discovered[17] seems to play no role in these excitations, probably because the dominant interfacial inhomogeneities contributing diffuseness have characteristic lengths like the coherence length ξ which is much smaller than the wavelengths and the depth of the moving layers in the excitations involved in these experiments. It seems unlikely that the diffuse interface structure has concealed a divergence of the shear viscosity since it could only introduce an alternative, diffusive relaxation mechanism in the interface that would be expected to vanish as $T \to T_c$, revealing any divergence of η.

We are pleased to acknowledge the use of the computer and essential guidance provided by Dr. Fred Hiltz and his colleagues in the Physical Biology Department at Cornell; helpful discussions with Professor Geoffrey Chester, Professor Michael Fisher, Professor John Wilkins, Dr. U. Felderhof, and Mr. Randy Watts; receipt of a preprint from the Paris group[11]; and the support of the National Science Foundation and the Advanced Research Projects Agency.

Subsequent to submission of this Letter, measurements of the shear viscosity η above T_c have been reported by Arcovito, Falaci, Roberti, and Mistura, who find a weak divergence of $\Delta\eta$ above T_c in a system (cyclohexane-analine) very similar to ours. It was analyzed as a logarithmically diverging term $\Delta\eta(T)$ superimposed on a monotonic term η'. We find that these data are also consistent with a weak-power-law divergence $\eta \sim \Delta T^{-0.07 \pm 0.015}$. Furthermore, we note that the form we chose, $\eta \sim \Delta T^{-0.04}$, to present our results below T_c can be represented accurately by a logarithmic divergence of $\Delta\eta = \eta - \eta' = -A \ln(1-T/T_c) + B$, with $A \cong 0.13$ and $B \cong -0.61$. We believe that neither set of data has distinguished between a logarithmically divergent term in η and a weak-power-law divergence.

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DIRECT MEASUREMENTS OF LINEAR GROWTH RATES AND NONLINEAR SATURATION COEFFICIENTS OF INSTABILITIES*

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An experimental method of perturbation about a nonlinear equilibrium state is employed to examine linear growth rates and nonlinear saturation coefficients. This procedure is illustrated by the experimental investigation of finite-amplitude unstable-collisional drift waves in a highly ionized plasma. Preliminary experimental results are compared with existing linear and nonlinear theories.

We wish to report an experimental method of measuring both the linear growth rate and the nonlinear saturation rates of an unstable mode. Recently, various authors have begun to investigate the nonlinear saturation mechanisms – such as mode-mode coupling,[1] wave-particle scattering,[2] or change in zeroth-order conditions.[3] The behavior of the energy of an unstable mode can be represented by[4]

$$\frac{dI}{dt}=\sum_n C_n I^n = 2\gamma I-\alpha I^2-\beta I^3+\sum_{n=4} C_n I^n, \qquad (1)$$

where I represents the wave energy, γ the linear growth rate, α and β the first and second nonlinear saturation coefficients, and C_n, the higher order expansion coefficients. While various nonlinear mechanisms can give rise to similar qualitative behavior, their contributions can be distinguished if a quantitative determination of C_n under various experimental conditions is possible. The method to be described in this paper is designed to perform this task.

When a single mode of well-defined frequency becomes unstable under steady-state conditions without external excitation, a nonlinear "dynamic equilibrium" is reached in which the nonlinear saturation rate balances the linear growth rate. If this nonlinear state is perturbed by an external excitation, then by observing the behavior in which the perturbed energy returns to its saturated value I_s defined as

$$\frac{dI_s}{dt}=0=\sum_{n=1} C_n I_s{}^n$$

we can derive information about the coefficients C_n. We shall first present our procedure of obtaining the first three coefficients γ, α, and β by specializing to the case of an unstable drift mode in an inhomogeneous plasma. The method, however, is generally applicable to a large number of unstable systems.

