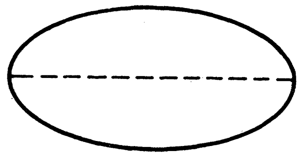

FIG. 2. The ultraviolet-divergent vacuum energy diagram.

and

$$H(g,\kappa)\theta_{\kappa} \to H(g)\theta. \qquad (6)$$

In fact, the construction of vectors θ satisfying Eqs. (5) and (6) exhibits, in an explicit fashion, the cancellation of the divergences in Figs. 1 and 2.

The approximate Hamiltonians $H(g,\kappa)$ are nonlocal, since they involve an ultraviolet cutoff. Thus it is necessary to establish the locality of the Heisenberg-picture dynamics [Eqs. (3) and (4)]. In other words we wish to prove that influence propagates in the Heisenberg picture no faster than the speed of light. We establish locality by introducing an approximate dynamics that obviously possesses the desired locality. Such an approximation is obtained by omitting from the interaction all parts that are localized in strips of width κ^{-1} about the light cones emerging from the space-time point $(x, 0)$. In this way we obtain approximate, but local, field operators $\hat{\psi}_{\kappa}(x,t)$ and $\hat{\varphi}_{\kappa}(x,t)$. We then show that suitable averages of $\hat{\psi}_{\kappa}$ and $\hat{\varphi}_{\kappa}$ converge to ψ and φ of [Eqs. (3) and (4)] as $\kappa\to\infty$. In this way we establish the locality of the limits ψ and φ.

One expects that the Yukawa theory has a vacuum vector Ω and a Hamiltonian H, but this has not yet been proved.

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LARGE-MASS TIMELIKE MUON PAIRS IN HADRONIC INTERACTIONS*

S. M. Berman,† D. J. Levy, and T. L. Neff‡
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
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Utilizing a simple field-theoretic model, we consider the production of timelike muon pairs via a massive virtual photon in high-energy hadronic collisions. We predict the differential cross section in the squared mass of the muon pair as well as the total cross section. We find that our form factors do not possess the analog of the Bjorken scaling associated with deep inelastic electron scattering, but are scale functions multiplied by the usual bremsstrahlung factor $\langle t_{\rm av}\rangle/s$.

We consider the interaction initiated by a hadron (proton or pion)

$$p(\pi)+\text{nucleon}\to(\mu^+\mu^-)+\text{hadronic states}, \qquad (1)$$

where the muons are produced via a very virtual photon. We investigate the asymptotic region analogous to that of deep inelastic electron scattering. In this kinematic region the invariant mass squared of the muon pair q^2 and the center-of-mass energy squared s of the initial hadrons are both very large. We take P^{μ} and $P_1^{\ \mu}$ to be the four momenta of the two initial hadrons and q^{μ} to be four momentum of the muon pair (see Fig. 1). We work in the kinematic region where the variables $q^2, q\cdot P, q\cdot P_1, P\cdot P_1\to\infty$, but the various ratios of these variables are held finite, i.e., the ratios $(q^2/P\cdot P_1)$, $(q\cdot P_1/q^2)$, etc. are all finite. It is our hope that these limits are relevant in the experimentally accessible kinematical regions.

If Process (1) is summed over all hadronic states and averaged over initial spins, then the differential cross section $d\sigma_{\mu^+\mu^-}$ is proportional

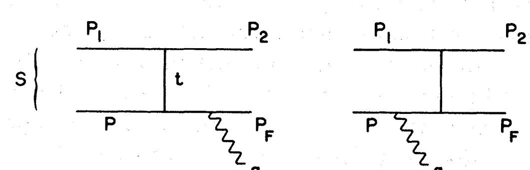

FIG. 1. Model for $p+N\to p+N+(\mu^+\mu^-)$.

