EFFECTIVE INTERACTIONS AND COLLECTIVE VIBRATIONS

Larry Zamick*
Rutgers, The State University, New Brunswick, New Jersey 08903,
and The Weizmann Institute of Science, Rehovot, Israel
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The calculated contribution of core polarization to the effective interaction between two nucleons in a nucleus is shown, for certain potentials, to be much larger in the random-phase approximation than in first-order perturbation theory. Collective quadrupole and (in one case) monopole excitations are responsible for this.

Consider the interaction of two nucleons beyond a closed shell. In a model space in which core excitation is not permitted one should use an effective interaction between these two nucleons which simulates the fact that, in reality, the core can be excited.

In Fig. 1(a) the direct or bare interaction between two nucleons is indicated. One may regard the interaction as a G matrix derived from a realistic two-nucleon interaction. Figure 1(b) represents the core-polarization correction to the effective interaction in first-order perturbation theory. At present nearly all calculations have been carried out in first order,[1,2] and it has been found that the magnitudes of those corrections are often comparable with, and sometimes even larger than, the bare matrix elements

It is naturally of interest to see if contributions of order greater than the first are important. We shall here consider the selected diagrams from all orders in which all bubbles, both forward and backward, are summed, with Fig. 1(c) as a typical example. This is of course the random phase approximation (RPA).

In diagram 1(b), the particle-hole pair (ph) can couple to various total angular momenta and isobaric spins $J''T''$. For each such combination we define the ratio of 1(b) to 1(a) as

$$\epsilon^0(\mathrm{ph})^{J''T''},$$

and we define

$$\epsilon^0(\mathrm{ph}) = \sum_{J''T''} \epsilon^0(\mathrm{ph})^{J''T''}.$$

The ratio of the sum of all diagrams such as 1(b) and 1(c) to 1(a) is defined as $\epsilon^{J''T''}$. We thus have the following relationship between the effective and bare interactions: In first order,

$$G_{\mathrm{eff}} = [1 + \sum_{\substack{\mathrm{ph} \\ J''T''}} \epsilon^0(\mathrm{ph})^{J''T''}]G_{\mathrm{bare}};$$

in random phase approximation,

$$G_{\mathrm{eff}} = [1 + \sum_{J''T''} \epsilon^{J''T''}]G_{\mathrm{bare}}.$$

In the case where the two nucleons are in a given shell n, l, j,[3] the quantity $\epsilon^{J''T''}$ satisfies the following equations:

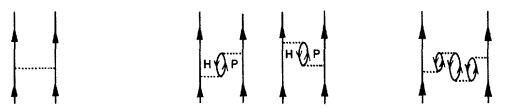
$$\epsilon^{J''T''} = \sum_{\mathrm{ph}} \epsilon^0(\mathrm{ph})[1+\sigma(\mathrm{ph})^{J''T''}], \quad \sigma(\mathrm{ph})^{J''T''} = \sum_{\mathrm{p'h'}} \sigma^{0,J''T''}(\mathrm{ph},\mathrm{p'h'})[1+\sigma^{J''T''}(\mathrm{p'h'})],$$

where

$$\sigma^0(\mathrm{ph},\mathrm{p'h'}) = \langle(\mathrm{ph}^{-1})^{J''T''}(V/\Delta E)(\mathrm{p'h'}^{-1})^{J''T''}\rangle\langle j\,V[j(\mathrm{p'h'}^{-1})^{J''T''}]^j\rangle/\langle jV(j(\mathrm{ph}^{-1})^{J''T''})^j\rangle.$$

Note that in the case $\mathrm{ph}=\mathrm{p'h'}$, the quantity σ^0 is simply the particle-hole interaction over the energy denominator.

It has been noted[4] that when the two-body interaction is the quadrupole-quadrupole potential, the quantity $\epsilon^{0,J''T''=0}$ becomes identical with the (first-order) isoscalar effective charge for $E2$ transitions and that σ^0 is identical to ϵ^0. It was noted also by Siegel and Zamick[5] that the effective charge was much larger in the RPA than in first order when the Kallio-Kolltveit[5] or Kuo-Brown (Hamada-Johnston[6]) G matrices were employed. We could thus anticipate that the effective interaction might be quite different in the RPA than in first order. Of course, there is one major difference; the $E2$ effective charge



(a) Bare matrix element (b) First order (c) Typical R.P.A.

