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†Present address: Rutgers—The State University, New Brunswick, N. J.

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GENERAL FORM OF REGGE TRAJECTORY AND RESIDUE FUNCTIONS IN THE SCATTERING OF PARTICLES WITH SPIN*

Loyal Durand, III, and Paul M. Fishbane
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

and

Stanley A. Klein
Joint Science Department, Claremont Colleges, Claremont, California 91711

and

L. M. Simmons, Jr.
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706
(Received 26 May 1969)

We present the most general forms for the trajectory and residue functions for daughter Regge poles which are consistent with analyticity and unitarity. Our results hold for arbitrary masses and spins of the external particles, all types of conspiracy, and general, nonparallel trajectories.

Two major problems are encountered in the construction of Regge-type expansions for two-body scattering amplitudes for general masses and spins of the external particles: (i) The individual terms in the expansion for an s-channel process $1+2\rightarrow 3+4$ contain spurious singularities at $s=0$ unless $m_1=m_2$ and $m_3=m_4$. (ii) The helicity amplitudes used to describe the scattering of particles with spin are not all independent at $s=0$ and at the pseudothresholds and thresholds, $s=(m_1\pm m_2)^2$ and $s=(m_3\pm m_4)^2$. The singularities at $s=0$ can be eliminated in the Regge expansion by the introduction of an infinite sequence of daughter Regge poles.[1] However, the conditions which ensure that the full scattering amplitude is analytic at $s=0$ impose nontrivial constraints on the behavior of the trajectory and residue functions near that point. Because of the kinematic restrictions on the helicity amplitudes at $s=0$ (conspiracy conditions) these constraints can connect the trajectory and residue functions for poles of opposite intrinsic parity (conspiracy).[2] The residue functions must also be adjusted to satisfy the kinematic constraints at pseudothresholds and thresholds.

The problem of determining the most general forms for Regge trajectory and residue functions which satisfy the constraints imposed by (i) and (ii) has been considered by many authors.[3-9] With the exception of very recent work,[7-9] the results which are available deal only with the first few terms in the Taylor series expansion of the trajectory and residue functions near $s=0$,[3,4] or are confined to the unphysical case of parallel Regge trajectories.[6] In this paper, we present a complete solution to the problem. We give the most general form for the trajectory and residue functions for daughter Regge poles which are consistent with the requirements imposed by analyticity and unitarity. Our results hold for nonzero (but otherwise arbitrary) masses and arbitrary spins of the external particles, all types of conspiracy, and general, nonparallel trajectories.

Our derivations of the most general forms for the Regge trajectory and residue functions near $s=0$ are based on the following requirements: (1) They must give Regge asymptotic behavior. We assume that the helicity amplitudes $f_{\lambda_3\lambda_4;\lambda_1\lambda_2}$ have asymptotic expansions of the Regge type for s small and t (or $\cos\theta_s$) large,[10]

$$f_{\lambda_3\lambda_4;\lambda_1\lambda_2}=(-1)^{\lambda-v+s_1-\lambda_1+s_3-\lambda_3}\sum_{SS'}[(2S+1)(2S'+1)]^{1/2}\begin{pmatrix} s_1 s_2 S \\ \lambda_1-\lambda_2-\lambda \end{pmatrix}\begin{pmatrix} s_3 s_4 S' \\ \lambda_3-\lambda_4-\lambda' \end{pmatrix}$$

$$\times\sum_{n=0}\left\{\beta_{n,S'\lambda'S\lambda}{}^{+}\left[\frac{e_{-\lambda,\lambda'}{}^{-\alpha_n{}^{+}-1}(-\cos\theta_s)+\tau_n(-1)^{\lambda-v}e_{-\lambda,-\lambda'}{}^{-\alpha_n{}^{+}-1}(\cos\theta_s)}{\cos\pi(\alpha_n{}^{+}-v)}\right]-[+\to-]\right\}$$

