MODIFIED-McINTYRE-MODEL ANALYSIS OF HEAVY-ION ELASTIC SCATTERING AT MEDIUM ENERGIES

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The A_l function of the McIntyre model was modified to fit the measured differential cross section of the elastic scattering ^{14}N on ^{12}C at $E_{lab}=22.5$ MeV, which shows extensive structure for large c.m.-system angles. The modified A_l function is a Woods-Saxon type function with a sharp drop for a certain l value. This drop simulates a great number of surface reactions, which diminish the elastic channel in the case of grazing collision.

In the last few years angular distributions of heavy-ion elastic scattering have been studied extensively. Theoretical analyses have been more or less successful. Especially, angular distributions with pronounced structure at large angles provide some difficulties. Complex potentials of Woods-Saxon type with or without a hard core were used to fit such distributions[1,2] although the physical meaning of optical potentials for heavy ions is somewhat doubtful. On the other hand, the mechanism of elastic transfer is used to explain oscillations at large angles.[3,4]

We have measured the angular distribution of ^{14}N on ^{12}C for $E_{lab}=22.5$ MeV at the Erlangen tandem accelerator. This distribution exhibits remarkable structure at large angles. In this Letter we want to show that it is possible to obtain a good fit for this distribution as well as for other distributions, that were not measured by us, with the use of a modified A_l function in the framework of a parametrized phase-shift analysis.

The experimental setup, the methods of detection by a gas-flow proportional counter, and the analysis with an on-line computer are described by Ischenko et al.[5] The absolute cross section was determined by measuring simultaneously the elastic scattering on ^{197}Au, which also evaporated onto the target. The ratio of ^{12}C to ^{197}Au was measured by elastic scattering at $E_{lab}=15$ MeV. By this method it is possible to quote the absolute cross section with an accuracy of $\pm 5\,\%$.

The measured angular distribution is shown in Fig. 1 (solid line). The dotted line shows our preliminary fit with the modified A_l function, and the broken line is a fit with the usual McIntyre model.[6] In the framework of this method the scattering amplitude is given by

$$f(\Theta)=f_C(\Theta)+\frac{1}{2ik}\sum_l(2l+1)e^{2i\sigma_l}(A_le^{2i\sigma_l}-1)\times P_l(\cos\Theta), \quad (1)$$

where $f_C(\Theta)$ and σ_l are the Coulomb scattering amplitude and the Coulomb phase shift, respectively. The A_l values are the ratios of the amplitudes of the outgoing to the incoming partial waves and simulate the absorption of the incident particles by the nucleus. Usually the A_l values are given by a Woods-Saxon-type function, which becomes 0 and 1 for small and large l values, respectively. The nuclear phase shift δ_l is assumed to be also a Woods-Saxon type function and can be written usually as

$$\delta_l=\delta(1-A_l), \quad (2)$$

where δ is the maximum value of the nuclear phase shift.

With this parametrization it is impossible to

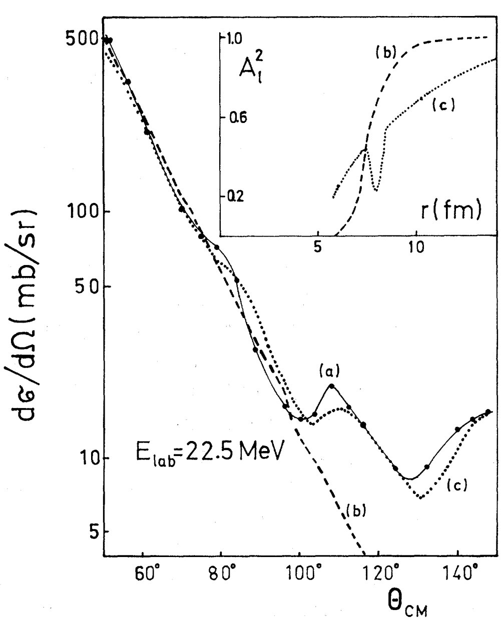

FIG. 1. Angular distribution of the elastic scattering reaction $^{12}C(^{14}N, ^{14}N)^{12}C$. (a) Experimental curve. (b) McIntyre fit. (c) Fit with the modified A_l function. The inset shows the A_l^2 function versus the nuclear distance r.

