istence of all possible surface reactions, is in agreement with the experimental data. This shows that for identical particle scattering the pronounced structure at large angles results partially from surface reactions other than the elastic transfer. It is to be expected that such surface reactions are also important for the elastic scattering of nonidentical particles.

An intensive study both of angular distributions with the modified A_l function and energy dependence of the parameters by measuring angular distributions in small energy intervals is in progress and will be published in a forthcoming paper

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THREE-BODY PARTIAL-WAVE ANALYSIS FOR THE FINAL-STATE SCATTERING OF (π^-, NN) FROM ^{12}C AND ^{16}O

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A nonrelativistic three-body treatment in the Born approximation is used for the final-state nucleon-nucleon scattering in the field of the residual nucleus, following the absorption of a bound pion in ^{12}C and ^{16}O. It is shown that the NN energy distributions represent a very sensitive probe of the nuclear two-body force and of the stronger optical potential which serves to descibe multiple-scattering effects.

The absorption of a bound negative pion from an atomic orbital is a rearrangement collision.[1] In principle, several channels are open: single-nucleon, deuteron, or unbound-nucleon-pair channels. Through energy-momentum conservation, however, the exit channels are limited by the transfer of small momentum and the high rest mass of the pion to the absorbing nucleons, hence low excitation of the residual nucleus. Thus single-nucleon emission is suppressed, but those double-nucleon channels in which two nucleons are emitted almost back-to-back qualify as exit channels. Each of the nucleons in this group of channels has high relative momentum but the total channel momentum is small.[2] Thus, in neglecting the single-nucleon and also cluster (e.g., deuteron and triton) channels, we circumvent the difficulties of rearrangement collisions, involving several complete sets of asymptotic scattering states.

The boundary condition of the group of nucleon-nucleon channels can be taken into account using appropriate coordinates. If, in a coincidence experiment, r_l are the distances of the two detectors from the residual nucleus, then only nucleon pairs with the specific channel partition energy $E=E_1+E_2$ and $E_1/E_2=(r_1/r_2)^2$ are picked up because, for equal time of flight t, the nucleons travel the distances $r_l=(2E_l/m)^{1/2}t$. This suggests using the coordinates[3] $r_1=r\cos\alpha$ and $r_2=r\sin\alpha$, where $0\le\alpha\le\frac{1}{2}\pi$.[4]

Three-body partial-wave analysis.—This choice of coordinates yields a discrete set of (asymptotic) channel-surface two-nucleon wave functions

$$\tilde{\psi}_c(s)=D_{nl_1l_2}(\alpha)\{[i^{l_1}Y_{l_1}(\hat{r}_1)i^{l_2}Y_{l_2}(\hat{r}_2)]_L \times[\chi_{1/2}(1)\chi_{1/2}(2)]_S\}_J{}^M|TM_T\rangle \quad (1)$$

with channel-surface coordinates $s=(\alpha,\hat{r}_1,\hat{r}_2)$ and $n=0,1,2,\cdots$. The energy-correlation component $D_{nl_1l_2}(\alpha)$ is related to Jacobi polynomials and yields a characteristic energy distribution[5] depending on n, l_1, and l_2. Expanding the nucleon-pair wave function $\psi_{\bar{c}}$ in terms of the channel-surface states $\tilde{\psi}_c$,

$$\psi_{\bar{c}}(r,s)=\textstyle\sum_c R_c{}^{\bar{c}}(r)\tilde{\psi}_c(s), \quad (2)$$

and using Clebsch-Gordan series for the group O(6) for the $\tilde{\psi}$ in the form

$$\tilde{\psi}_c(s)\tilde{\psi}_{c'}(s)=\textstyle\sum_{c''}B_{cc'c''}\tilde{\psi}_{c''}(s), \quad (3)$$

one finds the coupled-channel equations

$$\left[\frac{1}{r^5}\frac{d}{dr}\left(r^5\frac{d}{dr}\right)-\frac{(2n+l_1+l_2+2)^2-4}{r^2}+k^2\right]R_c^{\bar{c}}=\sum_{c'c''}B_{c''c'c}\nu_{c''}(r)R_{c'}^{\bar{c}} \tag{4}$$

