onance indicate $J^\pi = \frac{7}{2}^-$. The analogs of both levels decay predominantly to the respective 3^- core states. Finally, the 2.65-MeV $\frac{3}{2}^+$ state in ^{95}Zr is thought to arise from a neutron-particle excitation into the $d_{3/2}$ orbit. This result is in disagreement with a recent ^{96}Zr(p,d) study in which this level was suggested to share part of the $d_{5/2}$ neutron hole strength.[10]

In conclusion, we point out that the measurement of (p,p') angular distributions at isobaric analog resonances leading to an excited $J_f = 0$ state yields, in a simple, unambiguous way, the spins of the resonances. The usefulness of this method was demonstrated by determining the weak-coupling structure of states in ^{95}Zr.

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CRITERIA FOR FERROMAGNETISM IN DENSE NEUTRON FERMI LIQUIDS–NEUTRON STARS

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The ferromagnetic phase boundary and intrinsic neutron spin polarization are calculated as a function of the strength and range of the neutron forces, and the density of the neutron liquid. This system is discussed within the context of neutron stars.

A magnetically ordered state of fermion systems applies to some well-known examples of electronic systems: the ground-state configuration on an atomic scale, Hund's rule; "local-moment" impurity states in metallic alloys[1]; and itinerant electron ferromagnetic metals and alloys.[2] Here the underlying mechanism consists of spin-independent forces, the exclusion principle, and the energetic balance of increased kinetic energy at the expense of potential energy in relegating a majority of the fermions to a single spin band. Nuclear forces are a priori only partially understood, and phenomenological models have evolved in an attempt to explain nucleon-nucleon scattering data and nuclear binding in a nuclear matter environment. In this note, we treat a very simple but nontrivial model of the nuclear potential, viz. spin-independent, static, central forces consisting of a short-range repulsive core and a longer range (~1.5-F) attractive potential. Our results show that for a homogeneous neutron Fermi liquid, in the range of nuclear densities, the contributions to the energy from the singlet scattering of the core can dominate the effects of the attractive interaction and increased kinetic energy to cause stability of the "broken-symmetry" ferromagnetic state.[3,4] The force parameters and density range characterizing the ferromagnetic phase boundaries and the ferromagnetic regime are typical of what one would expect for neutron liquids in neutron stars. The existence of a ferromagnetic state in neutron stars would be of considerable interest as to (1) the consequences of the intense local magnetic fields generated, and (2) the substantial influence in the chemical-equilibrium composition and evolution of the star due to the increase in the Fermi energies of the baryon constituents. The effects of huge magnetic fields in such systems (previously assumed to have originated solely by the trapped flux in a collapsing star) have been extensively studied by several workers.[5] Using intense magnetic fields, these authors have, for example, proposed plausible mechanisms for both the radio and optical emission observed in pulsar radiation.

The many-body techniques used to compute the phase diagram and the polarization are typical of the methods used to analyze similar phenomena in itinerant ferromagnetic metals and alloys.[1,2] Here the relevant parameters are the interaction strength, range, and density of the liquid. The calculations have been performed in the self-consistent Hartree-Fock approximation. To calculate the phase diagram, Fig. 1, we impose a homogeneous magnetic field on the system and calculate the instability pole of the linear susceptibility in the paramagnetic regime. The results for the polarization in the ferromagnetic regime are obtained by equating the chemical potentials of the two spin bands in the ferromagnetic state. It is relevant to note that the ferromagnetic state is not necessarily characterized by complete polarization; rather the bands are split and one finds complete polarization only under specific conditions of the parameters. See Fig. 2.

We assume the following Hamiltonian for our system:

$$H=\sum_{\vec{k},\alpha}(\xi_k-\mu)n_{\vec{k},\alpha}+\sum_{\substack{\vec{k},\vec{k}',\vec{q}\\ \alpha,\beta}}\tfrac{1}{2}V_{\text{eff}}{}^{\alpha\beta}(\vec{q},\vec{k},\vec{k}')C_{\vec{k}+\vec{q},\alpha}{}^{\dagger}C_{\vec{k}'-\vec{q},\beta}{}^{\dagger}C_{\vec{k}',\beta}C_{\vec{k},\alpha}. \qquad (1)$$

Here ξ_k is the relativistic dispersion relation $c(m^2c^2+\hbar^2k^2)^{1/2}$; μ is the chemical potential; $V_{\text{eff}}{}^{\alpha\beta}$ is a general form of the "effective" two-body interaction in the Fermi liquid; i.e., we assume that all necessary vertex renormalizations have been performed. We now assume a simple model for the effective interaction: a spin-independent, central potential consisting of $V_1(r)$, a delta-function repulsive core, and $V_2(r)$, an attractive potential with a Yukawa tail (range λ). The "effective potentials" are assumed to be slowly varying functions of density in the regime of interest:

$$V_1(r)=I\delta(r);\quad V_2(r)=-(U\lambda/r)(e^{-r/\lambda}-e^{-2r/\lambda}). \qquad (2)$$

The results of the calculation are expressed in terms of the integrated strengths of these potentials times the free-particle density of states:

$$\bar{I}=I\rho^0(\epsilon_F);\quad \bar{U}=3\pi\lambda^3U\rho^0(\epsilon_F). \qquad (3)$$

