Table I. Gamma rays from the decay of the 5166-keV level in B^{10}.[a]

E_γ (MeV)	% of decays
5.166	5
4.449	30
3.012	65
2.154	14
1.437	18
1.023	33
0.717	81
0.414	33

[a]See Refs. 7 and 8.

in a calculated population of the 5166-keV level of <180. The measured intensity of the 3366-keV gamma ray taken from the full-energy peak only is 2480 ± 160 counts. Assuming no difference in detector efficiency from 3.0 to 3.5 MeV this means the observed intensity ratio of the 3366-keV level in Be^{10} to the 5166-keV level in B^{10} is >13.

In summary: There is evidence for the (γ, n) reaction leaving B^{10} in an excited state (717 keV); there is no evidence of population of the $T=1$ states in B^{10}; and the first excited state in Be^{10} at 3366 keV is seen clearly.

From these data it appears that the cross section integrated from threshold to 35 MeV for the $T=\frac{3}{2}$ giant resonance in B^{11} is considerably smaller than the cross section over the same range for the $T=\frac{1}{2}$ giant resonance.

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MAGNETIC SUSCEPTIBILITY OF NEUTRON MATTER*

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The magnetic susceptibility of a neutron gas at zero temperature, an idealization of neutron-star matter, is estimated for the realistic, soft-core nucleon-nucleon potential of Reid. For mass densities below twice that of ordinary nuclear matter, there is no sign of a ferromagnetically favorable instability of the normal ground state of the system, the effect of the interactions being to depress the magnetic susceptibility relative to its Fermi-gas value.

The most promising models of pulsars involve rapidly spinning neutron stars possessing intense magnetic fields.[1] It thus seems appropriate to investigate the magnetic properties of pure neutron matter at zero temperature using the most realistic nuclear forces available. There has been some speculation that as the density increases to one to two times that of the matter in the centers of heavy nuclei, the ground state of neutron matter might experience a ferromagnetic transition.[2] However, the exchange character (in this case simply the spin dependence) of the extra-core component of the two-nucleon potential is such as to oppose the onset of ferromagnetism; if the ground state of neutron matter ever does go ferromagnetic, the density at which the transition occurs is expected to be so high that current nuclear many-body theories[3] are probably inapplicable.[4]

Consider a uniform, extended system of neutrons at number density ρ, subject to nuclear forces alone (neutron gas).[5] In view of the aforementioned conclusions of Ref. 4, a proper study of the magnetic behavior of this system at zero temperature may begin with the calculation of its paramagnetic susceptibility χ. A standard derivation[6,7] yields the following working formula for the evaluation of χ:

$$\chi = \frac{\rho\gamma^2}{[\partial^2 E(s)/\partial s^2]_\rho|_{s=0}}. \tag{1}$$

Here s is the spin-polarization parameter $(\rho^{(+)} - \rho^{(-)})/\rho$, where $\rho^{(+)}$ and $\rho^{(-)}$, satisfying $\rho^{(+)} + \rho^{(-)} = \rho$, are, respectively, the partial densities of particles with spin up and spin down relative to a weak perturbing magnetic field; $E(s)$ is the energy per particle of the unperturbed system as a function of s; and γ is the particle magnetic moment. For an ideal Fermi gas of one species of spin-$\frac{1}{2}$ particles, Eq. (1) yields the familiar result

$$\chi_F = \gamma^2 \frac{1}{2\pi}\frac{M}{\hbar^2}\frac{2}{\pi} k_F, \tag{2}$$

where $k_F = (3\pi^2\rho)^{1/3}$ is the Fermi wave number of the unperturbed system. As is well known, the condition $\chi^{-1} = 0$ signals the onset of ferromagnetism.

A short cut in our development is allowed by the close formal parallel of the many-body methods required to treat liquid He^3 and the neutron gas. Østgaard[7] has carried out a careful evaluation of χ for the former system based on a reaction-matrix evaluation of the energy $E(s)$ [and associated self-consistent determination of the hole-line potential energy at $s=0$, $V(k_m)$]. His key formulas, to be applied here, are

$$(\chi/\chi_F)^{-1} = 1 + C_d + C_\sigma + C_r,$$
$$C_d = M/M^* - 1,$$
$$C_\sigma = (2/\pi)k_F\int_0^1 [a_e(x) - \alpha_o(x)]x dx. \tag{3}$$

