SHORT-RANGE CORRELATIONS AND ELASTIC ELECTRON SCATTERING*

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The effects of the short-range correlations on the form factor of elastic electron scattering have been calculated in the Born approximation for ^{16}O as an example. It turns out that in the high–momentum-transfer region it is essential to take into account the modifications of the nucleonic wave function for small internucleonic distances; the results indicate that from precise measurements details about the momentum exchange between bound nucleons can be deduced.

It has been pointed out by several authors,[1-4] that elastic electron scattering experiments in the high–momentum-transfer region represent a powerful tool to study the effects of the short-range correlations between bound nucleons. In this note their influence on the elastic form factor will be discussed systematically. In order to extract information from the experimental data and to understand the nuclear structure properties hidden in a cross-section measurement it is quite important to parametrize the unknown correlation function in a convenient and physically meaningful manner.

These correlations will be introduced using Jastrow's technique of correlated basis functions[5]: Starting from a Slater determinant $\varphi_0(1, \cdots, A)$, describing the ground state of a nucleus, the correlated wave function $\tilde{\varphi}_0(1,\cdots,A)$ is given by

$$\tilde{\varphi}_0(1,\cdots,A)=\varphi_0(1,\cdots,A)\prod_{\substack{i=1\\ j<i}}^{A} f(r_{ij}). \quad (1)$$

Here the correlation function $f(r)$ has to simulate the bounds due to the short-range part of the nucleon-nucleon interaction, i.e., $f(r)$ has to go to 0 and 1 for very small and very large internucleonic distances, respectively.

The electron scattering cross section or form factor is calculated either in Born approximation or by phase-shift analysis directly from the charge density

$$\rho_{\rm ch}(r)=\int d\tau_1\cdots d\tau_A\,\tilde{\varphi}_0(1,\cdots,A)\sum_{i=1}^{A} e\delta(r-r_i)\,[(1+\tau_i)/2]\,\tilde{\varphi}_0(1,\cdots,A). \quad (2)$$

Because of the rapid convergence of the cluster expansion in the actual calculations only terms linear in f have been taken into account. The effects of the short-range correlations will be demonstrated for the case of ^{16}O: The single-particle basis is defined by a Woods-Saxon potential, its parameters being fixed by the corresponding low-energy properties. The form factor can be calculated straightforwardly once the correlation function, $f(r)$, is given.

Using realistic single-particle wave functions the correlation function has to be transformed from relative to single-particle coordinates. To do this it is convenient to perform a Fourier decomposition of $f(r)$:

$$f(r)=\int dq\, w(q) f_q(r) \quad (3)$$

with

$$f_q(r)=1-j_0(qr).$$

Each Fourier component, $f_q(r)$, is already a presumably good approximation of the "true" correlation function[6] provided $1.25\ \text{fm}^{-1}<q<2\ \text{fm}^{-1}$.

In order to extract information on the unknown function $w(q)$ we will first use the approximation

$$w(q)=\delta(q-\bar{q}). \quad (4)$$

With this approximation the exchange of the momentum $\hbar\bar{q}$ is simulated between two otherwise independent nucleons in a nucleus. Varying the value of $\bar{q}$ the influence of $f(r)$ on the charge distribution, i.e., on the elastic electron scattering cross section, can be studied systematically.

In Fig. 1 the change of the charge distribution is shown which has been produced by several different correlation functions, i.e., several values of $\bar{q}$. The corresponding form factors calculated in Born approximation are presented in Fig. 2. Preliminary calculations have been done using phase-shift analysis and while the results show that the details of the form factors are changed by the Coulomb distortion the effects due to correlations remain. Obviously one can draw the

