$\omega\rho^0$ INTERFERENCE EFFECT IN THE $\pi^+p \to \pi^+\pi^-\Delta^{++}$ INTERACTION*

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(Received October 2 1969)

Evidence for a destructive interference effect between the ρ^0 and $\omega \to \pi^+\pi^-$ amplitudes has been observed in the channel $\pi^+p \to \pi^+\pi^-\Delta^{++}$ at 3.7-4.0 GeV/c. On the assumption of complete coherence between the ρ^0 and $\omega \to \pi^+\pi^-$ amplitude in a subsample, discussed in the text, a branching ratio $(\omega \to \pi^+\pi^-)/(\omega \to \pi^+\pi^-\pi^0) = 1.5\,\%$ is obtained with a 95% confidence-level lower limit of 0.25 %. The data do not lead to a significant upper limit. In terms of δ, the off-diagonal element of the $\rho^0\omega$ mass-mixing matrix, these values are equivalent to $\delta = 2.7$ MeV and a lower limit of 1.1 MeV. Under conditions stated in the text, a value $\beta = 1.5 \pm 0.3$ rad is obtained for the production phase of ρ^0 relative to ω in the channel studied.

We have observed evidence for destructive interference between the ρ^0 and the ω in the $\pi^+\pi^-$ final state, and thus for the two-pion decay mode of the ω.[1-4] The observation consists of a 3.5 standard deviation dip in the $\pi^+\pi^-$ mass spectrum at the ω mass, together with indications of change in this same mass interval of the decay angular distribution in the $\pi^+\pi^-$ center-of-mass system.

The experiment is part of a systematic study of the π^+p interaction spanning momenta between 3.7 and 4.0 GeV/c in which we have measured and analyzed 60 000 four-prong events. In particular the observation is based on studies of the reactions

$$\pi^+p \to \pi^+\pi^-\pi^+p \quad (13\,060 \text{ events}) \qquad (1)$$

and

$$\pi^+p \to \pi^+\pi^-\pi^0\pi^+p \quad (15\,814 \text{ events}). \qquad (2)$$

Here we estimate ρ^0 production in Reaction (1) as 8300 events and ω production in Reaction (2) as 4300 events. The data are taken from a recent exposure consisting of 180 000 pictures from the Lawrence Radiation Laboratory (LRL) 72-in. hydrogen bubble chamber in a separated π^+ beam at the Bevatron. The first and (when necessary) second measurements were carried out on the LRL flying spot digitizer (FSD). In cases where a further measurement was required we used the on-line Franckenstein system COBWEB. All processing of events was done with the SIOUX program. In the present note we consider in particular the channels

$$\pi^+p \to \pi^+\pi^-\Delta^{++} \quad (6844 \text{ events}) \qquad (1a)$$

and

$$\pi^+p \to \pi^+\pi^-\pi^0\Delta^{++} \quad (9596 \text{ events}) \qquad (2a)$$

for which we estimate ρ^0 production as 3200 events and ω production as 2000 events.[5]

We first consider the four-momentum transfer and decay angular distributions. The dN/dt_e distributions for the ρ^0 and ω events are shown in Figs. 1(a) and 1(b), respectively. The corresponding density matrix elements ρ^{00} (evaluated in the Jackson frame) are shown in Figs. 1(c) and 1(d). Here we define the "effective" four-momentum transfer squared t_e as $t_e = t - t_{\min} + t_{\min}^0$, where t is the four-momentum transfer squared, $t_{\min}$ is its physical boundary value, and $t_{\min}^0$ is a constant corresponding to the physical boundary for the central mass values of the two resonances under consideration, e.g., ρ^0 and Δ^{++}.[6] We find, in qualitative agreement with earlier results,[7] that the dN/dt_e distribution for the $\rho^0\Delta^{++}$ events is of the form $\exp(at_e)$ with $a = 12$ (GeV/c)$^{-2}$ and shows a break at $|t_e| = 0.4$ (GeV/c)^2. On the other hand the dN/dt_e distribution for the $\omega\Delta^{++}$ events is more complicated, exhibiting a