$$+\text{background terms.} \tag{1}$$

The trajectory and residue functions must be such that Eq. (1) is consistent with the known analytic properties of the complete helicity amplitudes, in particular, the kinematic constraints at $s=0$ and at the pseudothresholds.[11] (2) The Regge residues must factor. This property of the residue functions is a consequence of analyticity and unitarity as applied to the partial-wave S matrix, and must hold except at points where there are accidental collisions of different trajectories.[12] We assume that such collisions do not take place at $s=0$. In this case the trajectory and residue functions must be analytic functions of s (boson trajectories) or of $W=s^{1/2}$ (fermion trajectories) in some neighborhood of the points $s=0$ ($W=0$). (3) The Regge trajectories (leading trajectories, conspirators, and daughters) cannot be parallel.[7] This statement is a direct consequence of the analytic properties of the trajectory functions combined with theorems on their threshold behavior which follow from unitarity.[12] (4) The helicity amplitudes must be analytic functions of the external masses. The use of mass analyticity is crucial if one is to obtain the Regge expansion for equal-mass configurations as a smooth limit of the result for general masses.

Our basic results have been derived by two quite different methods. Details of the derivations will be published elsewhere.[13] In the present paper, we shall simply quote the solutions to the problem. As is well known,[2-6] the different possible solutions can be labeled by a parameter j_0 which assumes integer values for boson trajectories, and half-odd-integer values for fermion trajectories. This parameter is identified in the group-theoretical approach to high-energy scattering with one of the quantum numbers necessary to label an irreducible representation of the homogeneous Lorentz group.[14] In the present analytic approach, the value of j_0 may be associated with the dominant helicity: Only those s-channel amplitudes with $\lambda=\lambda'=\pm j_0$ contribute significantly to high-energy, small-momentum-transfer scattering in the t channel, $t\to\infty$, $s\to 0$.

The simplest constraints on the daughter Regge trajectories are well known: Successive trajectories have opposite signature, $\tau_n=(-1)^n\tau_0$, and $s=0$ intercepts spaced by integers, $\alpha_n{}^{\pm}(j_0,0)=\alpha_0{}^{+}-n$, $n=0,1,2,\cdots$.[1-3] The remaining constraints on the trajectory functions[7,9] are summarized for general values of j_0 and n by the formula

$$\alpha_n{}^{\pm}(j_0,s)+n=f_{n,1}(\alpha_n{}^{\pm},s)\pm s^{j_0}[\Gamma(\alpha_n{}^{\pm}+j_0+1)/\Gamma(\alpha_n{}^{\pm}-j_0+1)]f_{n,2}(\alpha_n{}^{\pm},s), \tag{2}$$

where

$$f_{n,i}(z,s)=\sum_{j=0}^{n}\frac{\Gamma(n+1)\Gamma(2z+2+n)}{\Gamma(n-j+1)\Gamma(2z+2+n-j)}s^j a_j{}^{(i)}(s). \tag{3}$$

For $j_0=0$ (the case of nonconspiring trajectories), the two terms in Eq. (2) are identical in form, and the trajectories of opposite parity are completely independent. For all other values of j_0, trajectories of opposite parity are correlated (parity-doubled conspiracy). The MacDowell symmetry of fermion trajectories appears automatically: For fermion trajectories, $2j_0$ is an odd integer, $s^{j_0}=W^{2j_0}$ is an odd function of W, and $\alpha_n^-(W)=\alpha_n^+(-W)$.

The same functions $a_j^{(i)}(s)$ appear in the expressions for all the $\alpha_n^\pm$. These functions are not arbitrary, but can be determined successively in terms of the $\alpha_n^\pm$ by using Eqs. (2). A given function $a_n^{(i)}$ will clearly depend on all the trajectory functions $\alpha_j^\pm$ for $j\leq n$. The relations among the trajectory functions $\alpha_n^\pm$ characteristic of (conspiring) daughter Regge trajectories arise from the fact that the $a_j^{(i)}$ must be analytic at $s=0$ if the helicity amplitudes are to be analytic at that point. It is clear from Eq. (2) that this condition implies that the mth derivatives of the functions $\alpha_n^+ + \alpha_n^-$ and $s^{-j_0}(\alpha_n^+ - \alpha_n^-)$ at $s=0$ can be expressed for $m<n$ in terms of the first m derivatives of the quantities $\alpha_j^+ + \alpha_j^-$ and $s^{-j_0}(\alpha_j^+ - \alpha_j^-)$ with $j\leq m$. The nth and higher derivatives of $\alpha_n^+ + \alpha_n^-$ and $s^{-j_0}(\alpha_n^+ - \alpha_n^-)$ are arbitrary.