The drift wave is an electrostatic mode occurring in a plasma possessing inhomogeneities in radial density, temperature, or potential gradients in the presence of an axial magnetic field B_0. We have chosen this electrostatic mode for illustration of our technique because its characteristics (standing wave in the axial direction and propagating wave in the azimuthal direction) permit a temporal measurement since the wave does not propagate out of the observation region. Furthermore, the saturation amplitude of this instability can be easily controlled by adjusting B_0. The frequencies of these modes are discrete because of the periodicity condition imposed in the azimuthal direction. As shown in Fig. 1 the unstable mode is detected by a Langmuir probe in a potassium plasma and then amplified and fed back through a variable phase shifter to a small exciton grid (1 cm^2). Through the adjustment of

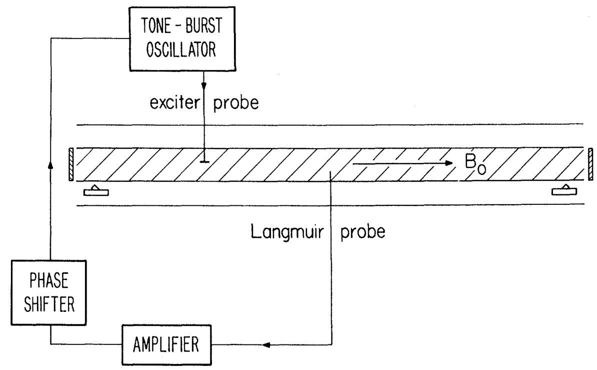

FIG. 1. Schematic of the experimental setup. The signal detected by a small Langmuir probe is amplified and fed back to a larger excitation grid through a phase shifter and a tone-burst oscillator. The plasma conditions are listed in Table I.

the phase of the exciting signal the amplitude of the unstable mode can be either enhanced or suppressed. After perturbing the nonlinear equilibrium state the excitation is withdrawn through a tone-burst generator and the subsequent approach to the equilibrium state is monitored as shown in Fig. 2. The plot of this variation in the form $F(I)=(1/I)dI/dt$ vs I (I is the square of the amplitude observed in Fig. 2) follows closely $F(I)=2\gamma-\alpha I-\beta I^2$, Eq. (1), with neglect of the higher order terms. For small observed values of I, a linear variation is observed which suggests that βI^2 in the preceding equation may be neglected and the values of 2γ and α can be obtained from the intercept and the slope of the plot $F(I)$ vs I. At large values of I, departure from a linear variation occurs which yields an estimate of the value of β. All three coefficients can also be measured by the curve-fitting technique. We have made measurements as functions of the most sensitive parameter, the axial magnetic field B_0, keeping constant other parameters such as density, axial length, and column radius. The results are listed in Table I along with our theory and Dupree's nonlinear calculation, to be discussed in the following.

The linear theory[5] for unstable drift waves predicts a net growth rate

$$\gamma=\gamma_0(K^2,d_I)-\delta_{ii}(d_i,d_I)-\sigma, \qquad (2)$$

where γ_0, the basic linear growth rate for the density-gradient drift waves which we are investigating, is proportional to the square of the normalized density gradient $K=-(1/n)(\partial n/\partial r)$; δ_{ii}, the ion viscous damping, is a function of the ion transverse diffusion rate[5] $d_i=\frac{1}{4}(k_\perp\rho_i)^2\nu_{ii}$ and the rate of parallel electron diffusion along B, d_I

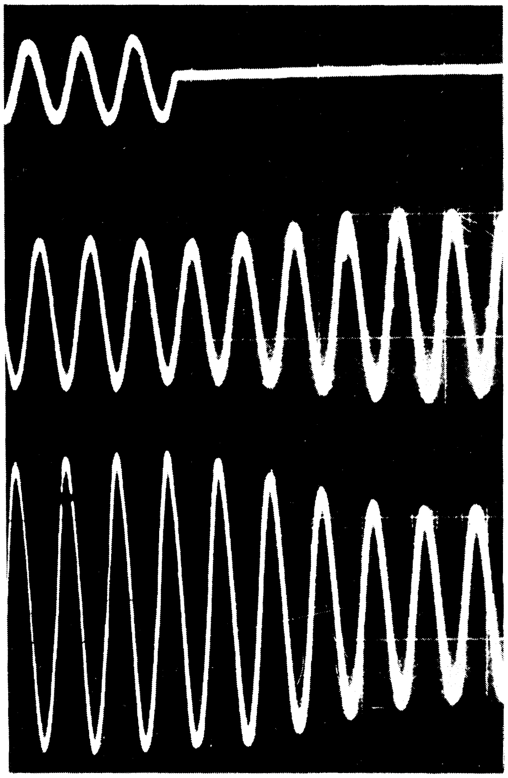

FIG. 2. Upper trace: Typical oscilloscope signal showing the signal applied to the feedback probe. Middle trace: Monitor of the instability by the Langmuir probe when the phase of the feedback signal is such than the unstable mode is initially suppressed, and then allowed to return to the nonlinear equilibrium value upon termination of the feedback excitation. Lower trace: Changing the phase of the feedback signal by 180° enhances the unstable amplitude initially and the subsequent decay to the nonlinear equilibrium state is observed. The vertical scales are 5 V/cm for the upper trace and 5 mV/cm for the middle and lower traces; the horizontal sweep is 2 msec/cm.