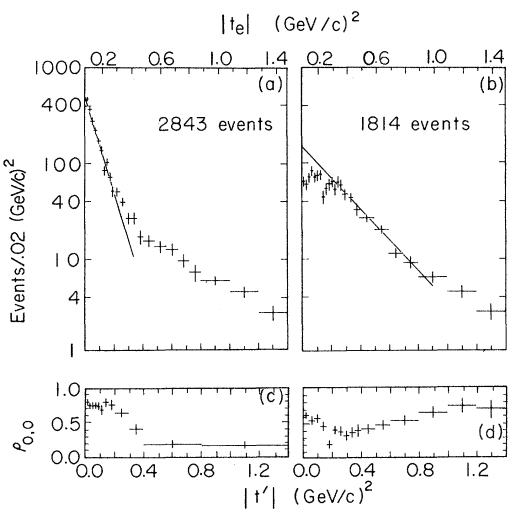

FIG. 1. The dN/dt_e and ρ^{00} distributions versus $|t'|$ (lower scale) and $|t_e|$ (upper scale) (see text for definitions): (a), (c) for the channel $\pi^+p \to \rho^0\Delta^{++}$; (b), (d) for the channel $\pi^+p \to \omega\Delta^{++}$.

fairly flat region up to $|t_e| = 0.22$ (GeV/c)^2 with a dip centered at $|t_e| \cong 0.24$ (GeV/c)^2 followed by a region with slope $a = 3.4$ (GeV/c)$^{-2}$. We note that at low t_e values both channels predominantly populate the state $|jm\rangle = |10\rangle$, suggesting π and B exchange, respectively. (Our experimental results do not depend on this interpretation.) In the case of the ρ^0 events, Fig. 1(c), the ρ^{00} values decrease gradually with increasing t_e. In the case of the ω events, ρ^{00} goes through a dip at $|t_e| = 0.24$ (GeV/c)^2 and then rises again gradually. These features will be discussed in detail in a later note. For the present considerations of the interference effect in the $\pi^+\pi^-$ mass spectrum we confine ourselves to the region $|t_e| < 0.22$ (GeV/c)^2 for which the average ρ^{00} values are

$$\langle \rho_\rho{}^{00} \rangle = 0.74 \pm 0.01 \text{ and } \langle \rho_\omega{}^{00} \rangle = 0.56 \pm 0.04.$$

The $\pi^+\pi^-$ invariant mass distribution for Reaction (1a) is shown in Fig. 2(a). A dip is seen at the $M(\pi^+\pi^-) = 780$-790 MeV mass bin (where $m_\omega = 783.4$ MeV). We find that this dip in the $\pi^+\pi^-$ mass spectrum is considerably enhanced for $|t_e| < 0.22$, Fig. 2(b); but does not occur for $|t_e| > 0.22$, Fig. 2(c). A background subtraction[8] from the two sides of the Δ^{++} band, Fig. 2(d), enhances the effect but leaves the statistical significance almost unaltered. Increased confidence that this dip is not merely a large statistical fluctuation which happens to occur at the ω mass is provided by examination of the distribution of the independent variable θ, the angle between incident π^+, and outgoing π^+ in the $\pi^+\pi^-$ center-of-mass system. At the mass of the ω, partial cancellation of the $\omega \to \pi^+\pi^-$ and ρ^0 amplitudes could lead to a change from the well-known characteristic distribution for ρ-decay; i.e., a $\cos^2\theta$ dependence together with a forward asymmetric term. Figures 3(a)-3(e) show the angular distributions, with the same background subtraction mentioned above,[8] as the $\pi^+\pi^-$ mass varies from below to above m_ω. At the bin corresponding to the dip, the $\cos\theta$ distribution does appear to be somewhat more isotropic, although on its own this effect is not of great statistical significance.

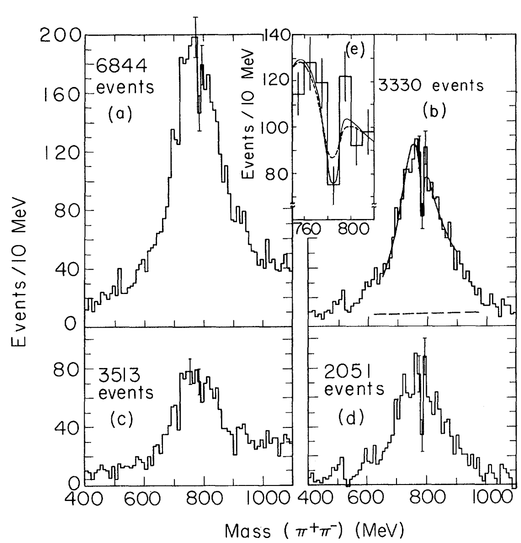

FIG. 2. The $\pi^+\pi^-$ invariant mass spectrum for the channel $\pi^+p \to \pi^+\pi^-\Delta^{++}$. (a) The entire data. (b) For $|t_e| < 0.22$ (GeV/c)^2. The solid curve corresponds to the fit to expression (3) in the text. The dashed line represents the incoherent background contribution. (c) For $|t_e| > 0.22$ (GeV/c)^2. (d) For $|t_e| < 0.22$ (GeV/c)^2, with background subtraction from both sides of the Δ^{++} band. (e) Enlargement of the $\rho^0\omega$ interference region from (b). The dashed curve represents a fit with our resolution function folded in and the result averaged over each bin.