The quantity M^* is the effective mass, to be computed via

$$M^* = \left[1 + \frac{M}{\hbar^2}\left(\frac{\partial V(k_m)}{\partial k_m}\right)_{k_m = k_F}\right]^{-1} M. \tag{4}$$

The quantities $a_e(x)$ and $a_o(x)$ are to be derived, respectively, from the two-body interaction in even- and odd-parity states. In particular, $2\pi(\hbar^2/M)a_2(x)$ is just the sum of even-partial-wave reaction-matrix elements for relative wave number $k_0 = k_F x$ [including for each element the statistical factors appropriate to a calculation of $E(0)$], and, in the absence of noncentral forces, $2\pi(\hbar^2/M)3a_o(x)$ is the corresponding odd-state sum. The three corrections C_d, C_σ, and C_r to the Fermi-gas value for $(\chi/\chi_F)^{-1}$ arise, respectively, from the modification in the density of single-particle states due to the interactions, from the spin dependence of the reaction matrix, and from the intrinsic dependence on s of the reaction matrix (rearrangement effect). An explicit approximate formula for C_r is given by Østgaard. The three-body contributions to $E(s)$ in the sense of Bethe[8] have been ignored; these will become more and more important as the density increases and eventually—along with other corrections—invalidate the present approach.

Using now standard methods,[9,10] Sprung and Banerjee have evaluated on-energy-shell diagonal reaction-matrix elements for the realistic soft-core two-nucleon potential of Reid[11] in the $^{2S+1}L_J$ partial waves $^1S_0$, $^1D_2$, $^3P_0$, $^3P_1$, and $^3P_2$-$^3F_2$. Higher-partial-wave contributions to the full reaction matrix were included in Born approximation, the interaction in these waves being identified with the one-pion-exchange potential (OPEP). The hole-line potential energy $V(k_m)$ was determined from the resulting full reaction matrix. Self-consistency was not complete. Calculations were performed for $k_0 = (0.2, 0.35, 0.45, 0.548, 0.65, 0.775, 0.9)k_F$, for the same values of k_m, and for k_F values in the range 0.504-2.02 fm^{-1}, corresponding to a factor of 60 variation in ρ. The calculations could probably be pushed to somewhat higher densities (optimistically, to $k_F = 3\ fm^{-1}$) without danger.

The Sprung-Banerjee matrix elements and potential energies supply the raw material for estimates of the corrections C_d, C_σ, and C_r. Results for C_d and for three different estimates $C_{\sigma e}$, C_σ', and C_σ'' of C_σ (to be defined below by their mode of computation) are summarized in Table I. With each k_F appears the corresponding mass density ρ_m and ratio ρ/ρ_0, where ρ_0 is the equilibrium particle density of (uniform, extended) symmetrical nuclear matter with only nuclear forces in action (ordinary nuclear matter). Each of the four densities selected has some intrinsic interest: (1) For pure neutron matter the S-state superfluid energy gap has a peak near $k_F = 0.756\ fm^{-1}$. Correspondingly, the even-state contribution to C_r is positive and sizable. (2) The even-state contributions to C_r are especially small near $k_F = 1.26\ fm^{-1}$. (3) At $k_F = 1.71\ fm^{-1}$, ρ coincides with ρ_0. Curiously, for this total baryon density, the partial density of the proton component of neutron-star matter (as computed in Ref. 9) corresponds to a Fermi wave number for this component at which the superfluid (superconductor) energy gap for this component in isolation is near its peak value. (4) Beyond $k_F = 2.02$

Table I. Results for the magnetic susceptibility of the ground state of pure neutron matter.

k_F (fm^{-1})	ρ_m (10^{14} g cm^{-3})	ρ/ρ_0	M^*	C_d	$C_{\sigma e}$	C_σ'	C_σ''	$(\chi/\chi_F)^{-1}$
0.756	0.244	0.086	0.950	0.052	0.720	0.702	⋯	1.8
1.26	1.13	0.40	0.873	0.145	0.772	0.736	0.760	1.9
1.71	2.83	1.0	0.779	0.283	0.750	0.742	⋯	2.0
2.02	4.66	1.6	0.719	0.391	0.698	0.736	0.798	2.1

fm^{-1}, i.e., $\rho \approx 2\rho_0$, no realistic reaction-matrix calculations have been carried out.