$=k_{\|}^2V_I^2/\nu_{ii}$; and σ, the end-plate damping rate, represents reflection loss of waves at the plasma end-plate boundaries.[6] The agreement in Table I between experiment and the theory of the linear growth rate provides a check on our technique. The present measurements ($\gamma>0$) serve as an extension of our previous measurements in the stable regime ($\gamma<0$).[6] After subtracting the end-plate damping rate (insensitive to change in B) we found the $\gamma_0-\delta_{ii}$ in both cases to agree with the linear theory[5,6] in order of magnitude and to behave similarly with change in B.

As the wave amplitude grows, nonlinear effects modify the above linear growth rate. First, we consider the nonlinear effects introduced by the change in the density gradient in the presence of drift waves. It has been shown theoretically and observed experimentally[3,7] that an increase in drift-mode amplitude is accompanied by an increase in the radial diffusion flux J causing a decrease in the radial density gradient. Using the steady-state continuity equation,

$$\nabla\cdot\vec{J}=-Rn^2+S,$$

where $J=J_c+J_\omega$ is the sum of the classical collisional diffusive flux J_c and the wave-induced diffusion flux J_ω, S is the plasma source term, and R the recombination coefficient, we can compute the square of the normalized density gradient under collisional and wave-induced diffusion as

$$(K^{NL})^2\cong K_0^2\left[1+\frac{J_\omega(I)}{J_c}\right]^{-1}$$
$$\cong K_0^2\left\{1-\frac{J_\omega(I)}{J_c}+\left[\frac{J_\omega(I)}{J_c}\right]^2+\cdots\right\}, \qquad (3)$$

where K_0 is the density gradient associated with classical diffusion only and we have assumed that $J_\omega \ll J_c$. Substitution of (3) into (2) yields the nonlinear modification of the basic growth rate γ_0 by the two terms αI and βI^2 with

$$\alpha=2\gamma_0 s,$$

$$\beta=-2\gamma_0 s^z, \qquad (4)$$

Table I. Linear and nonlinear saturation coefficients. Experimental conditions: $n_0=6\times10^{10}$ cm^{-3}; $\kappa^2=[(I/n_0)\partial n_0/\partial r]^2=0.36$, $k_\perp=1.4$ cm^{-1}, $k_{\|}=2.5\times10^{-2}$ cm^{-1}, $T_i=T_e\simeq0.2$ eV, potassium plasma. Experimental error: ±15%.

Axial magnetic field B (kG)	Nonlinear saturation amplitude, $10^2 n_{1S}/n_0$	Linear rate γ, $m=2$ (10^2 sec^{-1}) Experiment	Theory	Nonlinear saturation coefficient α, $m=2$ (sec^{-1}) Experiment	Theory (1)[a]	Theory (2)[b]	Nonlinear saturation coefficient β, $m=2$ (sec^{-1}) Experiment	Theory (1)[a]	Theory (2)[b]
1.07	1	4.5	4.5	9.0×10^6	2.9×10^6	5.8×10^4	-2.1×10^{10}	-2.3×10^9	-4.8×10^5
1.09	2	8.5	6.0	3.6×10^6	2.8×10^6	5.5×10^4	-4.4×10^9	-2.1×10^9	-4.5×10^5
1.15	3	10.0	8.9	2.2×10^6	2.4×10^6	4.6×10^4	-2.2×10^9	-1.6×10^9	-3.6×10^5

[a]Zeroth-order conditions. [b]Wave-particle scattering.

where

$$s=\frac{2(k_\perp\rho_i)^4v_i^2}{d_ID_c}$$

and D_c, the collisional diffusion coefficient, is $2\eta_\perp n_0KT/B^2$. The calculated values of α and β are found under "Theory (1)" in Table I.

