tal problems imposed on the experiment proposed here. The sensitivity of the apparatus in both groups is about 10^{-5} rad. With minor modifications of the optics and the detecting system, and replacing the reflecting mirror by a light trap, both groups should have no difficulty in performing this experimental test. Since $\bar{v}_n \propto \mathcal{E}^2$ and $\bar{v}_m \propto \mathcal{E}$, it may be difficult to observe θ_n if $\mathcal{E} < 10^8$ V/cm. However, there is always a good chance to measure θ_m. Even for $\mathcal{E} \simeq 10^6$ V/cm, $\theta_m \simeq \pm 3 \times 10^{-4}$ rad.

There have been some difficulties in acquiring a good estimation for the $\mathcal{E}$ value of a laser beam in a small region.[9] If the result of this proposed experiment turns out to be positive, the idea of this experiment will have a practical meaning. Further development of this apparatus may be used as an instrument to map the electric field in a laser beam. Undoubtedly, this is a crucial experiment. If the result turns out to be negative, it may mean a disaster in the present concept of classical fields, as all the calculations are a consequence of classical electrodynamics.

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CONTINUOUS BREAKING OF CHIRAL SYMMETRY*

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Continuity arguments for transitions between various subgroups of the chiral-symmetry group are presented. Some interesting sum rules are obtained in this way.

Recent investigations[1] have led to the suggestion that the physical manifestation of a chiral-invariant Lagrangian lies in the existence of Goldstone bosons, degenerate vacua, and nonperturbative solutions. In this note we utilize this idea within the framework of the Gell-Mann model[2] studied more recently by Gell-Mann, Oakes, and Renner (GMOR),[3] and investigate the general properties of the two-point functions. This approach leads in a natural manner to the introduction of many interesting subgroups of the chiral-symmetry group for different possible values of the symmetry-breaking parameter. We discuss the possibility of continuous transitions among different subgroups which leads to some interesting mass formulas and relations between other physically relevant quantities in a nonperturbative way. Especially, the relation[3] obtained by GMOR for the symmetry-breaking parameter is reproduced in this manner.

We start with the strong-interaction Hamiltonian density given by

$$H(x) = H_0(x) + \epsilon_0 S^{(0)}(x) + \epsilon_8 S^{(8)}(x), \tag{1}$$

where $H_0(x)$ is assumed to be invariant under the chiral group $W(3) \equiv \mathrm{U}^{(+)}(3) \otimes \mathrm{U}^{(-)}(3)$, broken by the scalar density terms. We assume[2,3] that the scalar density nonet $S^{(i)}(x)$ together with the nonet $P^{(i)}(x)$ $(i = 0, 1, \cdots, 8)$ transforms according to the $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ representation, satisfying the usual algebra.[2,3] These algebraic relations and the local generalization of the usual equations[3] of motion lead to the current divergences

$$\partial_\mu V_\mu^{(j)}(x) = \epsilon_8 f_{j8k} S^{(k)}(x), \quad \partial_\mu A_\mu^{(i)}(x) = (\epsilon_0 d_{j0k} + \epsilon_8 d_{j8k}) P^{(k)}(x). \tag{2}$$

Now, if we write the usual spectral representation for the commutator,

$$\langle 0|[A_\mu^{(i)}(x), A_\nu^{(j)}(y)]|0\rangle = \int_0^\infty dm^2 \left\{\left(\delta_{\mu\nu} - \frac{1}{m^2}\partial_\mu\partial_\nu\right)\rho_{ij}^{(1)}(m, A) - \rho_{ij}^{(0)}\frac{1}{m^2}\partial_\mu\partial_\nu\right\}\Delta(x-y, m) \tag{3}$$

and a similar one for the vector currents, we obtain, on taking divergences of both sides and setting $x_0 = y_0$, the relations

$$I_{ij} = \int_0^\infty dm^2 \rho_{ij}^{(0)}(m, A) = -(\epsilon_0 d_{0ik} + \epsilon_8 d_{8ik})(\xi_0 d_{0jk} + \xi_8 d_{8jk}),$$
$$K_{ij} = \int_0^\infty dm^2 \rho_{ij}^{(0)}(m, V) = -\epsilon_8 \xi_8 f_{8ik} f_{8jk}, \tag{4}$$

where

$$\xi_0 = \langle 0|S^{(0)}(0)|0\rangle, \quad \xi_8 = \langle 0|S^{(8)}(0)|0\rangle \tag{5}$$

are the only nonvanishing vacuum expectation values of the scalar density operator. Defining the real parameters a, b, and γ by

$$a = 2^{-1/2}\epsilon_8/\epsilon_0, \quad b = 2^{-1/2}\xi_8/\xi_0, \quad \gamma = -\tfrac{2}{3}\xi_0\xi_0, \tag{6}$$