Some detailed remarks concerning the entries in Table I follow. The numerical error in the C_d evaluation (due to the coarseness of the k_m grid) is perhaps 5%. [This error was estimated by fitting a linear function of k_m^2 to $V(0.775k_F)$ and $V(0.9k_F)$ and determining the corresponding M^* (cited in Table I), repeating this procedure for $V(0.65k_F)$ and $V(0.775k_F)$, and comparing the two M^* results.] Over the density range studied, the correction C_d depresses χ relative to its Fermi-gas value, the effect increasing with k_F. The evaluation of C_σ is rendered somewhat uncertain by the fact that no parity decomposition of the OPEP contribution to the full reaction matrix was available. More seriously, the presence of noncentral components in the Reid potential has the consequence that the ${}^{2S+1}L_J$ partial-wave reaction-matrix elements of Sprung and Banerjee no longer suffice for the evaluation of a_o. The three C_σ estimates $C_{\sigma e}$, C_σ', and C_σ'' differ in the manner in which these complications were dealt with; their spread should provide some idea of how drastic the corresponding approximations are. In $C_{\sigma e}$, a_o was simply omitted, along with contributions from partial waves approximated via OPEP. In C_σ', $2\pi(\hbar^2/M)a_o$ was approximated as one-third the sum of the statistically weighted odd-partial-wave reaction-matrix elements (this amounts to forgetting about the fact that for given LS, the reaction-matrix elements may depend on J), again disregarding the OPEP waves. Finally, C_σ'' was evaluated in the same way as C_σ', except that the OPEP contribution to the full reaction matrix, no longer ignored, was attributed wholly to odd-state interactions. The spread in these three estimates is seen to be at most 14%, at the highest density accessible. Over the density range studied, the correction C_σ, remarkably constant, again acts to depress χ relative to χ_F. The $(\chi/\chi_F)^{-1}$ entries in Table I refer to $1+C_d+C_\sigma$, differences among the C_σ estimates having been ironed out. The rearrangement correction C_r, while perhaps non-negligible, cannot be computed accurately, and even a rough estimate is tedious. However, preliminary work using the approximation of Ref. 7 (which involves a double quadratic interpolation, first in k_0 and then in k_F, from the reaction-matrix data supplied by Sprung and Banerjee) indicates that C_r enhances the correction C_d+C_σ by roughly 10% or less over the density range examined here, i.e., that its effect is to depress χ slightly further from χ_F. A thorough analysis of the rearrangement effect, and of the (possibly more important) error in the C_σ estimates C_σ', C_σ'' due to the noncentral character of the potential, is still in progress. A final note: If, following Ref. 9, we correct for incomplete self-consistency and "underbinding" of the Reid potential by attaching an overall adjustment factor to the reaction-matrix elements, our results for $(\chi/\chi_F)^{-1}$ will be increased by several percent.

The Reid soft- (Yukawa-) core potential is considered one of the more satisfactory potential models of the two-nucleon interaction, providing a good fit of the low-energy (deuteron and effective-range) data and the Livermore[12] phase-shift parametrization of the nucleon-nucleon scattering data in the laboratory energy range 0-350 MeV.[13] However, the current two-body data —even in the isospin-1 case of interest here, for which the most accurate scattering measurements can be made—do not determine the behavior of the short-range repulsion: The data may be fitted equally well using potentials with hard cores or finite, flat-topped cores.[14] In view of this uncertainty, which surely has profound consequences at high enough densities, it is noteworthy that Clark and Chao[4] arrived at basically the same conclusions regarding χ as those presently reported, using two hard-core potentials (the Hamada-Johnston and Iwamoto-Yamada potentials) whose short-range components differ substantially one from another as well as from that of the Reid soft-core potential. Incidentally, nuclear-matter results seem to favor soft cores,[8] although the evidence is not very strong.

In summary, a study of the normal ground state of the neutron gas with realistic forces reveals no sign of a ferromagnetically favorable instability at mass densities below twice that of ordinary nuclear matter. This conclusion enhances the importance of investigations of the role of superfluidity and/or superconductivity in neutron star matter.[15] As regards the (in some sense complementary) question of mechanisms for the origin of strong magnetic fields in neutron star models of pulsars, attention shifts from neutron ferromagnetism to other possibilities, notably that of self-consistent magnetization associated with the Landau orbital ferromagnetism state of the subsystem of electrons inevitably present in the star.[16]

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