Recently, Dupree[2] proposed that wave-particle scattering is another nonlinear saturation mechanism. For sufficiently large wave amplitudes the particle orbits are perturbed incoherently such that wave-particle interaction occurs, resulting in spatial diffusion of the particles. Although his assumption of incoherence for a single mode is still open to question, we have attempted to compare his results with our measured coefficients. Considering the interaction between drift waves and each species of particle as given by Eq. (2) of Ref. 2, we compute the wave-associated diffusion rates $d_\omega^{\ e}$ and $d_\omega^{\ i}$ (proportional to I) for electrons and ions which add to the collisional rates d_e and d_i, respectively. Substitution of these modified rates into (2) yields for the two lowest order saturation coefficients

$$\alpha\simeq2[h\gamma_0+2gd_i],$$

$$\beta\simeq-2h^2\gamma_0,$$

where γ_0 is defined in Eq. (2) and

$$g=\frac{(k_\perp\rho_i)^2[1-(k_\perp\rho_i)^2]k_\perp^2V_i^2}{\omega^2}$$

and

$$h=\frac{(k_\perp\rho_i)^2[1-(k_\perp\rho_i)^2](k_\perp V_i)^2}{d_e^2}.$$

The numerical results computed for our experimental conditions are listed under "Theory (2)" in Table I.

As expressed in Table I the measured nonlinear saturation coefficients α and β are sensitive functions of the magnetic field and are opposite in sign. Existing theories, however, do not predict sufficiently large coefficients and the same dependence on B, although the sign change and the general trend are predicted. Our measurements point out the need of other saturation mechanisms. Our findings of the dependence of α and β on the magnetic field reveals that the nonlinear saturation amplitude $I_s^{1/2}$ depends on the plasma parameter through both the linear growth rate and the nonlinear coefficients. This result differs from earlier theoretical conjectures[4,8] that the functional dependence of the saturation amplitude is derived from the linear growth rate only.

In conclusion, we have demonstrated a method of obtaining linear growth rates and nonlinear saturation coefficients. This method can be applied to a large number of systems where a single unstable mode is present. In the case of a spatially growing mode which saturates at a certain distance[9] a similar feedback scheme can be applied within a limited region to perturb the saturation amplitude. The observation of the subsequent approach to saturation should yield the same information as in the temporal case reported above.

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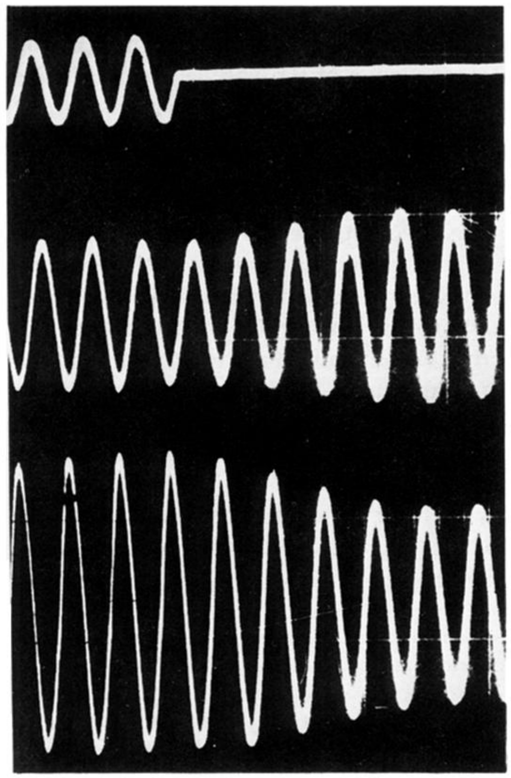

FIG. 2. Upper trace: Typical oscilloscope signal showing the signal applied to the feedback probe. Middle trace: Monitor of the instability by the Langmuir probe when the phase of the feedback signal is such than the unstable mode is initially suppressed, and then allowed to return to the nonlinear equilibrium value upon termination of the feedback excitation. Lower trace: Changing the phase of the feedback signal by 180° enhances the unstable amplitude initially and the subsequent decay to the nonlinear equilibrium state is observed. The vertical scales are 5 V/cm for the upper trace and 5 mV/cm for the middle and lower traces; the horizontal sweep is 2 msec/cm.