Eq. (4) leads to the following relations:

$$I_{33} = \gamma(1+a)(1+b), \quad I_{44} = \gamma(1-a/2)(1-b/2), \quad K_{44} = (9/4)\gamma ab, \quad I_{-1,-1} = \gamma(1+a)(1+b),$$
$$I_{-2,-2} = \gamma(1-2a)(1-2b), \quad I_{-1,-2} = I_{-2,-1} = 0, \tag{7}$$

where we have used the following combinations:

$$A_\mu^{(-1)}(x) = \frac{1}{\sqrt{3}}(A_\mu^{(8)}(x) + \sqrt{2}A_\mu^{(0)}(x)), \quad A_\mu^{(-2)}(x) = \frac{1}{\sqrt{3}}(A_\mu^{(0)}(x) - \sqrt{2}A_\mu^{(8)}(x)) \tag{8}$$

instead of $A_\mu^{(0)}$ and $A_\mu^{(8)}$.

We now investigate the consequence of the positivity conditions in the Hilbert space:

$$I_{jj} \geq 0, \quad K_{jj} \geq 0 \text{ (no summation over } j). \tag{9}$$

The requirement (9) on Eq. (7) can be shown to lead to solutions for a, b, and γ confined within the seven domains in the a-b plane shown in Fig. 1. Notice that a, b, and γ cannot assume arbitrary values. The boundaries of these domains are intimately related to symmetries under various subgroups of $W(3)$. Indeed, from Eq. (2) we find the following: (i) $a = 0$ implies $\partial_\mu V_\mu^{(j)}(x) = 0$ for $j = 0, 1, \cdots, 8$, i.e., exact validity of the usual group U(3). (ii) $a = -1$ leads to $\partial_\mu A_\mu^{(i)}(x) = 0$ for $j = 1, 2, 3$ as well as $\partial_\mu A_\mu^{(-1)}(x) = 0$. Since the usual isospin together with the hypercharge is a good symmetry, the point $a = -1$ corresponds to the subgroup $W(2) = U^{(+)}(2) \otimes U^{(-)}(2)$. (iii) $a = \frac{1}{2}$ gives $\partial_\mu A_\mu^{(-2)}(x) = 0$ which leads to the symmetry group $U_A^{(-2)}(1)$. Thus, $a = \frac{1}{2}$ corresponds to the symmetry $Z = U(2) \otimes U_A^{(-2)}(1)$, where U(2) represents the ordinary isospin and hypercharge group. (iv) $a = 2$ leads to $\partial_\mu A_\mu^{(j)}(x) = 0$ for $j = 4, 5, 6, 7$. If we set $x^{(\alpha)} = i\int d^3x\, V_4^{(\alpha)}(x)$ for $\alpha = 1, 2, 3, 8$ and $x^{(\alpha)} = i\int d^3x\, A_4^{(\alpha)}(x)$ for $\alpha = 4, 5, 6, 7$, then $x^{(\alpha)}$ $(\alpha = 1, 2, \cdots, 8)$ are easily shown to be generators of a new SU(3) group which we may call the chimeral SU(3) because of the parity mixup.

If the vacuum is nondegenerate, i.e., if no Goldston boson of zero mass appears when one of these group symmetries is reached as we change

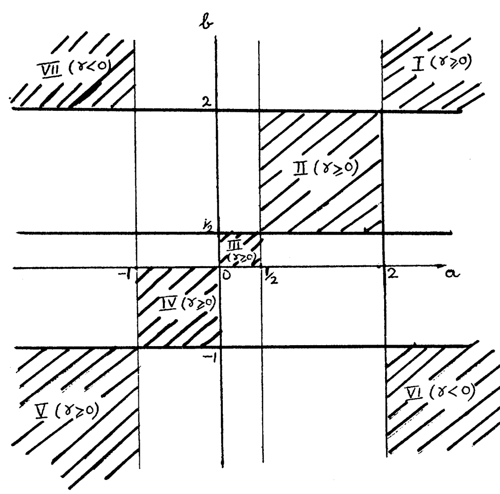

FIG. 1. Allowed domains for the parameters a and b.