get an acceptable good fit for the measured angular distribution. As there exists no a priori necessity to use a Woods-Saxon-type A_l function, we have modified this function to get a better fit. This modified A_l function is a Woods-Saxon-type function with a Gaussian dip for a certain l value l_s, which is determined by the best fit. The modified δ_l is defined by Eq. (2) in which the modified A_l has been inserted. Our preliminary fit with the modified A_l function shows a remarkable agreement with the experiment. This better agreement cannot be explained only by the number of parameters used because the fit with the standard parametrization fails, too, when using the maximum number of five parameters. The probability for elastic scattering A_l^2 as a function of the relative nuclear distance r is drawn in Fig. 1. It is remarkable, that the dip in the modified A_l^2 function occurs at a distance r, which is comparable with the interaction radius quoted by other methods. Therefore the dip seems to simulate the great number of surface reactions, which diminish the elastic channel in the case of grazing collision. The further increase of the A_l^2 function at smaller distance, i.e., a larger probability for elastic scattering, when the particles penetrate into the range of nuclear interaction appears to be strange at first thought, but it seems to reflect the operation of the Pauli principle. This explanation is supported by the fact that the δ_l values must be chosen negative to obtain a good fit. The shape of the A_l^2 function suggests the following picture: An incoming particle sees the target nucleus as a black disk surrounded by a grey zone and a black ring in it. This picture is in agreement with the molecularlike optical potentials. The physical meaning of the parameters in this parametrization is not obvious. Only the parameter l_s is surely connected with the interaction radius of the system involved. The width of the Gaussian dip Δl_s could be interpreted as a measure of the surface thickness. The large values of the diffuseness Δl_A and Δr_A (l and r notation, respectively) are comparable with the diffuseness for the imaginary part of the optical-model potential obtained by Krubasik et al.[7]

To be sure that this fit to our experimental data is not a singularity, we have examined angular distributions measured by Bromley, Kuehner, and Almqvist[8] (^{16}O on ^{12}C, $E_{\text{lab}} = 32$ MeV and ^{12}C on ^{12}C, $E_{\text{lab}} = 22.5$ MeV) and by Halbert, Hunting, and Zucker[9] (^{14}N on ^{12}C, $E_{\text{lab}} = 27.3$ MeV). Figure 2 shows the experimental angular distributions together with our preliminary fits, which reproduce both the absolute value of the cross section and the large-angle oscillations. Figure 2 exhibits a drawing of the A_l^2 values versus the nuclear distance, from which the fit parameters may be deduced.

Gobbi et al.,[4] and Barker[10] have calculated angular distributions of ^{12}C on ^{13}C and ^{14}N on ^{12}C, respectively, by adding coherently a contribution of the elastic transfer reactions $^{13}C(^{12}C, ^{13}C)^{12}C$ and $^{12}C(^{14}N, ^{12}C)^{14}N$, respectively, to the elastic potential scattering. This method suggests that oscillations at large angles are created only by the elastic transfer reaction. In the case of identical-particle scattering this reaction is impossible and the oscillations of the angular distribution should be described by the Mott interference. But the elastic-scattering data of ^{12}C on ^{12}C cannot be fitted by Mott scattering or with the McIntyre model. On the other hand, the fit with the modified parametrization, which implies the ex-