We wish to emphasize strongly that the analyticity constraints at $s=0$ relate only the intercepts and the first $n-1$ derivatives of $\alpha_n^\pm, \alpha_{n-1}^\pm, \cdots, \alpha_0^\pm$. They do not determine the complete behavior of these (analytic) functions. Consequently, Eqs. (2) and (3) do not constitute a set of polynomial equations for the $\alpha_n^\pm$,[7] as at first sight seems plausible. The quantities with a clear dynamical significance are the trajectory functions themselves, which give the location of the poles of the partial-wave S matrix in the complex j plane. The functions $a_j^{(i)}(s)$ have no dynamical significance, and may have singularities away from $s=0$.[7] Although we do not, therefore, have a parametrization for $\alpha_n^\pm$ in the usual sense,[15] Eqs. (2) and (3) can be used to construct Taylor series expansions for the trajectory functions valid near $s=0$. The fact that the trajectories cannot be parallel implies, however, that the $a_j^{(i)}$ cannot vanish identically. It is easily seen in this situation that the region in which the first few terms in the Taylor series provides a good approximation to the trajectory functions near $s=0$ shrinks rapidly for large values of n.

The Regge residue functions may be written in the form

$$\beta_{n,S'\lambda',S\lambda}^{j_0\pm}=\beta_{n,S'\lambda'}^{j_0\pm}(\alpha_n^\pm,s)\beta_{n,S,\lambda}^{j_0\pm}(\alpha_n^\pm,s). \tag{4}$$

The factor which describes the 1, 2 vertex is given by

$$\beta_{n,S,\lambda}^{j_0\pm}=[1-g_{n,\pm}'(\alpha_n^\pm)]^{-1/2}\sum_{j=0}^{n}\sum_{j_0'=-S}^{S}F(\alpha_n^\pm,n,j,j_0,j_0')s^{(j+|j_0-j_0'|)/2}a_{j,j_0'}(s)$$
$$\times[d_{\alpha_n^\pm,S,\lambda}^{j_0',\alpha_n^\pm+n-j+1}(\beta_1)\pm\eta_1\eta_2(-1)^{S-v+2s}2d_{\alpha_n^\pm,S,-\lambda}^{j_0',\alpha_n^\pm+n-j+1}(\beta_1)]. \tag{5}$$

The function $g_{n,\pm}'(z)$ is defined in terms of the f's in Eqs. (2) and (3):

$$g_{n,\pm}'(z)=\frac{d}{dz}\{f_{n,1}(z,s)\pm s^{j_0}[\Gamma(z+j_0+1)/\Gamma(z-j_0+1)]f_{n,2}(z,s)\}. \tag{6}$$

The factor F is given by

$$F(\alpha_n^\pm,n,j,j_0,j_0')=\left[\frac{\Gamma(n+1)\Gamma(2\alpha_n^\pm+2+n)}{\Gamma(n-j+1)\Gamma(2\alpha_n^\pm+2+n-j)}\right]^{1/2}\left[\frac{\Gamma(\alpha_n^\pm+j_0+1)\Gamma(\alpha_n^\pm-j_0'+1)}{\Gamma(\alpha_n^\pm-j_0+1)\Gamma(\alpha_n^\pm+j_0'+1)}\right]^{\gamma/2}, \tag{7}$$