to the following tensor:

$$W_{\mu\nu}=\left(\frac{E_1}{M}\right)\left(\frac{E_2}{M}\right)\int d^4x\, e^{-iqx}\langle P\cdot P_1|J_\mu(x)J_\nu(0)|P\cdot P_1\rangle=\frac{W_{2A}}{M^2}\left(P-\frac{q\cdot P}{q^2}q\right)^\mu\left(P-\frac{q\cdot P}{q^2}q\right)^\nu$$

$$+\frac{W_{2B}}{M^2}(P\leftrightarrow P_1)^{\mu\nu}+\frac{W_{2C}}{2M^2}\left\{\left(P-\frac{q\cdot P}{q^2}q\right)^\mu\left(P_1-\frac{q\cdot P_1}{q^2}q\right)^\nu+P\leftrightarrow P_1\right\}-\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)W_1, \quad (2)$$

where M is the nucleon mass and $J_\mu(x)$ is the full electromagnetic current. The matrix element involved in Eq. (2) is similar to the case considered by Bjorken for inelastic electron scattering except that the initial state contains two hadrons rather than one photon, and the mass of the virtual photon is timelike rather than spacelike.[1] It will turn out that the form factors anologous to νW_2 and W_1 of deep inelastic electron scattering, namely, $(q\cdot P)W_{2A}$, $(q\cdot P_1)W_{2B}$, $[(q\cdot P)(q\cdot P_1)]^{1/2}\times W_{2C}$, and W_1, do not scale in the Björken sense but are functions of the form

$$\frac{\langle t_{\rm av}\rangle}{s}F\left(\frac{q^2}{s},\frac{q\cdot P_1}{s},\cdots,\frac{q\cdot P}{q^2}\right),$$

where $\langle t_{\rm av}\rangle\approx(300\text{ MeV})^2$ is the average momentum transfer in purely hadronic interactions.

In an attempt to predict the total muon-pair cross section as well as the differential cross section in the photon mass q^2, we have abstracted certain "insights" gained by studying inelastic electron scattering. These may be summarized as follows.

The Bjorken scaling of the inelastic form factors, such that they are functions of the ratio $q^2/q\cdot P$ (P_μ = initial proton momentum) rather than independent functions of q^2 and $q\cdot P$ separately, is most simply understood in terms of pointlike scattering of the virtual photon on elementary constituent particles. These constituents may be partonlike (see Bjorken and Paschos[2]) or the more ordinary field-theoretic virtual nucleons and pions which make up hadronic structures (see Drell, Levy, and Yan[3]). Considering for the moment that this kind of model is capable of explaining processes involving both timelike and spacelike photons, then we note that there should be a significant qualitative difference between the timelike and spacelike cases. An elementary constituent may scatter elastically with a very massive spacelike photon (i.e., remain on its mass shell before and after the scattering). However, in the timelike case the initial constituent must be very massive (far off-shell) to decay into a very massive photon plus itself. This property reflects itself in the propagator of the internal nucleon (constituent) being evaluated far off shell for the timelike photon (see Fig. 1). [In this kinematical region, use of the Ward-Takahashi[4] identity for the proper proton-photon vertex plus the Deser, Gilbert, and Sudarshan[5] representation suggests that the proper vertex function $\Gamma(q^2, M_1^2, M_2^2)$, when $q^2\approx M_1^2\to\infty$, is indeed of order 1 or pointlike.]

Here we perform a model calculation for the process

$$p(\pi)+\text{nucleon}\to p(\pi)+\text{nucleon}+(\mu^+\mu^-). \quad (3)$$

We consider only the elastic part of the hadronic interactions, but since in pn interactions this is about one fourth the total cross section, we hope that our calculation gives a reasonable first approximation to the total process (1). This calculation will have two advantageous properties: (i) It will be explicitly gauge invariant, and (ii) it will involve no artificial cutoffs or extra parameters. We mock up the purely strong high-energy interactions with the exchange of a single neutral vector meson with its propagator, chosen such that it correctly reproduces the high-energy elas-