for the "radial" coupled-channel NN-components $R_c^{\bar{c}}$. For simplicity we assume that the residual nucleus is left in a singly excited state, i.e., we stay on the energy shell with respect to the residual nucleus so that the channel energy $E=k^2/2m$ is fixed and the (small) recoil momentum of the residual nucleus is neglected. The transition potentials $\nu_c(r)$ of Eq. (4) are defined by projection on the channel surface of either the two-body nuclear potential or the optical potential $V=V(r_1)+V(r_2)$, and $ds=\sin^2\alpha\cos^2\alpha\, d\alpha d\hat{r}_1 d\hat{r}_2$:

$$\nu_c(r)=\int ds V(r,s)\bar{\psi}_c^*(s). \tag{5}$$

Since the $\nu_c(r)$ in principle describes the dynamics of all three groups of channels, it is not surprising that they are long ranged. The long range of the ν_c associated with the two-body force describes the nuclear interaction between the bound nucleons of an emitted deuteron whose c.m.-system distance from the residual nucleus $R=r/\sqrt{2}$ approaches ∞ as it travels toward the asymptotic region. The long range of the central multipole potentials describes the momentum transfer between the residual nucleus and nucleon 1 which stays bound whereas nucleon 2 is emitted into the force-free ($r\to\infty$) region. In our model however, where the deuteron as well as the single-nucleon emission channels are closed, the long-range components do not contribute and can be cut off. Thus the effective transition potentials are all short ranged, implying the fast convergence of the partial-wave expansion.

Transition rates and results.—Using time-dependent perturbation theory, the transition rates are obtained from a coupled-channel wave $\varphi^{(-)}=\sum_{\bar{c}}A_{\bar{c}}(\beta,\theta)\psi_{\bar{c}}$. $\varphi^{(-)}$ is asymptotically normalized to a plane pair wave characterized by a nucleon-nucleon opening angle θ and partition-energy angle β subject to $E_1=(k^2/2m)\cos^2\beta$ and $E_2=(k^2/2m)\sin^2\beta$.

If we take into account the symmetry of the S matrix under nucleon exchange, each coherent contribution to the transition rate can be written as

$$\Big|\sum_{\substack{n'n''\\ l_1'\geq l_2'\\ l_1''\geq l_2'}}(-1)^{n''+l_2''}P_{l_1''}(\sin\tfrac{1}{2}\theta)P_{l_2''}(\sin\tfrac{1}{2}\theta)[D_{n''l_1''l_2''}(\beta)+(-1)^{1+n''+L'+S'+T'}D_{n''l_2''l_1''}(\beta)]$$
$$\times\frac{M(n'l_1'l_2')}{(1+\delta_{l_1'l_2'})(1+\delta_{l_1''l_2''})}\left(S^{*[L'S'J'T']}_{n''l_1''l_2'',n'l_1'l_2'}+(-1)^{1+n'+l_1'+l_2'+L'+S'+T'}S^{*[L'S'J'T']}_{n''l_1''l_2'',n'l_2'l_1'}\right)\Big|^2, \tag{6}$$

exhibitng explicitly the dependence of the partial π absorption probability on the NN opening angle θ and on the energy-partition angle β. By M we denote the reduced matrix elements of the pion-nucleon absorption operator.[6] It can be shown that, depending on $n''+L'+S'+T'\equiv 0$ or 1 (mod 2), the combination of D functions in (6) has a characteristic dip or a peak at half the channel energy E, i.e., $\beta=\frac{1}{4}\pi$. For instance, $D_{010}\pm D_{001}\propto\cos\beta\pm\sin\beta$ and $D_{021}\pm D_{012}\propto\sin2\beta(\cos\beta\pm\sin\beta)$.

For 1S-pion absorption on a neutron pair from the $S_{1/2}$ orbital of ^{16}O, two groups of partial waves with a $(\cos\beta-\sin\beta)^2$ and $(\cos\beta+\sin\beta)^2$ energy distribution, respectively, have a large nuclear matrix element M in the Born approximation and a large S-matrix–element combination in (6). Both groups together yield a broad peak a half-maximum energy $E_1=E_2=\frac{1}{2}E$ in the nn energy distribution of Fig. 1. For pn emission (Fig. 1) the spin-isospin components of the S matrix suppress one group of partial waves and thereby the maximum. A Serber force instead of a Gillet mixture for the nuclear two-body potential produces a dip both for nn and pn emission at $E_1=E_2=\frac{1}{2}E$ (Fig. 1). The angular distributions show roughly a $\sin^2\frac{1}{2}\theta$ dependence and are much less sensitive to the structure of the NN interaction.[7] The results for the $S_{1/2}P_{3/2}$, $(P_{3/2})^2$, etc. orbitals are similar. They differ from calculations which use asymptotic relative NN waves and experimental NN phase shifts[6] though.