FIG. 1. Contributions to the effective interaction.

picks out only the $J''=2$ collective mode, whereas the quantity ϵ^0 is the sum over all J''.

We consider two nucleons in the $0d_{5/2}$ shell plus an O^{16} core. These can couple to $T=0$, $J=1, 3, 5$ and $T=1$, $J=0, 2, 4$. The core polarization consists of particles being excited from the $0s$ to the $1s$-$0d$ shell and from the $0p$ to the $1p$-$0f$ shell. Thus the values of J'' range from 0 to 5, and $T''=0$ or 1.

The results are listed in Table I. As expected, the RPA results are quite different from the first-order ones. For example, for $J=0$, $T=1$ we find $\epsilon^0 G_{\text{bare}}=-0.755$ and $\epsilon G_{\text{bare}}=-1.955$ when the Kuo-Brown[2] interaction is used.

Next we analyze ϵ into its various $J''T''$ components. This is done in Table II for the cases $J=0$ and $J=4$. Looking first at the result for the Kallio-Kolltveit interaction[7] we note that the quadrupole mode ($J''=2$, $T''=0$) is the most important one. It is large in first order, and the difference between the RPA and first order is also very large.

For the Kuo-Brown[2] interaction two modes are important, the quadrupole mode ($J''=2$, $T''=0$) and the monopole mode ($J''=0$, $T''=0$). In the case where $J=0$ and $T=1$, they contribute coherently to give a large value of $\epsilon-\epsilon^0$, whereas for $J=4$, they largely cancel so that the RPA result is close to the first-order one.

Whereas the first order results of Kuo and Brown give a reasonable fit to the spectra of O^{18} and F^{18}, the RPA results are quite bad. For example, whereas the energy of the two valence nucleons in O^{18} as obtained from the mass tables is -3.9 MeV $[E(O^{18})-E(O^{17})+E(O^{16})-E(O^{17})]$, the results of a spectral calculation (including $s_{1/2}{}^2$ and $d_{3/2}$) gives a value about twice as large as this. The matrix elements are now much too large.

It was noted by Blomqvist[8] and also found by Goode[9] that in O^{16}, if one uses the Kuo-Brown interaction in an RPA calculation, one gets the lowest $J''=0$, $T''=0$ 1p-1h state near zero energy (the unperturbed position is $+2\hbar\omega\approx 30$ MeV). Goode also pointed out that the $J''=2$, $T''=0$ state comes at about 19 MeV. No quadrupole state has been found at this energy but recently it has been reported that quadrupole excitations may have been seen with a center at 27 MeV,[10] which is considerably higher than the above theoretical value. Clearly, the fact that these collective states come down too low in O^{16} is what causes the core polarization corrections in O^{18} and F^{18} to be too large.

Does the fault for the poor results lie with the G matrices that we employed, or with the random

Table I. Core polarization in first order and in the RPA.

Kuo-Brown						
J	T [a]	G_{BARE}	ϵ^0 [b]	$(\epsilon-\epsilon^0)$ [c]	$\epsilon^0 G_{\text{BARE}}$	ϵG_{BARE}
1	0	-0.296	0.844	3.159	-0.250	-1.185
3	0	-0.791	-0.120	0.018	0.095	0.081
5	0	-3.422	-0.023	0.207	-0.079	-0.787
0	1	-1.236	0.611	0.971	-0.755	-1.955
2	1	-1.012	-0.055	0.438	+0.056	-0.388
4	1	-0.434	-0.970	-0.102	+0.421	+0.465

Kallio-Kolltveit						
J	T	G_{BARE}	ϵ^0	$(\epsilon-\epsilon^0)$	$\epsilon^0 G_{\text{BARE}}$	ϵG_{BARE}
1	0	-2.608	0.045	0.640	-0.117	-1.786
3	0	-1.697	-0.170	-0.581	0.288	1.274
5	0	-3.919	0.001	0.243	-0.004	-0.955
0	1	-2.953	0.312	0.846	-0.921	-3.420
2	1	-0.803	0.034	0.254	-0.027	-0.231
4	1	-0.803	-1.246	-2.195	1.000	2.736

[a] The numbers J, T refer to the angular momentum and isospin of the two $d_{5/2}$ nucleons.