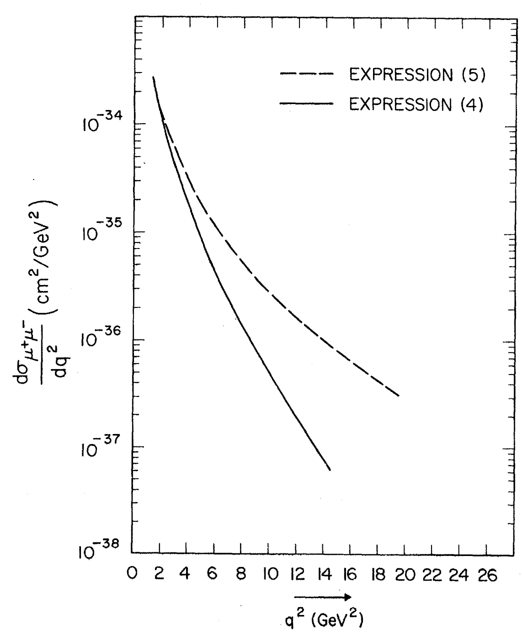

FIG. 2. Plots of q^2 distribution when masses of hadrons are taken equal to zero [Expression (5)] and when masses are nonzero [after integration of photon three-momentum in expression 4].

tic differential cross sections.[6] (The exchange of more vector mesons essentially leaves our final result unchanged and will be discussed in a longer forthcoming paper.) We introduce electromagnetic interactions through minimal pointlike couplings.

We find, after doing all integrations except over the virtual photon, the differential cross section for $p+N\to p+N+(\mu^+\mu^-)$,

$$d\sigma_{\mu^+\mu^-}=\sigma_{pn}\left(\frac{\alpha}{\pi}\right)^2\left(\frac{4}{3}\right)\frac{\langle t_{\rm av}\rangle}{s}\frac{1}{q^2}\frac{d^4q}{2\pi}\frac{[2P\cdot P_1-2(q\cdot P_1+q\cdot P)+q^2]}{(q^2-2q\cdot P)^2}\times\exp\left[\frac{-M^2(q^2-2q\cdot P)^2}{\langle t_{\rm av}\rangle(2P\cdot P_1-2q\cdot P_1)[q^2-2q\cdot(P+P_1)+2P\cdot P_1]}\right], \quad (4)$$

where the constant total elastic pn cross section σ_{pn} is taken as 10 mb and $\langle t_{\rm av}\rangle\approx M_p^2/12$.[7] The factor $\langle t_{\rm av}\rangle/s$ is analogous to the classical acceleration factor in bremsstrahlung emission.[8] To calculate the process $\pi+p\to\pi+p+(\mu^+\mu^-)$, we replace σ_{pn} by $\sigma_{\pi p}$ and $\langle t_{\rm av}\rangle$ by the appropriate average momentum transfer for πp elastic scattering. After integration over the photon three-momentum, $d\sigma_{\mu^+\mu^-}$ becomes a distribution in q^2 (see Fig. 2),

$$d\sigma_{\mu^+\mu^-}\approx\sigma_{pn}\left(\frac{\alpha}{\pi}\right)^2\left(\frac{4}{3}\right)\frac{\langle t_{\rm av}\rangle}{s}\frac{dq^2}{q^2}\frac{1}{q^2}\left[\frac{(s-q^2)(s+q^2)}{8s}-\frac{q^2}{4}\ln\frac{s}{q^2}\right]. \quad (5)$$

When we perform this integration in the overall c.m. system, the range of integration of $|\vec{q}|$ is $|\vec{q}|=0$ to $|\vec{q}|=(s-q^2)/2\sqrt{s}$. We stress that this expression is approximate and only appropriate in the region $q^2\to\infty$ and $s\to\infty$, with the ratio q^2/s finite. This formula is exact only if the masses of the hadrons are zero. Upon doing the integration over q^2, we find for the total cross section $\sigma_{\mu^+\mu^-}\approx 2.0\times10^{-34}$ cm^2, when $s=60$ GeV^2. Here we let the lower bound on q^2 be 1.5 GeV^2 as suggested by experiment.[9] We restate here that our structure functions $(q\cdot PW_{2A},\cdots,W_1)$ do not exhibit the analog of Bjorken scaling as suggested by other authors.[10] This is a consequence of using vector particles to mediate the strong forces and not allowing large momentum transfers in the purely hadronic interactions.

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†Guggenheim Fellow. Current address: Imperial College, London, England.

‡Danforth Graduate Fellow.

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[8]We thank J. D. Jackson for this comment.

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