispersion relation: (1) A strong t variation of one (or a few) of the dominant s-channel contributions (such as the Born term in $\gamma p \to \pi^+ n$),[9] and (2) an accumulative effect of the contributions at many energies, which may occur when we have an "almost divergent" dispersion integral for $|t| \leq \mu^2$. This happens when the large-ν behavior of the integrand is $\nu^{\alpha_\pi - 1}$, yielding a factor $\alpha_\pi(t)$ in the denominator of the integral.

We assume that if and only if one of these mechanisms is present, a forward structure should be found in the physical amplitude. The consistency of the t-channel approach with the s-channel analysis is tested by the simultaneous presence or absence of such mechanisms in both channels for any given amplitude.

We now briefly point out why the conclusions of the elementary OPE calculation with respect to a variation of $d\sigma/dt$ at $|t| \leq \mu^2$ are maintained by the various other models. The case of an evasive Reggeized pion is the closest to the elementary OPE. Both models give the same pole with the same residue at $t=\mu^2$ and the same properties at $t=0$. The difference between the two models is totally insignificant in the region $-\mu^2 \leq t \leq \mu^2$. The introduction of absorption corrections or Regge cuts adds an important (sometimes very large) term to the amplitude. However, this extra term is essentially constant over the $|t| \leq \mu^2$ region and it does not possess a $t=\mu^2$ pole. The effect of such a correction may be to turn a dip into a peak, or vice versa, but not to produce or cancel the very existence of a strong variation of $d\sigma/dt$.

The case of an $M=1$ Reggeized pion conspiring with a π_c trajectory can also be considered as a constant term added to the elementary OPE case. The π_c contribution is always practically constant for $|t| \leq \mu^2$. The π contribution is "rapidly varying."[3] This "rapid variation" is, however, precisely the same type of variation observed in the case of an evasive pion plus cuts or absorption. The t dependence of the conspiring π residue function is of the form $at+b$ while the corresponding function for an evasive pion has the form βt, where β is the residue function and t is a "kinematic factor." The derivative of $d\sigma/dt$ with respect to t in the $|t| \leq \mu^2$ region is of the same order of magnitude in both cases. We therefore conclude that absorption, cuts, or conspiracies may change $d\sigma/dt$, but they almost do not affect $|(d/dt)(d\sigma/dt)|$ for $|t| \leq \mu^2$.

The consistency between the t- and s-channel points of view is demonstrated in Table I, in which we present our conclusions with respect to the presence of forward structure[5] in π-exchange amplitudes. The following remarks are relevant to Table I:

(i) The elementary OPE term for $\Delta h=0$ is viewed as a real subtraction constant from the fixed-t dispersion-relation point of view. The dispersion relation itself tells us nothing about the t dependence of this term. We only know that the subtracted dispersion integral has no strong t variation at $|t| \leq \mu^2$ and that such a variation can be contained only in the subtraction term. In this case, the dispersion-relation point of view is consistent with, but does not predict, the existence of forward structure in $\Delta h=0$ OPE amplitudes. It is only the t-channel consideration which tells us here that the structure exists.

(ii) In the case of a Reggeized pion, the dispersion relation for $\Delta h=0$ is unsubtracted for $t \leq 0$ and the pion term is produced by the accumulative effect of the imaginary contributions to the high-

Table I. Consistency of s- and t-channel descriptions of possible forward structures in π-exchange amplitudes. "YES" and "NO" refer, respectively, to the predicted presence or absence of such a structure.