[b] The first-order perturbation-theory result.

[c] The difference between the RPA and first-order result.

Table II. The contribution of various multipoles to ϵ^0 and $\epsilon - \epsilon^0$.

		J = 0				J = 4			
		Kuo - Brown		Kallio-Kolltveit		Kuo - Brown		Kallio-Kolltveit	
J'	T'	ϵ^0	$(\epsilon-\epsilon^0)$	ϵ^0	$(\epsilon-\epsilon^0)$	ϵ^0	$(\epsilon-\epsilon^0)$	ϵ^0	$(\epsilon-\epsilon^0)$
0	0	0.04	0.29	0.01	0.00	0.11	0.83	0.06	0.02
0	1	0.01	0.00	0.00	0.00	0.02	-0.04	0.01	0.00
1	0	-0.06	0.00	-0.06	0.00	0.02	0.00	0.01	0.00
1	1	-0.02	0.01	-0.01	0.00	0.01	0.00	0.01	0.00
2	0	0.51	0.67	0.30	0.82	-0.73	-0.96	-0.84	-2.28
2	1	0.11	-0.02	0.07	-0.02	-0.15	0.03	-0.19	0.05
3	0	-0.06	0.00	-0.02	0.00	-0.08	0.00	-0.05	0.01
3	1	-0.05	0.01	-0.06	0.01	-0.07	0.01	-0.16	0.03
4	0	0.17	0.02	0.07	0.02	-0.08	-0.01	-0.07	-0.02
4	1	0.05	-0.01	0.02	-0.00	-0.03	0.00	-0.02	0.00
5	0	-0.07	0.00	-0.01	0.00	-0.01	0.00	0.00	0.00
5	1	-0.01	0.00	-0.05	0.00	0.00	0.00	0.01	-0.00
TOTAL		0.61	0.97	0.31	0.84	-0.97	-0.10	-1.25	-2.20

phase approximation? Probably both contribute. The goodness of the RPA was studied by Siegel[4,11] in a problem analogous to this one—the effective charge in O^{17}. By doing a complete second-order calculation he found that the totality of non-RPA graphs for the isoscalar part of the effective charge were of opposite sign to the RPA graphs—the cancellation was not enough to reverse the trend of the RPA but was definitely significant. M. Kirson and R. Barrett have done third-order perturbation-theory calculations of the effective interaction, and Kirson informed the author that the cancellation of the RPA graphs is much stronger for the effective interaction than for the effective charge.

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[1]G. Bertsch, Nucl. Phys. 74, 234 (1965).

[2]T. T. S. Kuo and G. E. Brown, Nucl. Phys. 95, 40 (1966), A114, 24 (1968).

[3]The equation for the general case, where the two nucleons are in different shells, was obtained by P. Ellis.

[4]S. Siegel and L. Zamick, "Importance of Higher Order Corrections to the Effective Charge" (to be published).

[5]S. Siegel and L. Zamick, Phys. Letters 28B, 450 (1969).

[6]S. Hamada and T. Johnston, Nucl. Phys. 34, 382 (1962).

[7]A. Kallio and K. Kolltveit, Nucl. Phys. 53, 87 (1969).

[8]J. Blomqvist, Nucl. Phys. A103, 644 (1967).

[9]P. Goode, thesis and private communication.

[10]R. J. Stewart, R. C. Morrison, and D. E. Frederick, Phys. Rev. Letters 23, 323 (1969).

[11]S. Siegel, Bull. Am. Phys. Soc. 14, 604 (1969).