MAGNETIC DIPOLE GAMMA-RAY STRENGTH FUNCTION IN DEFORMED NUCLEI, AND NEUTRON-CAPTURE GAMMA RAYS*

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The magnetic dipole gamma-ray strength function for some strongly deformed heavy nuclei is computed on the basis of the Nilsson model, pairing, and a residual spin-spin interaction. The strength of the spin-spin interaction is adjusted to give average agreement with experiment of effective spin g factors for low-energy $M1$ transitions and moments. Predictions are made for average $M1$ widths of primary transitions following neutron capture; the predictions are in reasonable agreement with recent data.

Existing experimental data on the reduced widths of primary magnetic dipole transitions in the (n,γ) reaction have recently been summarized by Bollinger.[1] The reduced widths for γ-ray transitions of 4- to 9-MeV energy are found to be, on the average, some 10 to 20 times larger than "single-particle" estimates.[2,3] It is the purpose of this note to present some results of a simple computation, with no ad hoc assumptions, which can account for some of these results.

By reduced width we mean the quantity[3]

$$k_{M1}=\Gamma_\gamma(M1,i\rightarrow f)E_\gamma^{-3}D^{-1}, \tag{1}$$

where, for a transition from a resonance i to a final state f, Γ_γ is the partial radiative width (in eV), E_γ is the γ-ray energy in MeV, and D is the average spacing in the region of the initial state of levels of the same spin and parity, also in MeV. The experimental values of k_{M1} summarized by Bollinger are average values, obtained either by averaging results for several individual resonances or by finding the average ratio of $\Gamma_\gamma(M1)/\Gamma_\gamma(E1)$ by the "average-spectrum" method[4] and normalizing by use of Carpenter's results[5] for average $\Gamma_\gamma(E1)$ values. The experimental values are considered uncertain by a factor of about 2.

In order to get a reasonable estimate of average reduced widths we start with the independent-particle model, modified by a pairing interaction, to get the energies and strengths of two-quasiparticle (2QP) $M1$ excitations. The strengths corresponding to these excitations will be spread over many nuclear states, of course. For spherical nuclei only a small number of 2QP excitations will have any strength, and thus any prediction of a strength function depends sensitively on the details of the damping, due to residual nuclear interaction, of the $M1$ excitations into the sea of actual nuclear states of the same spin and parity. For nuclei with an equilibrium quadrupole deformation, however, the strength is already split into many more 2QP excitations; the deformation spreads the strength naturally, and if the damping width is neither very large nor very small compared with the splitting width, then a simple smoothing of the results should be realistic.

The strengths predicted by such an independent-quasiparticle model are of the right order of magnitude, but peak at too low an energy to account for the experimental data. It is known, however, that the effective spin g factors for nucleons in individual-particle models are smaller than their free values, and that this may, at least in part, be attributed to a spin polarization of the nuclear core. Following a line of thought due to Fujita and collaborators,[6] we can relate the reduction of the g factors for low-energy (seniority-conserving) transitions and moments to an upward shift in the strength function for transitions in which the seniority changes by 2 units. Indeed, calculations of the g-factor renormalization for deformed rare-earth nuclei have already been done by Bochnacki and Ogaza[7]; we simply use their model to see the effects on the strength function for $M1$ transitions in the energy region of interest to us.

In the long-wavelength limit, the partial width for $M1$ γ-ray emission from an initial state with total angular momentum I_i to a final state with angular momentum I_f can be expressed as

$$\Gamma_\gamma(M1,I_i\rightarrow I_f)=2.76\times10^{-3}E_\gamma^{3}\sum_{M_f,\kappa}|\langle I_fM_f|M(M1,\kappa)|I_iM_i\rangle|^2, \tag{2}$$

with Γ_γ in eV, E_γ in MeV, and

$$M(M1,\kappa)=\sum_{j=1}^{A}[g_l(j)l_\kappa(j)+g_s(j)s_\kappa(j)]. \tag{3}$$

We then, in the spirit of Blatt and Weisskopf,[2] estimate the average $M1$ partial width by replacing the individual strength (squared matrix element) by a strength function (sum of squared matrix elemets per MeV) multiplied by the experimental average level spacing (in MeV):

$$\langle \Gamma_\gamma (M1, I_i \to I_f) \rangle = 2.76 \times 10^{-3} E_\gamma{}^3 D S(E_\gamma, I_i \to I_f), \tag{4}$$

and thus,

$$\langle k_{M1} (I_i \to I_f) \rangle = 2.76 \times 10^{-3} S(E_\gamma, I_i \to I_f). \tag{5}$$