	case (a): $\Delta h=0$	case (b): $\Delta h \geq 1$	case (c): $\Delta h=1$; $\gamma x \to \pi^{\pm}y$
Elementary OPE t-channel	A pole at $t=\mu^2$. YES	No pole at $t=\mu^2$. NO	s and u channel Born terms have to be added to π-exchange to guarantee gauge invariance. Combined contribution has a $(t-\mu^2)^{-1}$ factor in the full amplitude YES
s-channel (Fixed-t dispersion relations (DR)).	ν^0 energy dependence. DR requires subtraction. No s-channel resonance has strong t-variation. Only the subtraction term may contain a forward structure. ?	$\nu^{-\Delta h}$ energy dependence. No subtraction. DR converges rapidly. No s-channel term has strong variation in $\lvert t\rvert \leq \mu^2$. NO	ν^{-1} energy dependence. No subtraction. s and u channel Born terms in DR are rapidly varying in $\lvert t\rvert \leq \mu^2$. YES
Reggeized OPE t-channel	$(\sin\pi\alpha)^{-1}$ term is proportional to $(t-\mu^2)^{-1}$. Pole at $t=\mu^2$. YES	Nonsense factor α kills pole at $t=\mu^2$. $(\alpha/\sin\pi\alpha)$ does not vary strongly at $\lvert t\rvert \leq \mu^2$. No $t=\mu^2$ pole. NO	An extra $(t-\mu^2)^{-1}$ factor multiplies the usual $\Delta h=1$ Regge amplitude, to guarantee gauge invariance. YES
s-channel (Fixed-t dispersion relations (DR)).	$\nu^{\alpha(t)}$ energy dependence. At $0 \geq t \geq -\mu^2$, $\alpha(t) \leq 0$. No subtraction needed. Dispersion integrand behaves like $\nu^{\alpha-1}$. Integral has (ν^{α}/α) term giving a $(t-\mu^2)^{-1}$ factor. YES	Same as in elementary OPE NO	Same as in elementary OPE. YES

energy part of the dispersion integral. Since α_π is only slightly smaller than zero, the dispersion integral "almost diverges" and the pion term is "almost a subtraction term." It is perhaps misleading to consider this as an s-channel argument since, in this particular case, it is the high-energy $\nu^{\alpha_\pi-1}$ term in the dispersion integrand which produces the effect. Note that the connection between the $\alpha_\pi \to 0$ limit of a Reggeized pion and the elementary pion corresponds here to the relation between the "almost divergent" dispersion integral and the subtraction term mentioned above.

(iii) The Reggeized π-exchange amplitude for type-(c) processes ($\gamma x \to \pi^{\pm}y$) must contain an extra $(t-\mu^2)^{-1}$ factor which follows from gauge-invariance requirements.[10] It is this term which restores the pion pole to these $\Delta h=1$ amplitudes.

What are the direct experimental consequences of our analysis?

(1) The $\Delta h=1$ amplitudes for $\pi^-p \to \rho^0 n$ are of class (b). We therefore predict that $\rho_{11}{}^{\mathrm{GJ}} d\sigma(\pi^-p \to \rho^0 n)/dt$ will remain practically constant in the $0 \geq t \geq -\mu^2$ region ($\rho_{11}{}^{\mathrm{GJ}}$ is the ρ-meson density matrix element in the Gottfried-Jackson frame).[11] This predicted behavior is entirely different from the t dependence of $d\sigma(\gamma p \to \pi^+ n)/dt$, and we expect that the VDM will not hold for these processes at[12] $\lvert t\rvert \leq \mu^2$. The s-channel point of view (both the FESR and the fixed-t dispersion relations) provides a simple explanation for this phenomenon since the t dependence of the nucleon Born term in the $\lvert t\rvert \leq \mu^2$ region is entirely different for γp

$\rightarrow \pi^+ n$ and $\pi^- p \rightarrow \rho_{tr}{}^0 n$. This indicates that the π-exchange mechanism which operates here depends very strongly on the mass of the external vector particle, contrary to the spirit of the VDM hypothesis.

(2) Processes such as $\pi^+ p \rightarrow \rho^+ p$, $\pi^- p \rightarrow \rho^- p$, $pp \rightarrow p\Delta$, and $\pi^+ p \rightarrow \rho^0 \Delta^{++}$ involve $\Delta h = 0$ and $\Delta h \geqslant 1$ amplitudes. They should obey our criteria for the existence of forward structures. Present data[13] on $\pi^+ p \rightarrow \rho^0 \Delta^{++}$ indicate that our conclusions are consistent with experiment. For other reactions, the $|t| \leqslant \mu^2$ data are not sufficiently good to test our prediction.