S-matrix elements involving the optical potential are, for ^{12}C and ^{16}O, by about a factor $\xi=4V_0R_0^2/(V_{00}\rho_0^2)$ larger than those of the two-body force because of the longer range of the former. We denote by V_0, V_{00} and R_0, ρ_0 the strength and range of the one- and two-body potentials with Gaussian shape, respectively. Using $V_0\approx\frac{1}{2}V_{00}$ for the energy-dependent optical po-

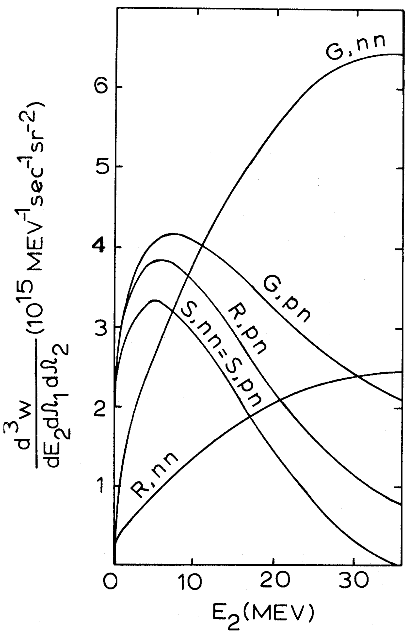

FIG. 1. 1S-pion transition rates for neutron-neutron (nn) and proton-neutron (pn) emission from the $S_{1/2}$ orbit of ^{16}O, using a two-body potential of Gaussian shape with Gillet (G), Rosenfeld (R), or Serber (S) mixtures and no optical potential. The rates are plotted versus the energy E_2 of one nucleon for the angle $\theta = 180°$ between the two outgoing nucleons.

tential, $\xi \approx 6$ results for ^{12}C and ^{16}O. A more realistic value of ξ is somewhat lower because the Gaussian shape underestimates the high-momentum components of the two-body interaction which are relevant for the (π^-, NN) process. The central potential is too strong to be adequately treated by Born approximation. Calculations with a weak central potential of strength $V_0 = 2$ and 4 MeV are shown in Fig. 2, from which we draw the following qualitative conclusions:

Multiple-scattering effects in the final-state scattering of the (π^-, NN) reaction, which may to some extent be described by an optical potential, are not only non-negligible but stronger than the final-state NN interaction because of their longer range. The NN energy distributions—especially the occurrence of a peak or a dip at $\frac{1}{2}E$—are very sensitive to the detailed structure of both the one- and two-body forces.

The central potential enhances mainly the direct S-matrix elements $S_{c,c}$. Thus the pion absorption rates depend on V_0 essentially in terms of the relative strength of the diagonal S-matrix elements for the central and two-body potentials. In the Born approximation, this dependence on V_0 is not monotonic because the contributions of the central and two-body potentials are both signifi-

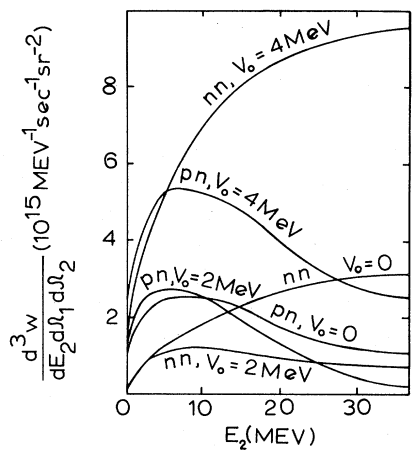

FIG. 2. 1S-pion transition rates for nn and pn emission at $\theta = 180°$ from the $S_{1/2}$ orbit of ^{12}C as a function of the energy E_2 of one nucleon, using a two-body potential with Gillet mixture and central potentials of strength $V_0 = 0$, 2, and 4 MeV.

cant.

Finally, we observe an important feature of this three-body partial-wave analysis of the (π^-, NN) process. The structure of the experimental energy distributions is directly related to nuclear and S-matrix elements via the characteristic energy distribution of the partial-wave D functions in (6).

