HIGH-ENERGY MUON-PROTON SCATTERING: MUON-ELECTRON UNIVERSALITY*

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(Received 10 April 1969)

Measurements of the μ-p elastic cross section in the range $0.15 < q^2 < 0.85$ $(\mathrm{GeV}/c)^2$ are compared with similar e-p data. We find an apparent disagreement between the muon and electron experiments which can possibly be accounted for by a combination of systematic normalization errors.

We have performed a test of muon-electron universality by comparing our muon-proton elastic scattering cross sections (see Table I) with similar electron-proton results. Since the influence of the nucleon vertex and of the photon propagator on the cross section is the same in both cases, any difference in the results must be blamed solely on the lepton vertex. In considering possible modifications to the lepton vertex, the usual form factor may be replaced by a product of the nucleon form factor and a lepton form factor $L_l(q^2)$. The Rosenbluth formula then becomes[1]

$$\left.\frac{d\sigma}{dq^2}\right|_l = \left.\frac{d\sigma}{dq^2}\right|_{\mathrm{NS}} \frac{L_l^{\,2}(q^2)}{\cot^2\frac{1}{2}\theta} \times\left[2\tau G_M^{\,2} + \frac{G_E^{\,2}+\tau G_M^{\,2}}{1+\tau}\cot^2\tfrac{1}{2}\theta\right]$$

and the ratio of muon to electron cross sections is simply $L_\mu^{\,2}(q^2)/L_e^{\,2}(q^2)$.

Measurements of the muon-proton cross sections for μ^+ and μ^- at 6 and 11 GeV/c and μ^- at 17 GeV/c were used. Some details of the experimental setup and data analysis are given in the preceding Letter.[2] The one-photon-exchange approximation to the interaction was found to be adequate as both the μ^+ and μ^- data satisfied the Rosenbluth straight-line test and the μ^+ and μ^- cross sections are the same. For the present analysis, we combine the μ^+ and μ^- data and assume the cross section depends linearly on $\cot^2\frac{1}{2}\theta$.

It is important that the muon and electron data be treated on an equal basis in the comparison. In the muon experiment, $\cot^2\frac{1}{2}\theta$ is always large enough that the second (slope) term completely dominates the first (intercept), and the electric and magnetic form factors may not be separately determined. On the other hand, the electron experiments were performed at lower energies and thus have a sizable contribution to the cross section from the intercept term. Both experiments do measure the same slope and we choose this quantity as the basis for our comparison.

To facilitate the comparison, we extract a single form factor from the slope, assuming that

$$G_E = G_M/\mu \equiv G.$$

This definition in no way influences the results as it is applied to the electron data as well as to the muon data. The form factor $G(q^2)$ is merely a more physical variable and has a convenient parametrization as a function of q^2.

A plot of $G(q^2)$ vs q^2 is shown in Fig. 1 for the muon data and for the electron data of Janssens et al.[3] While our analysis uses only the Janssens data, the results are unchanged by the inclusion of all relevant electron-proton scattering data.[4]

Table I. Measurements of the μ-p elastic cross section $d\sigma/dq^2$ at $6^\pm$, $11^\pm$, and 17^- GeV/c as a function of q^2. Cross sections are presented in units of 10^{-30} cm^2/(GeV/c)^2.

q^2 (GeV/c)^2	6^-	6^+	11^-	11^+	11^-	Typical error
0.175	4.30	4.40	4.36	4.27	4.22	0.19
0.225	2.14	1.97	2.03	2.06	1.98	0.12
0.275	1.16	1.16	1.20	1.18	1.31	0.09
0.325	0.86	0.79	0.74	0.76	0.78	0.07
0.375	0.54	0.56	0.46	0.56	0.54	0.06
0.425	0.25	0.30	0.46	0.38	0.33	0.06
0.475	0.18	0.18	0.23	0.24	0.22	0.04
0.525	0.20	0.11	0.28	0.12	0.19	0.03
0.575	0.13	0.14	0.11	0.17	0.18	0.03
0.625	0.07	0.13	0.08	0.05	0.11	0.02
0.675	0.08	0.04	0.06	0.02	0.08	0.02
0.725	0.05	0.06	0.06	0.05	0.05	0.02
0.775	0.06	0.04	0.04	0.06	0.05	0.02
0.825	−0.01	−0.01	0.02	0.04	0.05	0.02