We now proceed to estimate the branching ratio $R = (\omega \to \pi^+\pi^-)/(\omega \to \pi^+\pi^-\pi^0)$. We obtain a lower limit to R by fitting the interference effect, for the sample in Fig. 2(b), on the assumption that for this sample the ρ^0 and ω populate the same helicity states and their $\pi^+\pi^-$ decay amplitudes are coherent. To obtain an upper limit to R we would have to find a sample for which the ρ^0 amplitude vanishes while the ω amplitude is still finite or for which one knows the degree of coherence (e.g., complete incoherence) between the ρ and the $\omega \to \pi^+\pi^-$ amplitudes. Since no such sample is available in our data, we cannot give a significant upper limit to R.

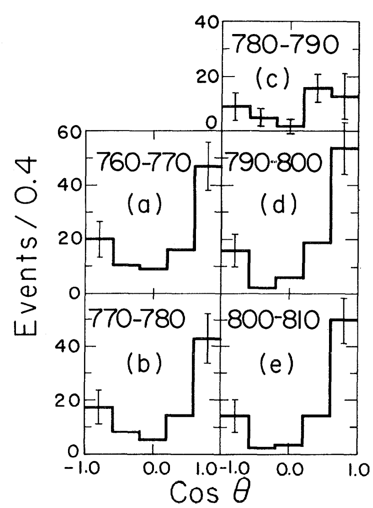

FIG. 3. (a)-(e) The $\cos\theta$ distribution for the same events as in Fig. 2(d) in a series of 10-MeV $\pi^+\pi^-$ mass intervals. The mass intervals in MeV are indicated on each histogram.

It is clear from our data that we observe a destructive interference effect between the ρ^0 and ω amplitudes. Thus the overall phase difference between the two amplitudes must be close to π. This phase difference is composed of two parts: (a) the relative production phase β between ρ^0 and ω, which is characteristic of our particular production process (i.e., through channels 1a and 2a), and (b) the phase β' of the G-parity violating amplitude for ω decay into $\pi^+\pi^-$ which is common to all reactions in which this decay can be studied. The observed interference effect thus implies $\beta+\beta'\approx\pi$. Using a phenomenological parametrization invoking only a few well-tested assumptions (described below), the phase for ω decay becomes $\beta'=\arg[b_\rho(M)]$, permitting the $\pi^+\pi^-$ mass spectrum dN/dM to be expressed as

$$dN/dM = n[|c_\rho b_\rho(M) + c_\omega \alpha e^{i\beta}(\Gamma_\rho/2)^{1/2} \times b_\rho(M) b_\omega(M)|^2 + I(M)]. \quad (3)$$

The expression $b_\lambda(M)$ is the mass dependence of a Breit-Wigner amplitude:

$$b_\lambda(M) = (\Gamma_\lambda/2)^{1/2}/(M_\lambda - M - i\Gamma_\lambda/2).$$

The two terms within the absolute-value signs represent the amplitude for ρ^0 production $\to\pi^+\pi^-$ decay, and for ω production $\to\pi^+\pi^-$ decay, averaged over angles and the chosen t_e interval. Here the form of the ω decay amplitude is motivated by the knowledge that the "intrinsic" $\omega\to\pi\pi$ decay amplitude is small[9] and hence the observed decay must proceed through a ρ^0 intermediate state.[10] In this phenomenological treatment, c_λ are taken as real numbers proportional to the square root of the number of events of particle λ produced in the selected t_e interval. In this sample these numbers are $N_\rho=2130$ and $N_\omega=540$. The term $I(M)$ represents the background, considered here as incoherent, and of the form $a+bM$ multiplied by a phase-space factor. The quantity n is an overall normalization. Under the assumption of complete coherence between the ρ^0 and $\omega\to\pi^+\pi^-$ amplitudes in this sample, the value of α gives the branching ratio by $R=\alpha^2$. The expression $\frac{1}{2}\alpha(\Gamma_\rho\Gamma_\omega)^{1/2}b_\rho(M)$ describes the G-parity nonconserving amplitude for ω decay into $\pi^+\pi^-$, provided that R is "large" ($\gtrsim 1\%$). This form of the vertex function follows from unitarity (Watson's theorem) and from ρ^0 dominance of the p-wave $\pi^+\pi^-$ scattering amplitude.[11] Since the ρ and ω masses are very close, this yields[12] an $\omega\to 2\pi$ decay phase of $\beta'\approx\pi/2$ at $M=M_\omega$.