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RATIO OF STATIC ELECTRIC-QUADRUPOLE MOMENTS OF $K=3$ AND $K=0$ STATES IN ^{172}Yb*

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The ratio of the electric-quadrupole interaction frequencies between the 3^+ 1174-keV state and the 79-keV 2^+ state has been measured using time-differential perturbed angular correlation techniques. The measured ratio $\omega_0(1174)/\omega_0(79) = 0.53 \pm 0.06$ corresponds to a ratio of the static quadrupole moments of $Q(1171)/Q(79) = 1.33 \pm 0.15$, which, combined with the value of $Q(79)$ derived from Coulomb-excitation transition probabilities, yields a value of $Q(1174) = 2.92 \pm 0.33$ b.

The observation of the time-differential perturbed angular correlation (pac) of nuclear gamma rays has been used in a small number of cases to observe static electric-quadrupole interactions of nuclei in solid sources.[1] As in Mössbauer-effect measurements, extraction of detailed information concerning the magnitude of either the nuclear moment or the electric field gradient (EFG) involved must rely on data from other experiments or from calculations; however, ratios of the interaction frequencies of different states in the same element can be measured by observing these states in the same solid environment. This Letter describes the application of this principle to time-differential pac in ^{172}Yb, and the measurement of the ratio of the quadrupole moment of the 3^+ state at 1172 keV to that of the 2^+ 79-keV first-excited state. Since the latter is known from Coulomb-excitation measurement of $B(E2)$,[2] the quadrupole moment of the 1174-keV 3^+ state can be found. It should be pointed out that the direct measurement of quadrupole interactions in such highly excited and long-lived states is not accessible to Mössbauer measurements or any other conventional method.

The decay of ^{172}Lu has recently been studied by many authors,[3-5] and most of the features of the level scheme are now fairly well understood. In particular, the γ-γ cascades 91-1095 and 1095-79 keV (see Fig. 1) have been found to have very large anisotropies ($A_2 = 0.42 \pm 0.02$, $A_4 = 0.012 \pm 0.025$, and $A_2 = -0.41 \pm 0.01$, $A_4 = -0.042 \pm 0.012$, respectively). Furthermore, both the 79-keV ($T_{1/2} = 1.8$ nsec) and the 1174-keV levels ($T_{1/2} = 8.3$ nsec) have lifetimes that, although short, are amenable to time-differential pac studies under favorable conditions. The present experiment makes use of the fact that the 1095-keV γ ray is common to both cascades, and that the 79- and 91-keV γ rays are sufficiently close to one another to be included in a common energy window. Thus, with appropriate timing and logic, the time-differential pac of one cascade will appear on one flank of the measured time spectrum, and that of the other cascade on the other flank. The measurements were carried out using NaI(Tl) detectors on RCA 8575 photomultiplier tubes; these fed conventional "slow" linear circuitry and a "fast" time spectrometer consisting of Oak Ridge Technical Enterprises Corporation (ORTEC) fast discriminators and time-to-amplitude converter (TAC). The output from the TAC was stored in a multichannel analyzer gated by the "slow"

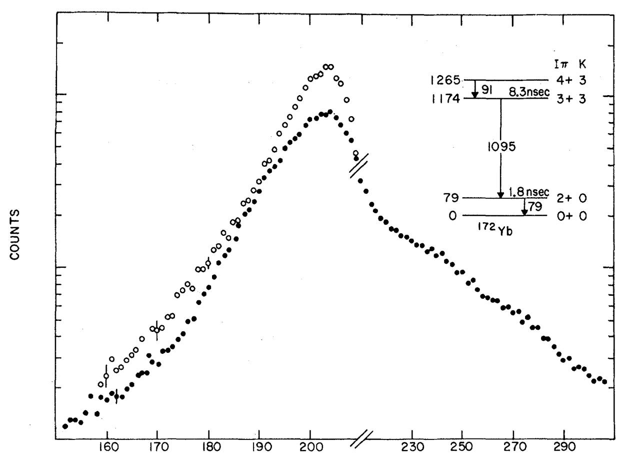

FIG. 1. Time spectrum of the 91- to 1095-keV (right-hand side) and 1095- to 79-keV cascades in ^{172}Yb in thulium metal. The data taken at $\theta = 90°$ (open circles) are displaced upward slightly from the 180° data (solid circles) for clarity.