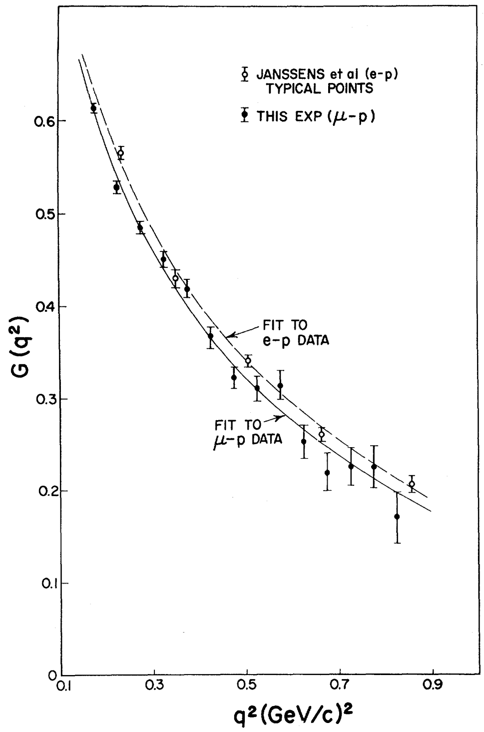

FIG. 1. Measurements of the form factor $G(q^2)$ vs q^2 for this experiment and for the e-p data of Janssens et al. Not all of the electron data are shown. The solid and dashed curves represent fits to the muon and electron data, respectively.

A quantitative comparison was made by fitting the electron and muon data separately with the function

$$G_{e,\mu}(q^2)=F(q^2)\frac{N_{e,\mu}}{1+q^2/\Lambda_{e,\mu}^2}. \tag{1}$$

The factor $F(q^2)$ is common to both fits and contains most of the q^2 dependence of the form factors. Any μ-e structure difference is contained in the remainder of the expression, where $\Lambda_{e,\mu}$ is a cutoff parameter and $N_{e,\mu}$ is an arbitrary normalization. A number of forms for $F(q^2)$ were tried: (a) the dipole fit,[5] $C(1+q^2/\Lambda^2)^{-2}$, (b) the Mack fit,[6] $\exp\{-A[\ln^2(aq^2+4am_\pi^2)-\ln^2(4am_\pi^2)]\}$; (c) the three-pole fit given by Janssens et al.; and (d) the polynomial in $1/q$, $-0.49536+0.86018/q-0.22805/q^2+0.027391/q^3$. The parameters in $F(q^2)$ were determined by fitting to the electron data above. The data were fitted best by the polynomial in $1/q$, and this fit was used. The results are, however, independent of the particular form used for $F(q^2)$. Then Eq. (1) was fitted separately to the electron and muon data, yielding values for $N_{e,\mu}$, $\Lambda_{e,\mu}$, and their uncertainties. With this procedure, $N_e=1$ and $1/\Lambda_e^2=0$ but the electron and muon errors are treated in a symmetrical fashion. To compare these two fits, we form the ratio

$$\frac{G_\mu}{G_e}=\frac{N_\mu}{N_e}\frac{1+q^2/\Lambda_e^2}{1+q^2/\Lambda_\mu^2}\approx\frac{N}{1+q^2/\Lambda^2},$$

where $N=N_\mu/N_e$ and $1/\Lambda^2=1/\Lambda_\mu^2-1/\Lambda_e^2$.