where $\gamma=(j_0-j_0')/|j_0-j_0'|=\pm1$. The functions $d_{JJ'\mu}^{j_0\sigma}(\beta)$ are the representation coefficients for the homogeneous Lorentz group,[14] expressible for the case of interest as polynomials in $e^{-\beta}$. The hyperbolic angle β_1 is given by $\cosh\beta_1=E_1/m_1$, $\sinh\beta_1=|\vec{p}_1|/m_1$, where E_1 and $\vec{p}_1$ are the center-of-mass energy and momentum of particle 1 in the s channel. For the 3, 4 vertex, β_1 is replaced by β_3, $\cosh\beta_3=E_3/m_3$, $\sinh\beta_3=|\vec{p}_3|/m_3$. The factors η_1 and η_2 give the intrinsic parities of particles 1 and 2. The functions $a_{j,j_0'}(s)$ are independent of the daughter number n, and may be different for the 1, 2 and 3, 4 vertices. Proper behavior of the helicity amplitudes at $s=0$ is assured if these functions are analytic in the neighborhood of that point. The asymmetry in the treatment of particle 1 (or 3) relative to particle 2 (or 4) in Eq. (5) is only apparent. By appropriate rearrangements of the d function, and redefinition of the a's, Eq. (5) can be brought into an equivalent form with β_1 (β_3) replaced by β_2 (β_4).

Equation (5) summarizes all the constraints on the residue functions at $s=0$ and the pseudothresholds which follow from the conditions imposed by analyticity and factorization. The interpretation of the equation is similar to that of Eq. (2): The actual residue functions are the quantities of dynamical interest (residues of the partial-wave S matrix at the Regge poles). The functions $a_{j,j_0'}(s)$ have no direct dynamical significance, but are determined by Eqs. (5). We may nevertheless use these equations to parametrize the Regge residue functions near $s=0$. Care must be taken, however, to adjust the a's to eliminate any spurious singularities introduced by the factor $(1-g_n')^{-1/2}$, and to enforce the proper behavior of the residue functions at nonsense points.[7]

The results for the trajectory and residue functions given in Eqs. (2), (4), and (5) have several important features:

(1) The behavior of the residue and trajectory functions near $s=0$ is determined, as expected, by the value of j_0. The form of the α's is otherwise independent of the nature of the coupling to the external particles. Information on the couplings appears only in the β's. Explicit expressions for the helicity amplitudes for small s will be given elsewhere.[13]

(2) The expression for the β's in Eq. (5) is valid for all mass configurations (mass analyticity) and all values of S, λ, and j_0. Note that the sum over j_0' in Eq. (5) is present even for the leading trajectories, $n=0$. The β's contain precisely the number of free parameters at $s=0$ that is expected from discussions which treat the equal-equal, equal-unequal, and unequal-unequal mass configurations separately.[4]

(3) In the special case of equal external masses, $m_1=m_2$ and $m_3=m_4$, and zero total energy, $s=0$, the Lorentz group is a symmetry group of the scattering amplitude. As expected, only the single term in Eq. (5) which corresponds to the irreducible representation of the Lorentz group characterized by the parameters $\sigma=\alpha(0)+1$ and j_0 survives in this limit.

(4) For general external masses, or $s\neq 0$, the trajectory and residue functions display an intrinsic mixing of different irreducible representations of the homogeneous Lorentz group. The set of Regge poles $\alpha_n^{\pm}$, $n=0,1,\cdots$, cannot be identified at nonzero values of s with a single pole in a Lorentz amplitude labeled by parameters j_0 and σ, as has sometimes been proposed.[5] This identification would require that the trajectories be strictly parallel, $\alpha_n^{\pm}(j_0,s)=\sigma(s)-n-1$, and that the sum over j_0' in Eq. (5) be limited to the single term with $j_0'=j_0$. We note first that the trajectories <u>cannot</u> be parallel (analyticity and unitarity).[7] Second, the sum over j_0 is necessary if one is to obtain the most general behavior of the residue functions at $s=0$ and at the pseudothresholds. Because of this sum, the dominant behavior of a residue function of a particle pole may correspond to a value of j_0 different from that which characterizes its behavior at $s=0$ [e.g., the residue of a pion pole classified by $j_0=1$ at $s=0$ could behave as a $j_0=0$ residue at the pion mass, $\alpha_\pi=0$]. The expressions in Eqs. (2) and (5) may be interpreted from the group theoretical point of view[3] as including the terms which break the equal-mass $s=0$ Lorentz symmetry to all orders in perturbation theory.