We have carried out a least-squares fit of expression (3) above to our data in Fig. 2(b) in which we left α, β, M_ρ, and Γ_ρ as free parameters.[13] The best values give $\alpha=0.12\pm0.04$, $\beta=1.5\pm0.3$ rad, $M_\rho=766\pm3$ MeV, and $\Gamma_\rho=161\pm10$ MeV when fitting to the data between the $\pi^+\pi^-$ mass of 650 and 900 MeV. Here the background intensity was first estimated from the entire mass spectrum shown in Fig. 2(b) and amounts to 11% in the mass region fitted. This fit is shown as the solid curve in Fig. 2(b). The dashed line represents the background term. The enlarged insert of the region of the dip, Fig. 2(e), shows the fit in greater detail (solid curve) as well as a fit with our resolution function of full-width at half maximum (FWHM) of 10 ± 1 MeV folded in (dashed curve).[14] We find that inclusion of this resolution function has negligible effect on the fitting parameters and the quality of the fit. For the present we have ignored the $\pi^+\pi^-$ S-wave effect, in that we have fitted mass distributions only. In the future, if large amounts of high-resolution data become available, it is an intriguing possibility to study the S-wave effects in the ω-ρ^0 interference region.

Under the assumptions of the above fit, the value of α leads to a ratio $R=1.5\%$ and, in particular, allows us to quote the 95% confidence-level lower limit of R as 0.25%. As we cannot measure the actual degree of coherence between ρ^0 and $\omega\to\pi^+\pi^-$ decay, expression (3) does not lead to a useful upper limit to R.

In conclusion, we have observed evidence for destructive interference between ρ^0 and $\omega\to\pi^+\pi^-$ amplitudes in the channel $\pi^+p\to\pi^+\pi^-\Delta^{++}$. If we accept the above-mentioned theoretical value for the $\omega\to\pi^+\pi^-$ decay-phase β', this leads to a measure of the relative ρ^0 and ω production phase $\beta=1.5\pm0.3$ rad, a result which has been interpreted by Goldhaber, Fox, and Quigg[11] as evidence for partial π-B exchange degeneracy. This interpretation then leads to predictions for β of $-\pi/2$ for the $\pi^-p\to n\pi^+\pi^-$ reaction and 0 (π) for K^-- ($\overline{K}^0$-) induced reactions. Furthermore, these authors point out that universal coupling of photons to vector mesons implies $\beta=0$ for the $e^+e^-\to\pi^+\pi^-$ formation.[15] Thus, for the different reactions and β values, the interference pattern at the ω mass is expected to go through the entire gamut of possible shapes from destructive to constructive interference. The value we quote for R from expression (3) will tend to be an underestimate as any incoherent component in the data sample of Fig. 2(b) will increase R. The related quantity δ (the off-diagonal element of the ρ^0,ω

mass-mixing matrix)[16] is given by $\delta = (\alpha/2) \times (\Gamma_\rho \Gamma_\omega)^{1/2}$. We thus obtain a corresponding value of $\delta = 2.7$ MeV and a 95% confidence-level lower limit on δ of 1.1 MeV.

We wish to acknowledge our indebtedness to the Alvarez Group bubble chamber crew under Robert Watt and the Bevatron crew under Walter Hartsough for help in carrying out this experiment. We would like to thank John Kadyk for his generous contributions and participation in the early stages of this work. Furthermore, we would like to acknowledge the important contributions made by the meticulous efforts of our scanning and measuring staff and the FSD staff under Howard White, as well as the programming work of Emmett Burns. We have benefited from discussions with A. S. Goldhaber, J. D. Jackson, and C. Quigg.

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*Work supported by the U. S. Atomic Energy Commission.

