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ARE ABSORPTIVE CORRECTIONS INCOMPATIBLE WITH UNITARITY?*

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We find that absorptive corrections to production amplitudes can generate, through unitarity, absorptive corrections to two-body amplitudes.

Theoretical and phenomenological arguments favor the existence of Regge cuts; this kind of singularity is spontaneously generated in the framework of some models by the iteration of "input" Regge poles, and therefore does not spoil the predictivity of the theory with the introduction of new parameters.

Two well-known models of this kind, the rescattering model[1,2] and the absorptive[3]-eikonal[4,5] model, have recently been compared by Finkelstein and Jacob[6] (FJ); they found that, to good approximation, the cuts predicted by the two models have the same structure, but opposite sign!

Let us briefly recall the results of the two models.[7]

(A) Absorptive-eikonal model.—The amplitude for an inelastic reaction is obtained by correcting the "Born" term (which we assume to be the exchange of a Regge pole R_{ab}) for the elastic rescattering in the initial and final states[3]:

$$T_{ab} = (S_{aa})^{1/2} \times R_{ab} \times (S_{bb})^{1/2}. \tag{1}$$

Assuming that $S_{aa} = S_{bb} = 1 + 2iP$, where P is the amplitude for Pomeranchuk exchange, we obtain

$$T_{ab} = R_{ab} + 2i(R_{ab} \times P). \tag{2}$$

A possible way of extending Eq. (2) to elastic scattering is to identify the eikonal phase with pure Pomeranchuk exchange,[4] obtaining

$$T_{aa} = P + i(P \times P) + \cdots. \tag{3}$$

(B) Rescattering.—The unitarity relation for an elastic scattering amplitude can be written, separating out the contribution of the elastic intermediate state, as

$$\mathrm{Im}\,T_{aa} = T_{aa}^* \times T_{aa} + \sum_{n \neq a} \int d\Phi_n\, T_{an}^* T_{an'}, \tag{4a}$$

the inelastic analog of (4a) being

$$\mathrm{Im}\,T_{ab} = \mathrm{Re}(T_{ab}^* \times T_{aa} + T_{bb}^* \times T_{ab}) + \sum_{n \neq a,\, b} \int d\Phi_n\, T_{bn}^* T_{an}. \tag{4b}$$

Assuming a multiperipheral model (MPM) for the production amplitude, the sum over the multiparticle intermediate states behaves at large s as a Regge pole[1,11]:

$$\sum_{n \neq a} \int d\Phi_n\, M_{an}^* M_{an} = \mathrm{Im}P + \text{a low-lying cut}, \tag{5a}$$

$$\sum_{n \neq a,\, b} \int d\Phi_n\, M_{an}^* M_{bn} = \mathrm{Im}R_{ab} + \text{a low-lying cut}, \tag{5b}$$

where M_{an} is the multiperipheral production amplitude. In the approximation of neglecting $\mathrm{Re}\,T_{aa}$ and $\mathrm{Re}P$ with respect to $\mathrm{Im}\,T_{aa}$ and $\mathrm{Im}P$, on inserting Eqs. (5) in the unitarity relations (4) it is possible to solve them by iteration.[2] The solution has the form of a power series expansion in

P^n in the space of the Fourier-Bessel transforms, which corresponds to an expansion in inverse powers of ln$s$ (if the intercept of the P trajectory is $1-\epsilon$, it corresponds to an expansion in powers of $s^{-\epsilon}/\ln s$). It turns out[2] that when the series converges (i.e., $s \gtrsim 10$ GeV), only the first correction is important. Therefore we can write

$$\mathrm{Im}T_{aa} \cong \mathrm{Im}P + P^* \times P, \tag{6a}$$

$$\mathrm{Im}T_{ab} \cong \mathrm{Im}R_{ab} + 2\,\mathrm{Re}(P^* \times R_{ab}). \tag{6b}$$

The derivation of Eq. (6) is valid also in the multi-Regge model (MRM) in the approximation of disregarding the coupling of the Pomeranchuk to inelastic processes.[8] In a coupled-channel formalism,[9,10] it is possible also to take into account this effect: The discontinuity of the relevant[11] cut presents, in addition to the AFS term, an extra contribution due to the multiple Pomeranchuk exchange in the production amplitude. In the forward region this term has the opposite sign of the AFS term, and at the edge of the cut is about twice as large, but it decreases rapidly with a width (in J) of the order of the inelastic coupling constant of the Pomeranchuk g^2, and we expect that it becomes important only at energies of the order of e^{1/g^2}. Therefore at present energies Eqs. (6) seem valid also in the MRM.