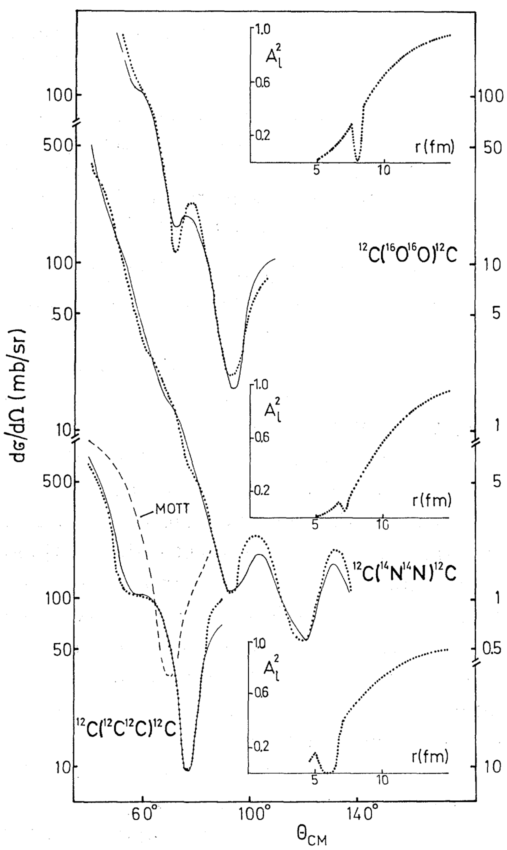

FIG. 2. Angular distributions of the elastic scattering of ^{16}O on ^{12}C ($E_{\text{lab}} = 32$ MeV, Ref. 8), ^{14}N on ^{12}C ($E_{\text{lab}} = 27.3$ MeV, Ref. 9), and ^{12}C on ^{12}C ($E_{\text{lab}} = 22.5$ MeV, Ref. 8). Dotted lines represent the experimental data; solid lines are fits with the modified A_l function. The insets show the A_l^2 function versus the nuclear radius r.

istence of all possible surface reactions, is in agreement with the experimental data. This shows that for identical particle scattering the pronounced structure at large angles results partially from surface reactions other than the elastic transfer. It is to be expected that such surface reactions are also important for the elastic scattering of nonidentical particles.

An intensive study both of angular distributions with the modified A_l function and energy dependence of the parameters by measuring angular distributions in small energy intervals is in progress and will be published in a forthcoming paper

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THREE-BODY PARTIAL-WAVE ANALYSIS FOR THE FINAL-STATE SCATTERING OF (π^-, NN) FROM ^{12}C AND ^{16}O

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A nonrelativistic three-body treatment in the Born approximation is used for the final-state nucleon-nucleon scattering in the field of the residual nucleus, following the absorption of a bound pion in ^{12}C and ^{16}O. It is shown that the NN energy distributions represent a very sensitive probe of the nuclear two-body force and of the stronger optical potential which serves to descibe multiple-scattering effects.

The absorption of a bound negative pion from an atomic orbital is a rearrangement collision.[1] In principle, several channels are open: single-nucleon, deuteron, or unbound-nucleon-pair channels. Through energy-momentum conservation, however, the exit channels are limited by the transfer of small momentum and the high rest mass of the pion to the absorbing nucleons, hence low excitation of the residual nucleus. Thus single-nucleon emission is suppressed, but those double-nucleon channels in which two nucleons are emitted almost back-to-back qualify as exit channels. Each of the nucleons in this group of channels has high relative momentum but the total channel momentum is small.[2] Thus, in neglecting the single-nucleon and also cluster (e.g., deuteron and triton) channels, we circumvent the difficulties of rearrangement collisions, involving several complete sets of asymptotic scattering states.

The boundary condition of the group of nucleon-nucleon channels can be taken into account using appropriate coordinates. If, in a coincidence experiment, r_i are the distances of the two detectors from the residual nucleus, then only nucleon pairs with the specific channel partition energy $E=E_1+E_2$ and $E_1/E_2=(r_1/r_2)^2$ are picked up because, for equal time of flight t, the nucleons travel the distances $r_i=(2E_i/m)^{1/2}t$. This suggests using the coordinates[3] $r_1=r\cos\alpha$ and $r_2=r\sin\alpha$, where $0\leq\alpha\leq\frac{1}{2}\pi$.[4]

Three-body partial-wave analysis.—This choice of coordinates yields a discrete set of (asymptotic) channel-surface two-nucleon wave functions

$$\tilde{\psi}_c(s)=D_{nl_1l_2}(\alpha)\{[i^{l_1}Y_{l_1}(\hat{r}_1)i^{l_2}Y_{l_2}(\hat{r}_2)]_L \times[\chi_{1/2}(1)\chi_{1/2}(2)]_S\}_J{}^M|TM_T\rangle \quad (1)$$

with channel-surface coordinates $s=(\alpha,\hat{r}_1,\hat{r}_2)$ and $n=0,1,2,\cdots$. The energy-correlation component $D_{nl_1l_2}(\alpha)$ is related to Jacobi polynomials and yields a characteristic energy distribution[5] depending on n, l_1, and l_2. Expanding the nucleon-pair wave function $\psi_{\bar{c}}$ in terms of the channel-surface states $\tilde{\psi}_c$,

$$\psi_{\bar{c}}(r,s)=\sum_c R_c{}^{\bar{c}}(r)\tilde{\psi}_c(s), \quad (2)$$

and using Clebsch-Gordan series for the group O(6) for the $\tilde{\psi}$ in the form

$$\tilde{\psi}_c(s)\tilde{\psi}_{c'}(s)=\sum_{c''}B_{cc'c''}\tilde{\psi}_{c''}(s), \quad (3)$$