HYDROMAGNETIC PLASMA ACCELERATION BY RAPIDLY ROTATING ASTROPHYSICAL OBJECTS*

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We have solved analytically for the acceleration of plasma away from a rotating magnetized object. Application of that analysis to rotator models for the radio, optical, and x-ray pulsar NP0532 suggests that it has a magnetic moment of order 3×10^{28} G cm^3 and is losing mass at a rate order 2×10^9 g/sec. These parameters would provide particles in excess of 10^{11} eV and a 6×10^{-4}-G field in the Crab Nebula at 1 lt yr from NPO532.

We would like to point out that rapidly rotating magnetized stars (e.g., pulsars[1]) can accelerate particles to relativistic energies. Basically, we have solved for the hydromagnetic flow of plasma away from a rotating magnetized plasma source. This treatment is complementary to the idea that particles could be accelerated in any rotation-induced radiation field[2] in that the assumed fields can be axisymmetric, and hence would not generate a radiation field at all. Originally such solutions were discussed by Dicke[3] and developed analytically by Modisette[4] and Weber and Davis[5] for the (nonrelativistic) case of the sun. We find that the analysis can be made relativistically covariant and have solved the relevant equations to illustrate relativistic acceleration.

For analytic simplicity the analysis was, as before,[4,5] restricted to the equatorial plane, azimuthal symmetry is assumed, the frozen-in-flux approximation is employed, and finally, the magnetic field is assumed to be radial at the surface (e.g., an aligned magnetic quadripole moment). The treatment directly parallels that of Weber and Davis except that relativistically covariant expressions are used throughout. There are five equations that completely specify the flow equations: conservation of angular-momentum density, energy flux, and mass, together with the metric condition and the field equations. The resultant flow solutions can be parametrized in terms of the angular momentum carried away per unit mass (L). There is a minimum value for L that gives a flow solution that can reach to infinity. These solutions correspond to the smallest energy loss rate and are presumed to represent the correct physical solution for expansion into a vacuum. Solutions for the boundary conditions $u\rightarrow 0$ as $r\rightarrow 0$, where $uc\equiv dr/ds$, together with neglect of gravitation and plasma pressure, tell us how effective a rotating magnetic field can be at accelerating a cold plasma. We then obtain the equation of motion

$$x^4[u^2(1+\eta^3\lambda)-(1+u^2)(u+\eta^3)^2]+x^2[\eta^3+2u^2(u+\eta^3)-\eta^3u^2\lambda^2]-u^2=0. \quad (1)$$

The dimensionless quantities are defined by $x=r/a$, where $a^2=\Phi^2/4\pi fc$, $\eta^3=(\Omega a/c)^2$, $L=\lambda\Omega a^2$, f is the mass loss rate, Φ is the radial magnetic flux per steradian, and Ω is the angular rotation frequency. The total energy loss per particle is given by $1+\eta^3\lambda$ times the particle rest mass. The minimum-L condition is obtained when

$$(1+\eta^3\lambda)^2=(1+\eta^2)^3, \quad (2)$$

which admits large values of η, corresponding to relativistic radial proper velocities. Equation (2) has two solutions which become equal at the critical point

$$x_c^2=\lambda(1+\eta^3\lambda), \quad (3)$$

where

$$u_c=1/\lambda.$$

For large values of η, $\lambda\rightarrow 1$ and the critical point approaches the radial distance

$$r_c=c/\Omega, \quad (4)$$

namely the distance at which rigid corotation would equal the velocity of light. As a matter of fact, $dr/ds=c$ at this point, and the (physical) solution near the critical point is, approximately,

$$r=\frac{1}{\Omega}\frac{dr}{ds}. \quad (5)$$

Here the proper velocity increases as Ωr. If the ordinary velocity had this behavior, the system would be in rigid rotation; thus the system can corotate "rigidly" beyond r_c only in the sense that each element executes a given angular displacement in the same amount of proper time.