We conclude with a few remarks:

(a) All forward structures in type-(c) amplitudes ($\gamma x \rightarrow \pi^\pm y$) should be accounted for by the s- and u-channel Born terms in the fixed-t dispersion relations. We know that this is the case in $\gamma p \rightarrow \pi^+ n$.[9] We predict that it also happens in $\gamma p \rightarrow \pi^- \Delta^{++}$.

(b) The qualitative difference between $\gamma p \rightarrow \pi^+ n$ and $\pi^- p \rightarrow \rho_{tr}{}^0 n$ at small t raises the question of the explicit dependence of these cross sections on the mass of the external vector particle. This can be studied, in principle, by measuring $\pi^- p \rightarrow e^- e^+ n$ for various $e^- e^+$ invariant masses.

(c) If we consider the reaction $\pi^- p \rightarrow \rho^0 n$ in models in which the ρ meson couples to a conserved current, an s-channel Born term has to be added to the π-exchange term in order to guarantee current conservation. In such models all our conclusions are still valid since neither the t-channel pion nor the s-channel poles create a significant forward structure in $\Delta h = 1$ amplitudes.

(d) Since the small-t, π-exchange contribution is predominantly real, it is difficult to discuss its construction from s-channel resonances or background in the FESR sense. The role played by the pion within the framework of the duality idea therefore seems to be exceptional. We do not know to what extent it is meaningful to consider resonance-dominance assumptions for building t-channel pion terms in bootstrap-type FESR calculations.

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[4]We must specify the frame in which the ρ mesons are transversely polarized. Our arguments apply directly to the t-channel c.m. (Gottfried-Jackson) frame, but our negative conclusion with respect to the existence of a forward structure in $\pi^- p \rightarrow \rho_{tr}{}^0 n$ is likely to remain valid in other frames. See G. Zweig, Nuovo Cimento 32, 689 (1964); J. S. Ball and M. Jacob, Nuovo Cimento 54A, 620 (1968).

[5]Throughout our discussion we refer to forward dips or peaks as "forward structure." Our analysis can determine whether or not such a structure exists at all in a given amplitude, but we cannot foresee whether the structure will have the form of a dip or of a peak.

[6]Resonances do not dominate the real part of an amplitude and the resonance-background separation in the real part is totally obscured. See, e.g., H. Harari, Phys. Rev. Letters 22, 562 (1969).

[7]The FESR enables us to calculate the specific t dependence of the term which multiplies $(t-\mu^2)^{-1}$ in the amplitude, but it does not shed any light on the question of the very existence of a forward structure.

[8]Continuous-moment sum rules can be viewed as linear combinations of FESR for the real and the imaginary parts. Consequently, they do not add any new information to our discussion.

[9]H. Harari, rapporteur's talk, in Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, 1967 (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968).

[10]Zweig, Ref. 4; Ball and Jacob, Ref. 4.

[11]A similar conclusion follows for $\rho_{11}{}^H d\sigma(\pi^- p \rightarrow \rho^0 n)/dt$, where $\rho_{11}{}^H$ is defined in the helicity frame, provided that $\rho_{11}{}^{GJ} \gtrsim 0.1$. To show this we rotate from the GJ to the helicity frame. $\rho_{11}{}^H d\sigma/dt$ at $|t| \leqslant \mu^2$ is mostly given by the transverse terms in the GJ frame with a contribution of order $\mu^2/m_\rho{}^2$ of the longitudinal term in the GJ frame. This last term has a forward structure, but its effect on $\rho_{11}{}^H d\sigma/dt$ is of the order $(\mu^2 \rho_{00}{}^{GJ}/m_\rho{}^2 \rho_{11}{}^{GJ})$. For $\rho_{11}{}^{GJ} \gtrsim 0.1$, $\rho_{11}{}^H d\sigma(\pi^- p \rightarrow \rho^0 n)/dt$ should not vary by more than 10-15% at $|t| \leqslant \mu^2$.

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