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[1]Preliminary results on these data were presented at the Conference on $\pi\pi$ and $K\pi$ Interactions, Argonne National Laboratory, 11-16 May 1969 (unpublished), and at the International Conference on Elementary Particles, Lund, Sweden, 25 June-1 July 1969 (unpublished).

[2]For theoretical background see, for example, J. Bernstein and G. Feinberg, Nuovo Cimento 25, 1343 (1962); J. Harte and R. G. Sachs, Phys. Rev. 135, B459 (1964).

[3]For earlier experimental results see S. M. Flatté, D. O. Huwe, J. J. Murray, J. B. Shafer, F. T. Solmitz, M. L. Stevenson, and C. G. Wohl, Phys. Rev. 145, 1050 (1966). Also S. M. Flatté, Phys. Rev. 155, 1517 (1967), and University of California Radiation Laboratory Report No. UCRL-18687 (unpublished), and references quoted therein.

[4]For compilations and interpretation of earlier data see G. Lütjens and J. Steinberger, Phys. Rev. Letters 12, 517 (1964); M. Roos, Nucl. Phys. B2, 615 (1967); and J. Pisut and M. Roos, Nucl. Phys. B6, 325 (1968).

[5]The number of events in a double-resonance region, e.g., $\rho^0\Delta^{++}$, are estimated by considering the entire width of the meson resonance above an incoherent background for a cut on the $p\pi^+$ mass. We have used a rather narrow mass band for Δ^{++} (1160-1280 MeV). When we refer to the number of events in channels 1a and 2a, we mean the number of events obtained for the above Δ^{++} mass cut. When events in the meson bands are given, we use 680 to 860 MeV for the ρ^0 band and 760 to 820 MeV for the ω band.

[6]Thus t_e has the same property as $t' = t - t_{\min}$ in eliminating boundary effects but also gives the effective distance to the pole of the exchange particle, just as t does.

[7]See for example, D. Brown et al., Phys. Rev. Letters 19, 664 (1967), and earlier references quoted therein.

[8]The background subtraction consisted of subtracting the events in the π^+p mass regions 1100-1160 and 1280-1340 MeV from the Δ^{++} band, defined here as 1160-1280 MeV.

[9]R. Gatto, Nuovo Cimento 28, 658 (1963).

[10]S. Coleman, S. L. Glashow, H. J. Schnitzer, and R. Socolow, in Proceedings of the Twelfth International Conference on High Energy Physics, Dubna, U.S.S.R., 1964 (Atomizdat., Moscow, U.S.S.R., 1966), Vol. 1, p. 785.

[11]A. S. Goldhaber, G. C. Fox, and C. Quigg, Phys. Letters 30B, 249 (1969).

[12]More accurately, $\beta' = \pi/2 + \tan^{-1}[(M_\omega - M_\rho)/(\Gamma_\rho/2)]$.

[13]In the fit we have actually used a slightly different Breit-Wigner amplitude for the ρ meson; viz., $b_\rho(M) = (2\Gamma_\rho M_\rho/\pi)^{1/2}/(M_\rho^2 - M^2 - iM_\rho\Gamma_\rho)$ with $\Gamma_\rho(M) = \Gamma_0(p/p_0)^3(M_\rho/M)$, where p is the π momentum in the ρ center of mass and p_0 corresponds to its value for $M = M_\rho$. See J. D. Jackson, Nuovo Cimento 34, 1644 (1964). Furthermore, we have included a phase-space factor for the production process.

[14]We have confirmed by independent checks that (a) our resolution is well determined and reproduces the mass distributions for physical δ functions such as the η in Reaction (2), and that (b) this resolution (±4 MeV in average σ) does not seriously deteriorate the significance of a dip of ω-like width. The significance would be seriously affected for a FWHM $\approx$ 20 MeV.

[15]While this paper was in preparation, evidence for a $\rho^0, \omega \to \pi^+\pi^-$ interference effect has been reported for the $e^+e^- \to \pi^+\pi^-$ reaction and interpreted by the authors as showing $\beta + \beta' \approx \pi$ and thus $\beta \approx \pi/2$ [J. E. Augustin et al., Nuovo Cimento Letters II, 214 (1969)]. This interpretation is thus at variance with the one given in Ref. 11.

[16]See, for example, S. Coleman and S. Glashow, Phys. Rev. Letters 6, 423 (1961); Phys. Rev. 134, B671 (1964); J. Yellin, Phys. Rev. 147, 1080 (1966); and others quoted in Ref. 11.