the value of a, then it is easy to show that we must have $b=a$ at $a=-1, 0, \frac{1}{2}$, and 2. For example, it is easy to see that $\sqrt{2}S^{(0)}(x)+S^{(8)}(x)$ belongs to a $(\underline{2}, \underline{2}^*)\oplus(\underline{2}^*, \underline{2})$ representation of the group $W(2)$, so that its vacuum expectation value is zero when $W(2)$ is an exact symmetry group, provided that the vacuum state is nondegenerate. Thus, $a=-1$ in this case leads to $b=-1$. Indeed, one can prove the converse also. For instance, if $b=2$, we obtain from Eq. (7) $I_{44}=0$. The positivity condition of the spectral weight then implies $\partial_\mu A_\mu^{(4)}(x)=0$ which leads to exact chimeral SU(3) invariance or $a=2$. Notice in particular that the converse statements do not require the assumption of nondegenerate vacuum.

We shall now appeal to a continuity argument. We may regard b as a continuous function of a, $b=f(a)$, except possibly at a few points. In fact these isolated points can be shown to correspond to those cases where we encounter group symmetries realized via Goldston bosons. At $a=-1$, since $W(2)$ is presumably such a symmetry group, b will not reach the value -1, so that as a is decreased below -1, the value of b will jump discontinuously as is evident from the Fig. 1. Thus $a=-1$ is an essential singular point of the theory. Similarly, it is tempting to conjecture that $a=2$ is also an essential singularity. In this case, the chimeral SU(3) symmetry would be realized through the emergence of zero-mass Goldstone kaons. One can also convince oneself that the points $a=0$ and $a=\frac{1}{2}$ in all likelihood are not singular. One reason is that if this were not the case, it would be difficult to understand the success of the GMOR theory which utilizes a perturbative approach and leads to the determination $a\simeq-0.9$, suggesting that the perturbation theory makes sense with a radius of convergence $|a|\simeq 0.9$.

An application of the $U_A^{(-2)}(1)$ symmetry at $a=\frac{1}{2}$ now implies, for instance, that corresponding to the K meson there may exist a scalar counterpart κ, which at $a=\frac{1}{2}$ has the same mass as the K. It is quite interesting that although the original chiral $W(3)$ symmetry may well be realized through the existence of pseudoscalar Goldstone bosons without the necessity of introducing scalar mesons, the above consideration at $a=\frac{1}{2}$ independently suggests the existence of the scalar κ meson. Then the validity of the usual SU(3) group at $a=0$ demands the existence of a scalar $I=1$, $Y=0$ particle δ, the counterpart of the pion. Now, with respect to the $U_A^{(-2)}(1)$ group, exact at $a=\frac{1}{2}$, we have two possibilities: (i) π and δ transform separately as singlet representations of this group; (ii) π and δ transform as a parity doublet so that at $a=\frac{1}{2}$ we would expect $m_\pi=m_\delta$. The first possibility is probably the the simplest but also the least interesting one, as we shall see presently. The possibility (ii) however, requires a constraint. This arises because it is easy to see that the current $A_\mu^{(j)}(x)$ $(j=1,2,3)$ must be invariant under the gauge transformation generated by $F_5^{(-2)}=-i\int A_4^{(-2)}(x)d^3x$ so that the coupling of the pion to $A_\mu^{(j)}$ must vanish at $a=\frac{1}{2}$, if the pion does not transform according to the singlet representation of $U_A^{(-2)}(1)$. However, we shall adopt the possibility (ii) for the remainder of this paper.