We first allow no relative normalization difference ($N_\mu=N_e=1$) and find $1/\Lambda^2=0.148\pm0.024$ $(\mathrm{GeV}/c)^{-2}$, a value distinctly different from zero. However, in this fairly restrictive range of q^2, a finite $1/\Lambda^2$ is difficult to distinguish from a normalization error. Indeed, the muon data in Fig. 1 appear to lie systematically below the electron results. Accordingly, if we constrain the shapes to be the same ($1/\Lambda_e^2=1/\Lambda_\mu^2=0$), the relative normalization N is 0.960 ± 0.006, some 6 standard deviations from unity. This 4% suppression of the form-factor ratio represents an 8% difference in the cross sections since $d\sigma/dq^2 \propto G^2$.

One source of systematic error in the cross section is the determination of the momentum transfer q. In our q^2 region, the cross section varies approximately as $1/q$.[5] Thus, any error in q results in five times that error in the cross section. But, we believe our measurement of q to have a systematic error of less than $\frac{1}{2}$%, yielding at most a $2\frac{1}{2}$% error in the cross section. A few of the proton trigger counters were found to have low efficiencies and all data passing through them were eliminated. Any remaining counter inefficiency is estimated to be less than 2%. We are unable to find any single systematic error that could account for the 8% suppression of the muon cross section, but do admit that an optimum combination of systematic effects might remove the disagreement.

Figure 2 shows our results together with the muon data of Ellsworth et al.[7] Their data also lie below the electron results. In fact, our fit to their data, forcing the shape to be that of the electron data ($1/\Lambda_\mu^2=0$), yields $N=0.940\pm0.019$, consistent with our findings. On the other hand, if we deny any normalization problems in the experiments, blaming any discrepancies on a failure of μ-e universality, we find $1/\Lambda^2=0.10\pm0.03$ $(\mathrm{GeV}/c)^{-2}$ for the experiment of Ellsworth et al., similar to our value of 0.148 ± 0.024 $(\mathrm{GeV}/c)^{-2}$.

From these data, one might conclude that there is a difference between the muon and the electron, characterized by a cutoff parameter Λ $=2$-3 GeV/c. However, it is clear that a simple normalization shift of either the muon or elec-

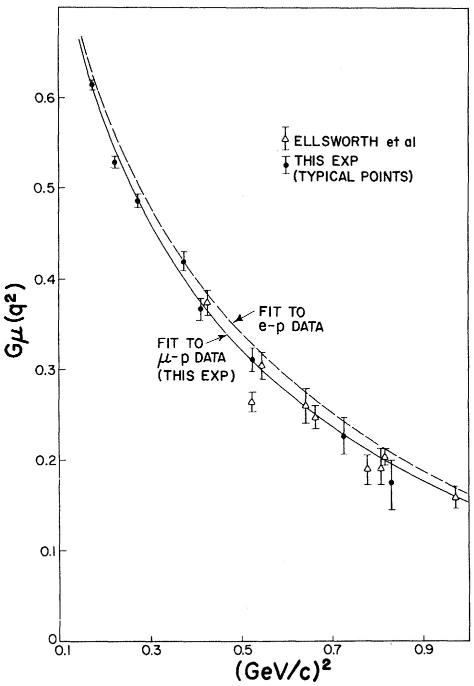

FIG. 2. Measurements of $G(q^2)$ vs q^2 for this experiment and the μ-p results of Ellsworth et al. A fit to the electron data is shown for comparison.

tron data would eliminate this discrepancy. In view of the systematic uncertainties discussed above and the possibility of similar effects in the electron data, we prefer a conservative comparison that leaves both N and $1/\Lambda$ as free parameters. Now we find $N = 0.976 \pm 0.017$ and $1/\Lambda^2 = 0.064 \pm 0.056$ $(\mathrm{GeV}/c)^{-2}$. In this case, the cutoff parameter $1/\Lambda$ is not significantly different from zero. With 95% confidence, we claim that $\Lambda > 2.4$ GeV/c, a result similar to that of Ellsworth et al.

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*Research supported in part by the U. S. Atomic Energy Commission.

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