The linear susceptibility in the paramagnetic regime is

$$\chi=(\tfrac{1}{2}g_N\mu_N)^2\rho(\epsilon_F)\left(1-\left\{I-\frac{2\pi^2}{k_{F0}{}^2}\left[\frac{\partial\gamma(k)}{\partial k_{F0}}\right]_{k=k_{F0}}\right\}\rho(\epsilon_F)\right)^{-1}, \qquad (4)$$

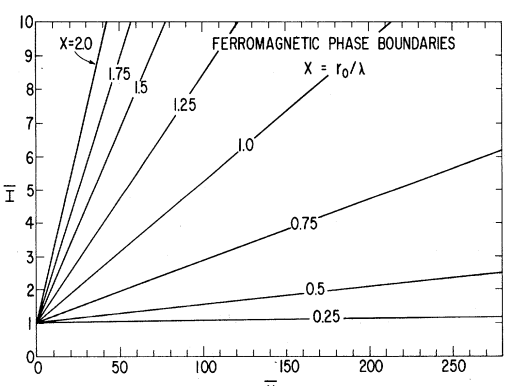

FIG. 1. Ferromagnetic phase boundaries as a function of the potential strengths $\bar{I}$ and $\bar{U}$, and x, the ratio of the interparticle spacing to the range of the attractive potential λ, for $\lambda=1.5$ F.

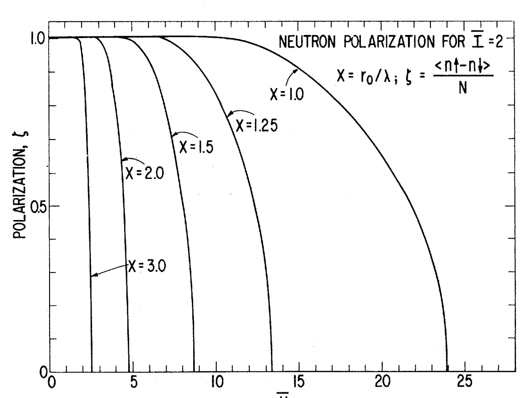

FIG. 2. The polarization ζ, as a function of decreasing $\bar{U}$ for $\bar{I}=2$ for various values of $x=r_0/\lambda$, for $\lambda=1.5$ F.

where $\gamma_\alpha(k)$ and the renormalized density of states are given by

$$\gamma_\alpha(\vec{k}) = -\sum_{\vec{q}} V_2(q)\langle n_{\vec{k}+\vec{q},\alpha}\rangle, \tag{5}$$

$$\rho(\epsilon_F) = \rho^0(\epsilon_F)\left\{1 + \frac{2\pi^2\rho^0(\epsilon_F)}{k_{F0}{}^2[\partial\gamma(k)/\partial k]_{k=k_{F0}}}\right\}^{-1}. \tag{6}$$

Performing the relevant Fermi integral (5), we find the condition for ferromagnetism as described by the pole of χ, Eq. (4), to be

$$\bar{I} - 1 \geq f(z)\bar{U}, \quad f(z) = \tfrac{1}{6}z^4 \ln\frac{(1+1/z^2)^4}{1+4/z^2} \tag{7}$$

and

$$z = (\lambda k_{F0})^{-1} = (4/9\pi)^{1/3} r_0/\lambda,$$

where r_0 is the mean interparticle spacing $(4\pi N/3)^{-1/3}$. The phase diagram is given in Fig. 1. To the left of the phase boundary for a given x the system will be ferromagnetic for all values of $\bar{I}$ and $\bar{U}$. As these results are given in terms of integrated strengths, we can estimate the required square-well (range λ_c) core strength at the phase boundary line,

$$I \sim U(\lambda/\lambda_c)^3(9/4)(\bar{I}/\bar{U})_{\text{phase boundary}}. \tag{8}$$

As an illustration of our calculated curves, taking $\lambda/\lambda_c = 3.5$ at a density of 3.5×10^{13} g/cm^3, we obtain a required square-well strength for ferromagnetism of $I \gtrsim 16U$.[6]

In Fig. 2 we show a typical result of our calculations of the polarization as a function of r_0/λ, $\bar{I}$, and $\bar{U}$. These results are obtained by numerical solution of the implicit equations resulting from equating the chemical potentials of the two spin bands:

$$\xi_{kF\uparrow} + I\langle n\downarrow\rangle + \gamma_\uparrow(k_{F\uparrow}) = \xi_{kF\downarrow} + I\langle n\uparrow\rangle + \gamma_\downarrow(k_{F\downarrow}). \tag{9}$$

The polarization $\zeta = \langle n\uparrow - n\downarrow\rangle/N$, and $k_{F\pm} = k_{F0}(1 \pm \zeta)^{1/3}$. The magnitude of the saturation moment per unit volume is given by $1.91\mu_N(4/3\pi r_0{}^3)^{-1}\zeta$, which is in the range 10^{13}-10^{15} emu in the density regime studied. As the system is a liquid, there is no crystalline anisotropy and the net moment would be in the direction of H_0, the intrinsic magnetic field. The internal B field is enhanced over H_0, but the degree of enhancement is difficult to ascertain because of the unknown demagnetizing factors associated with such a body.[7]

In conclusion, we would like to emphasize the following additional points: (1) The consideration of more complicated nucleon force models would not preclude the existence of the ferromagnetic state,[6] and (2) in a model of rotating neutron stars there is no reason, a priori, that the direction of magnetization should coincide exactly with the axis of rotation—hence an enhanced rotational damping due to magnetic dipole radiation[8] and a "searchlight beam" from radiative mechanisms influenced by the large magnetic fields.

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