We now apply this formulation to obtain sum rules. If we regard the mass of a particle as a function of the symmetry-breaking parameter a, we have for the meson masses the following constraints: (i) $m_\pi^2=0$ at $a=-1$, (ii) $m_K^2=0$ at $a=+2$, (iii) $m_\pi^2=m_K^2$ and $m_\kappa^2=m_\delta^2$ at $a=0$, and (iv) $m_K^2=m_\kappa^2$ and $m_\pi^2=m_\delta^2$ at $a=\frac{1}{2}$. Simple mass relations satisfying these constraints can be written down as

$$\begin{aligned} &m_\pi^2=(1+a)m_0^2(a), \quad m_K^2=(1-a/2)m_0^2(a),\\ &m_\kappa^2=(1-2a)K(a)+\tfrac{3}{2}am_0^2(a),\\ &m_\delta^2=(1-2a)K(a)+3am_0^2(a), \end{aligned} \tag{10}$$

where m_0^2 and K are some unknown functions of a. From Eq. (10), we obtain readily

$$m_\pi^2/m_K^2=2(1+a)/(2-a) \tag{11}$$

$$m_\kappa^2-m_\delta^2=m_K^2-m_\pi^2. \tag{12}$$

The relation Eq. (11) is identical to the result[3] obtained by GMOR and leads to a determination $a\simeq-0.89$. In the mass formula (12) if we use the experimental value $m_\delta=960$ MeV, we obtain $m_\kappa=1070$ MeV which is quite compatible[5] with recent experiments and also agrees with values obtained from recent theoretical analyses of the K_{l3} problem. It is also interesting to remark that Eq. (12) has been obtained before[6] from perturbative $SW(3)$ arguments with a different $SW(3)$ breaking interaction. Now the coupling constants of (π, K) and κ mesons to axial vector and vector currents, respectively, must obey the following constraints: (i) $f_\pi=0$, $f_K=f_\kappa$ at $a=\frac{1}{2}$; (ii) $f_\pi=f_K$, $f_\kappa=0$ at $a=0$. Proceeding as before, we obtain

$$f_\kappa=f_K-f_\pi. \tag{13}$$

Taking $f_K/f_\pi\simeq1.2$ we get $f_\kappa\simeq0.2f_\pi$ in reasonable agreement with several estimates[5,7] of f_κ.

Finally, we would like to mention that one can easily write down the analogs of Eqs. (12) and (13) in case of vector and axial vector mesons to obtain

$$m_{K_A}{}^2 - m_{A_1}{}^2 = m_{K^*}{}^2 - m_\rho{}^2 \tag{14}$$

$$G_{K_A} - G_{A_1} = G_{K^*} - G_\rho. \tag{15}$$

It is interesting to note that Eq. (14) leads to $m_{K_A} \simeq 1200$ MeV. Also, Eq. (15) is quite compatible with the Weinberg second sum-rule predictions for $SW(2)$ symmetry.

Details of this work with more applications will be published elsewhere.

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VERY HIGH-ENERGY COLLISIONS OF HADRONS

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Proposals are made predicting the character of longitudinal-momentum distributions in hadron collisions of extreme energies.

Of the total cross section for very high-energy hadron collisions, perhaps $\frac{1}{3}$ is elastic and 10% of this is easily interpreted as diffraction dissociation. The rest is inelastic. Collisions involving only a few outgoing particles have been carefully studied, but except for the aforementioned elastic and diffractive phenomena they all fall off (probably as a power of the energy at high energy). The constant part of the total inelastic cross section cannot come from them. And we know that at such energies, the majority of collisions lead to a relatively large number of secondaries (perhaps the multiplicity increases logarithmically with energy). These collisions have not been studied extensively because, with the large number of particles, so many quantities or combinations of quantities can be evaluated that one does not know how to organize the material for analysis and presentation.

It is the purpose of this paper to make suggestions as to how these cross sections might behave so that significant quantities can be extracted from data taken at different energies. These suggestions arose in theoretical studies from several directions and do not represent the result of consideration of any one model. They are an extraction of those features which relativity and quantum mechanics and some empirical facts[1] imply almost independently of a model. I have difficulty in writing this note because it is not in the nature of a deductive paper, but is the result of an induction. I am more sure of the conclusions than of any single argument which suggested them to me for they have an internal consistency which surprises me and exceeds the consistency of my deductive arguments which hinted at their existence.

Only the barest indications of the logical bases of these suggestions will be indicated here. Perhaps in a future publication I can be more detailed.[2]

Supposing that transverse momenta are limited in a way independent of the large z-component momentum of each of the two oncoming particles in the center-of-mass system (so $s = 2W^2$), an analysis of field theory in the limit of very large W suggests the appropriate variables to use for the various outgoing particles in comparing experiments at various values of W in the c.m. system. They are the longitudinal momentum P_z in ratio to the total available W, i.e., $x = P_z/W$, and the transverse momenta Q in absolute units.