If we have a situation in which $\eta\gg1$, then it would appear that neglect of thermal properties and gravitational effects should be a reasonable approximation (except, of course, very near the surface). The torque per steradian in the equatorial plane is given by

$$\Omega\Phi^2/4\pi c \quad (\eta \gg 1), \tag{6}$$

and therefore the torque is independent of the mass loss rate (f) for relativistic flow, that condition being

$$f < \Omega^2\Phi^2/4\pi c^3. \tag{7}$$

Let us illustrate these equations for the pulsar NP0532 in the Crab Nebula. The repetition rate is about once every 33 msec; thus $\Omega \approx 2\times 10^2$ sec^{-1} if each rotation of the object gives one pulse pattern. From the slowing down rate[6] ($\Omega/\dot{\Omega} = -2484 \pm 2$ yr) and typical neutron-star moments of inertia ($I \approx 10^{45}$ g cm^2), we infer a radial magnetic flux (Φ) of order 3×10^{22} G cm^2, hence a magnetic moment of about 3×10^{28} G cm^3, provided that the mass loss is less than about 2×10^{16} g/sec. As a point of reference, the solar wind carries away about 10^{13} g/sec. This value of Φ corresponds to a field of only about 6 G at $r = r_\odot$, comparable with typical solar fields. The asymptotic magnetic field is azimuthal and is given by

$$B_\varphi = \Phi\Omega/cr \quad \text{(relativistic limit)}, \tag{8}$$

corresponding to about 2×10^{-4} G at 1 lt yr from the pulsar (if unmodified by compression or field-line reconnection).

The frozen-in-flux approximation must fail at such small values of f that the plasma is unable to provide the currents necessary to freeze in the magnetic fields. This condition requires

$$f = \gtrsim m\Phi\Omega/4\pi ce \quad \text{(frozen-in flux)}, \tag{9}$$

or about 2×10^9 g/sec for our NP0532 example. One can show quite easily that the vacuum fields outside a rotating magnetized object are not such as to cause a test charge to corotate at the same angular velocity. Consequently, plasma particles at any interface will see a rapid change in electric field as they cross that interface. The electrostatic force on the particles is of order $e\Omega\Phi/r$, whereas the gravitational force is of order GMm/r^2, and taking $M \approx M_\odot$, $m = m_{\text{proton}}$, and $r = 10^4$ m gives for NP0532 electrostatic forces about 5×10^8 times stronger than surface gravitation. The electrostatic force is exactly balanced by the magnetic force on the moving particle if the plasma corotates. But corotation demands a nonzero space-charge distribution. For example, an axially symmetric magnetic field gives axial corotation if the field lines are equipotentials; hence for a dipole field the electrostatic potential is

$$A_0 = \Omega \sin^2\theta / r, \tag{10}$$

requiring a space-charge density

$$4\pi\rho = -\nabla^2 A_0 = -3\Omega \sin\theta \cos\theta / r^3. \tag{11}$$

Since the vacuum fields do not provide corotation, the overall effect should be to draw up plasma from the surface until sufficient plasma is available to maintain the corotational fields near the pulsar and, in the same way, maintain the plasma loss far from the pulsar.[7] In both cases the work against gravitational forces is taken out of the rotational energy.

An entirely self-consistent formulation appears feasible, in which case f can be derived from first principles. For our simplified analysis here, we will be content to note that if we estimate f to be the minimum value consistent with (9), the particles would be accelerated to energies of at least 2×10^2 times their rest-mass energy. Thus protons could be accelerated to more than 10^{11} eV (and electrons to similar energies in any shock transition to subsonic flow, e.g., where where the pulsar plasma flow interacts with the interstellar medium). The magnetic field would be compressed at least threefold in such a shock. The pulsar could thereby provide both the magnetic field and the energetic particles required to account for the continuum emission from the surrounding nebula.

This analysis is complementary to that of Gunn and Ostriker[2] in that no radiation field is required. Taken together then, these two approaches suggest that rapidly rotating magnetized objects (pulsars, presumably) can be sources of relativistic particles under a wide variety of circumstances.

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[7]It has been brought to my attention that similar conclusions have been reached by P. Goldreich (Ref. 14 quoted in Ref. 2 above), but I have not yet seen a copy of this reference. See also A. J. Deutsch, Ann. Astrophys. 18, 1 (1955).

EXPERIMENTAL TEST OF UNIVERSALITY IN $\Sigma^- \to nl^-\nu$*

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In a 540 000-frame stopping-K^- exposure at the Brookhaven National Laboratory alternating-gradient synchrotron, the branching ratio $\Gamma(\Sigma^- \to n\mu^-\nu)/\Gamma(\Sigma^- \to \text{all})$ is found to be $(0.43 \pm 0.09) \times 10^{-3}$. Combining this with the world-average $\Sigma^- \to ne^-\nu$ branching ratio of $(1.10 \pm 0.05) \times 10^{-3}$ gives $\Gamma_\mu/\Gamma_e = 0.39 \pm 0.08$, which agrees with the universal $V-A$ prediction of 0.45.