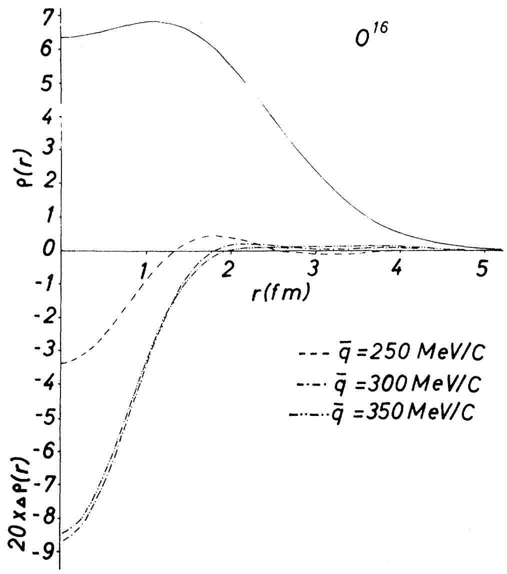

FIG. 1. The shell-model charge distribution $\rho(r)$ of ^{16}O (solid line), together with the deviations, $\Delta\rho(r)$, produced by the short-range nucleon-nucleon correlations (broken lines); the proton charge distribution has been included.

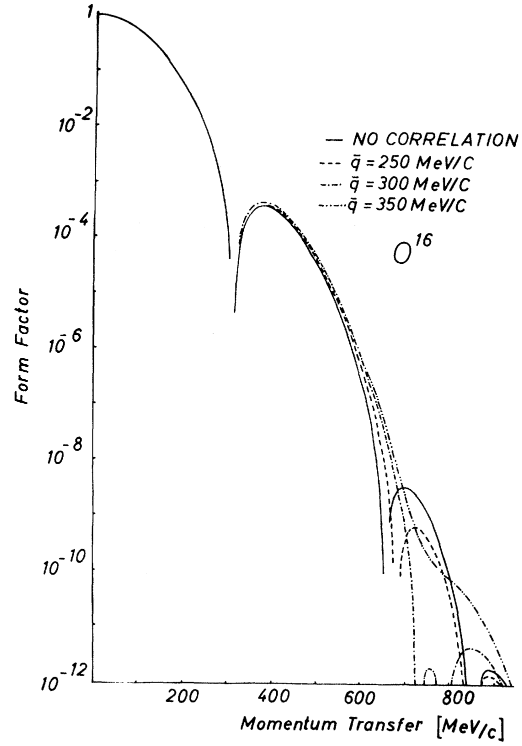

FIG. 2. The form factors of elastic electron scattering on ^{16}O calculated in the independent-particle model without (solid line) and with various correlation functions (broken lines).

following conclusions: (1) The short-range correlations do have a negligible effect on the form factor for small momentum transfer – in this region the shell-model parameters and the effects of the conventionally used residual interactions determine the cross section (long-range correlations). (2) For large momentum transfer ($q > 400$ MeV/c), however, the form factor depends sensitively on the correlation function; this in turn leads to the possibility of measuring the correlation function, i.e., $w(q)$. (3) The assumption that only one momentum $\hbar\bar{q}$ is being exchanged, is a caricature of the true situation; actually the momentum to be exchanged between the two nucleons will be spread. Calculations have been performed with a Gaussian spread of $\hbar\bar{q}$ equal to 90 MeV/c and no qualitative changes result. This point is being investigated further. (4) To use conventional correlation factors, i.e., Gaussians or step functions, does not seem to be very appropriate – though very handy for calculations with harmonic-oscillator wave functions – to describe the effect of the short-range correlations. By the use of such a function the exchange of both small and large momenta is simulated, thus simultaneously mixing up the effects due to long- and short-range correlations with the properties of the independent particle motion. Before drawing serious conclusions on the shell-model parameters of a nucleus, one first has to separate clearly the effects exclusively due to the short-range correlations.

From the present analysis it seems extremely desirable to have measurements of the elastic electron scattering form factor in the high-momentum region. There the effects of the short-range correlations are dominant and at the same time unmasked by problems of the final-state interaction and/or the reaction mechanisms. In order to avoid mixing up long- and short-range correlations it seems to be important to analyze the experimental data in terms of the momenta, $\hbar\bar{q}$, exchanged between bound nucleons.

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