The strength function we compute from the strength of 2QP excitations at energy E_γ. We thus assume a form of Brink's hypothesis[8] that the strength due to 2QP excitations is independent of the detailed structure of the final state.[9] We assume that the deformed nuclei are axially symmetric, and thus that K, the component of total angular momentum along the symmetry axis, is a good quantum number, at least for the final states. Then the $M1$ strength function may be written as

$$S(E_\gamma, I_i \to I_f K_f) = S_1(E) + [C(I_i, 1, I_f; K_f, 0, K_f)]^2 [S_0(E) - S_1(E)], \tag{6}$$

where

$$S_\kappa(E_\gamma)\Delta E = \sum_{E\gamma, \Delta E} |\langle 2\mathrm{QP}, K = \kappa | M(M1, \kappa) | \mathrm{g.s.} \rangle|^2. \tag{7}$$

The branching ratios implicit in Eq. (6) will be discussed elsewhere; from now on in the present note we set the Clebsch-Gordan coefficient equal to its average value of $(\frac{1}{3})^{1/2}$.

The 2QP wave functions and energies are taken to be the eigenfunctions and eigenvalues of the following intrinsic Hamiltonian:

$$H(\text{intrinsic}) = H(\text{Nilsson}) + H(\text{pairing}) + H(\text{spin-spin}), \tag{8}$$

where the Nilsson[10] Hamiltonian is taken between basis states in the asymptotic representation,[11] the pairing Hamiltonian is the usual one,[12] and we use, as did Bochnacki and Ogaza, the schematic spin-spin interaction

$$H(\text{spin-spin}) = \tfrac{1}{2} V_0 \sum_{i,j} \vec{s}_i \cdot \vec{s}_j, \tag{9}$$

with the exchange matrix elements neglected. Bochnacki and Ogaza give evidence that the effective neutron-proton spin-spin interaction is considerably weaker than that between like nucleons. We have neglected the neutron-proton interaction, but as long as the like-nucleon interaction strength is adjusted to fit the experimental g-factor renormalization the predicted strength functions are essentially unaffected by this neglect. We found that the average proton-proton interaction required to fit the effective g factors was somewhat larger than the average neutron-neutron interaction, and thus adopted $V_0 = 2100 \times M^{-5/3}$ MeV, where M represents the number of protons for the p-p interaction and the number of neutrons for the n-n interaction.[13]

The eigenvalue problem was solved in the usual Tamm-Dancoff and random phase approximations, and the strength functions were found by smoothing over 1.25-MeV regions. The random-phase-approximation results for Er^{166} are shown in Fig. 1. The exact locations of the peaks in the strength functions must be considered somewhat uncertain because of uncertainties in the strength of the spin-spin interaction and an uncertainty in the scale factor (we have used $\hbar\omega_0 = 41 A^{-1/3}$ MeV). The $\langle k \rangle$ values predict-