One of the most straightforward tests of μ-e universality and the V–A interaction[1] in strange-particle leptonic decays is a measurement of the ratio of the Σ^- decay rates: $\Gamma(\Sigma^- \to n\mu^-\nu)/\Gamma(\Sigma^- \to ne^-\nu)$. If we assume μ-e universality, but not the V–A form, the matrix element for $\Sigma \to nl\nu$ can be written

$$M \propto \sum_{i=1}^{5} C_1(\bar{u}_n \Gamma_i u_\Sigma)[\bar{u}_l \Gamma_i (1+\gamma_5) u_\nu],$$

where $l = \mu$ or e; the Γ_i are the five Lorentz covariant operators 1 (S), γ_5 (P), γ_α (V), $\gamma_\alpha\gamma_5$ (A), and $\sigma_{\alpha\beta}$ (T); and the C_i are the coupling constants. In this general case, the μ-e ratio is given by[2]

$$R = \frac{\Gamma(\Sigma^- \to n\mu^-\nu)}{\Gamma(\Sigma^- \to ne^-\nu)} = 0.45 + 0.60\gamma, \qquad (1)$$

with

$$\gamma = \frac{\mathrm{Re}C_S C_V^* - 6\,\mathrm{Re}C_A C_T^* + 6\delta\,\mathrm{Re}C_V C_T^* - \delta\,\mathrm{Re}C_P C_A^*}{|C_S|^2 + |C_V|^2 + 3|C_A|^2 + 12|C_T|^2},$$

and

$$\delta = \frac{M_\Sigma - M_n}{M_\Sigma + M_n}.$$

Equation (1) is valid up to $O(\delta^2)$.

The quanity γ clearly vanishes if the interaction is pure V and A. It could also vanish due to an accidental cancellation, which is unlikely a priori. If we assume V–A, then the ratio R provides a test of μ-e universality.

We have scanned 540 348 frames of stopping-K^- film, obtained at the Brookhaven alternating-gradient synchrotron 30-in. hydrogen bubble chamber, for the decays $\Sigma^- \to n\mu^-\nu$.[3] The events were identified by seeing the μ^- stop in the chamber and decay to an electron. All events were required to satisfy the following criteria: (1) Σ^- produced by at-rest K^-, (2) $0.1 \leq \Sigma^-$ length ≤ 0.95 cm, (3) $|\mu \text{ dip}| \leq 60°$, and (4) $32 \leq \mu$ momentum ≤ 80 MeV/c.

The upper length cut on the Σ^- removes background due to Λ's produced by stopping Σ^-'s. The upper μ momentum cut removes background due to $\Sigma \to n\pi$, $\pi \to \mu\nu$, where the π-μ decay is undetected. There is a small background due to the radiative decays $\Sigma \to n\pi\gamma$, $\pi \to \mu\nu$. A Monte Carlo calculation shows that this background is <1 event. Since we expect <1 genuine $\Sigma^- \to \mu^-$ event to be rejected because of a sizable plural scattering kink, we make no subtraction for radiative background.

A total of 56 events were found which satisfy all of our criteria. In order to determine the scanning efficiency, a special second scan was undertaken, in which only $\Sigma \to \mu$ and $\Sigma \to \pi \to \mu$ events were recorded. About half the film was scanned in this way. The first scan was found to have a momentum-dependent efficiency. In order to take this into account with minimum increase in statistical error, the events were divided into momentum regions: (I) $32 \leq p_\mu \leq 55$ MeV/c and (II) $55 < p_\mu \leq 80$ MeV/c. The scanning efficiencies, based on 26 events found in the double-scanned film, were the following:

	Region I	Region II
First scan	(80 ± 13) %	(43 ± 12) %
Second scan	(89 ± 10) %	(86 ± 13) %

The 56 events were divided into groups according to their momentum and to the scan or combination of scans in which they were found. Each event was weighted by the inverse of the efficiency for its group. Each event was also weighted by the inverse of the probability that a μ of that