As already remarked by FJ, absorptive corrections [Eqs. (2) and (3)] look more appealing than rescattering ones [Eqs. (6)], from both a theoretical and a phenomenological point of view. To the arguments presented by FJ, we can add the following:

(A) Shrinkage.—In the framework of the MPM or of the MRM, in which Regge poles are overlap functions, it is hard to understand how the Pomeranchuk can have a smaller slope than the other trajectories[10]; in fact, a value $\alpha_P' \simeq 1$ is usually suggested, and it is supported by the recently established existence of a 2^+, $I=0$ $\pi\pi$ resonance at 1 GeV.[12] This value of α_P' is not in disagreement with the present data on p-p scattering, if the corrections to the pole contribution are of the form suggested by absorption.[5,13] In fact in this approach the addition of the broader negative-cut correction sharpens the diffraction peak, and its logarithmic disappearance can mask the shrinkage to a large extent, whereas if rescattering corrections are used, the apparent shrinkage should be considerably larger at the present energies than the one due to pure pole.[2]

(B) Alternation in signs.—Alternating signs in the corrections to the imaginary parts, in agreement with (2) and (3), are strongly suggested, both theoretically[14] and phenomenologically: In particular, Huang and Pinsky,[15] in a phenomenological fit to p-p scattering data in which the phases relative to the pole of the first cut ($\Delta\varphi_1$) and of the second cut ($\Delta\varphi_2$) were left free, found from the best fit $\Delta\varphi_1 = -2.89$ and $\Delta\varphi_2 = -0.354$.

Therefore we are in the unpleasant situation of having a not deeply motivated model (especially in the case of elastic scattering) which gives good results, in apparent contrast to another more reliable procedure. We want to remark here that this contradiction does not necessarily arise; in fact, in order to obtain the first order (in $1/\ln s$) corrections to the dominant Regge pole, we must take into account corrections of the same order to the production amplitude T_{2n}. In order to evaluate these corrections, we remark that the absorptive model has a reasonable motivation for inelastic scattering, and assume that

$$T_{2n} = (S_{22})^{1/2} M_{2n} (S_{nn})^{1/2}, \tag{7}$$

where M_{2n} is the multiperipheral or the multi-Regge production amplitude.

We can only guess what the effect of S_{nn} is going to be: In the following we are going to neglect it in a first approximation. A partial justification for this simplification can be the following: It is reasonable to assume that the main effect of final-state rescattering is given by the elastic scattering of two of the final particles. The scattering of adjacent particles in the multiperipheral chain can be taken into account in a multimeson Reggeized model substituting the pure meson pole exchange with a pole and a cut. This substitution is not likely to change the structure of the integral equation[8,16] in a substantial way; therefore we still expect to obtain from the integration over $d\Phi_n$ and the summation over n a leading pole and a cut which still starts from $2\alpha_{M-1}$, even if with a different discontinuity. The scattering of two nonadjacent particles i and j in the multi-Regge chain is depressed by a factor

$$\frac{(p_i + p_j)^2}{(p_i + p_{i+1} + \cdots + p_{j-1} + p_j)^2},$$

and we can hope that this is enough to prevent these corrections from being important. Therefore we write Eq. (7) to first order as

$$T_{2n} \simeq M_{2n}(1 + iP). \tag{8}$$

Inserting (8) in the unitarity equation (4a) yields

$$\mathrm{Im}T_{aa} \simeq T_{aa}^* \times T_{aa} + \left(\sum_{n \neq a} \int d\Phi_n M_{an}^* M_{an}\right) \times (1 + iP - iP^*).$$

Performing the integration over $d\Phi_n$ and the summation over n in order to obtain a leading Regge pole, as in the usual MRM or MPM calculations, [Eq. (5)] we obtain, instead of Eq. (6a),

$$\mathrm{Im}T_{aa} \cong P^* \times P + \mathrm{Im}P - 2\,\mathrm{Im}P \times \mathrm{Im}P, \tag{9a}$$

which coincides with the first-order eikonal assumption (3).[17] In the case of two-body inelastic scattering T_{ab}, the relevant correction to the production amplitude is

$$T_{an} = M_{an} + iM_{an} \times P + iM_{bn} \times R_{ab},$$

which, inserted in the unitarity equation (4b) and using again Eq. (5), yields

$$\mathrm{Im}T_{ab} \cong \mathrm{Im}R_{ab} + 2\,\mathrm{Re}(R_{ab}^* \times P) - 4\,\mathrm{Im}R_{ab}\,\mathrm{Im}P$$
$$= \mathrm{Im}R_{ab} + 2\,\mathrm{Re}(R_{ab} \times P), \tag{9b}$$

in complete agreement with (2)

In conclusion we have found that the MRM or the MPM models for the production amplitude are not reliable for the computation, through unitarity, of the cuts, because reasonable corrections to M_{2n} can change even the sign of the predicted cut. In particular we have found that the assumption of absorptive corrections to inelastic amplitudes is consistent with unitarity; absorptive corrections to production amplitudes and unitarity produce absorptive corrections in the two-body inelastic processes, and a cut which is equivalent for phenomenological purposes to the one provided by the eikonal model, in the elastic scattering amplitude.

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