Applications of these results to high energy scattering will be discussed elsewhere.[13]

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one must use the more general result given in Eq. (5) of the present paper. We (DFS) wish to thank Dr. J. H. Weis for calling this problem to our attention.

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$$e_{\lambda\mu}^{j}(z)=\tfrac{1}{2}e^{-i(\pi/2)(\lambda-\mu)}[\Gamma(j+\lambda+1)\Gamma(j+\mu+1)\Gamma(j-\lambda+1)\Gamma(j-\mu+1)]^{1/2}\left(\frac{z+1}{2}\right)^{(\lambda+\mu)/2}$$
$$\times\left(\frac{z-1}{2}\right)^{-j-1-(\lambda+\mu)/2}\frac{1}{\Gamma(2j+2)}\,{}_2F_1\left(j+\lambda+1,j+\mu+1;2j+2;\frac{2}{1-z}\right).$$

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[14]The irreducible unitary representations of the Lorentz group are labeled by parameters j_0 and σ which give the values of the Casimir operators $\vec{J}^2-\vec{K}^2=j_0^2+\sigma^2-1$, $\vec{J}\cdot\vec{K}=-ij_0\sigma$, $j_0=0,\frac{1}{2},1,\cdots$, σ pure imaginary. The representation coefficients are given by matrix elements of the boost operators, $d_{ll'\mu}^{j_0\sigma}(\beta)=\langle j_0\sigma l\mu|e^{-i\beta K_3}|j_0\sigma l'\mu\rangle$. These functions are discussed, for example, by S. Ström, Arkiv Fysik 29, 467 (1965), and 33, 465 (1966); and by W. H. Greiman, thesis, Iowa State University, 1969 (unpublished), who use the notation $A_\mu^{ll'}(\beta,j_0,-i\sigma)$ for the same functions.

[15]Our interpretation of Eq. (2) differs profoundly in this respect from the interpretations given their (equivalent) results by Bronzan, Taylor, Kuo, and Suranyi, Ref. 9.

CROSSING-SYMMETRIC REGGE-BEHAVED AMPLITUDES WITH NONLINEAR TRAJECTORIES

Mahiko Suzuki
Department of Physics, University of Tokyo, Tokyo, Japan
(Received 12 May 1969; revised manuscript received 16 June 1969)

An explicit example of crossing-symmetric Regge-behaved amplitudes is constructed for nonlinear trajectories. The Regge behavior is proved for Re$t\to+\infty$, except on the real axis, as well as for Re$t\to-\infty$.

In this Letter we present an explicit example of crossing-symmetric, Regge-behaved amplitudes (so-called Veneziano representation[1]) for nonlinear trajectories. It is given in the form of an integral representation involving parameters associated wtih Regge trajectories. Both Regge asymptotic behavior and fixed-u behavior are proved along any direction on the complex plane except on the real axis.

Let us first construct an amplitude having poles at desirable locations with residues of the correct angular dependence, that is, an amplitude in which (i) poles should be located at $\alpha(s)$ and $\alpha(t)=n$, where n is a non-negative integer, and (ii) the residue at $\alpha(s)=n$ [or $\alpha(t)=n$] should be an nth polynomial of t (or s), barring ancestors. It is easily seen that the following integral representation meets these two conditions:

$$\mathcal{B}(-\alpha(s),-\alpha(t))=\int_0^1 dz\, z^{-\alpha(s)-1+\Delta\alpha(s)f(z)}\times(1-z)^{-\alpha(t)-1+\Delta\alpha(t)f(1-z)},\qquad(1)$$

where α is a Regge trajectory which obeys a dispersion relation[2]

$$\alpha(s)=as+b+\frac{s}{\pi}\int ds'\frac{\mathrm{Im}\alpha(s')}{s'(s'-s)},\qquad(2)$$

and $\Delta\alpha$ is the deviation of α from a linear trajectory, for which we assume in our representation

$$\lim_{|s|\to\infty}\Delta\alpha(s)/s=0.\qquad(3)$$

The function $f(z)$ is to satisfy the following im-