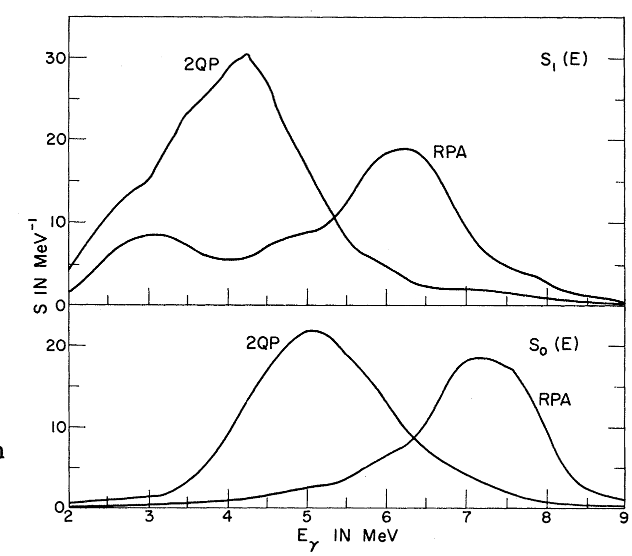

Fig. 1. The $K=1$ and $K=0$ magnetic-dipole strength function results for Er^{168}. The curves labeled 2QP are based on the Nilsson model and pairing. For the curves labeled RPA a residual schematic spin-spin interaction has been introduced and the random-phase approximation used.

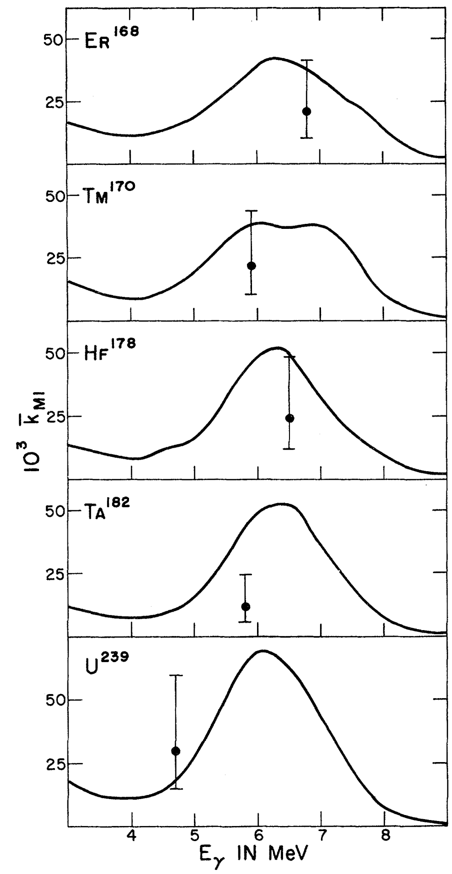

Fig. 2. Comparison of computed values for average reduced width $\langle k_{M1}\rangle$ with the experimental values summarized by Bollinger (Ref. 1). The experimental values for Er^{168}, Tm^{170}, and Ta^{182} are from the work of L. M. Bollinger and G. E. Thomas [Phys. Rev. Letters 21, 233 (1968), and Bull. Am. Phys. Soc. 13, 721 (1968), and private communications]. That for Hf^{178} is from D. J. Buss and R. K. Smither (Bollinger, Ref. 1). The value for U^{239} was derived by I. Bergqvist [Nucl. Phys. 74, 15 (1965)] from his results and from those of H. E. Jackson [Phys. Rev. 134, B931 (1964)]. Recent results on capture in the lowest eight resonances lead to a value of 40 [D. L. Price, R. E. Chrien, O. A. Wasson, M. R. Bhat, M. Beer, M. A. Lone, and R. Graves, Nucl. Phys. A121, 630 (1968)].

ed for several deformed nuclei are compared with the experimental data in Fig. 2. Since neither the experimental numbers nor the results of the computation can be considered very precise, the agreement seems satisfactory and to imply that the experimental strength is due to seniority-two excitations which are shifted upward in energy by a residual interaction. In the sense that the strength functions show peaks which exhaust most of the available strength, one may speak of them as $M1$ giant resonances.

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[13]The g-factor renormalization computations done in this work, as in Ref. 7, involve only $K=0$ 2QP excitations; that the same value of V_0 is at least approximately appropriate for $K=1$ excitations is shown in further work by Bocknacki and Ogaza in connection with the parameter b_0 which occurs in the analysis of $M1$ transitions within bands with $K=\frac{1}{2}$ [Nucl. Phys. 83, 619 (1966), and Acta Phys. Polon. 27, 649 (1965)].