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ELECTRODISINTEGRATION OF ^{3}H AND ^{3}He

D. R. Lehman
The George Washington University, Washington, D. C. 20006
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Two- and three-body electrodisintegration cross sections for ^{3}H and ^{3}He are calulated with nucleon-nucleon interactions represented by nonlocal, separable forms. Good agreement with experiment is obtained with Tabakin's ground state for the range of incident energies 250 to 550 MeV.

During the past several years, considerable attention has been given to the study of inelastic electron scattering from ^{3}H and ^{3}He in order to investigate their structure. Experimental effort has involved high- and low-energy electrons at various electron-scattering angles,[1] including an experiment where the fast proton was detected in coincidence with the scattered electron.[2] Theoretical effort has been devoted mainly to the coincidence cross section for two-body breakup of ^{3}He with the ground state represented by an analytic form containing one or two parameters which are determined from photodisintegration and/or form-factor data.[3] However, a pole-model calculation for comparison with the first high-energy data where only the electron is detected (hereafter called the inelastic cross section as opposed to the coincidence cross section) has been carried out.[4] The constant vertex parameters of this model were determined from the coincidence data with the result that the inelastic cross sections were 30 to 70% too high. A reanalysis of the coincidence data resulted in qualitative agreement, but is limited due to the ambiguities in interpreting the experimental data and the lack of an accurate calculation of the three-body coincidence cross section. Thus these circumstances, along with the availability of more recent inelastic cross-section data,[5] indicate the importance of an accurate three-body coincidence calculation and of a calculation of the inelastic cross sections without use of the coincidence data. The purpose of this Letter is to report the results of such work for the processes

$$e+{}^3\mathrm{He}\to e'+p+\mathrm{D} \tag{1a}$$

$$\to e'+p+n+p \tag{1b}$$

and

$$e+{}^3\mathrm{H}\to e'+n+\mathrm{D} \tag{2a}$$

$$\to e'+n+n+p, \tag{2b}$$

where the nucleon-nucleon interaction is described by a separable potential.[6]

The model we use to describe the electrodisintegration process is similar to that used previously. The incident electrons are treated in the extreme relativistic limit. We treat the electron-nucleus interaction in the impulse approximation, retaining only those terms corresponding to the electron interacting with the ejected nucleon. The electron-nucleon interaction is treated in Born approximation using the McVoy-Van Hove interaction.[7] In both the two-body and three-body final states, we neglect interactions between the ejected nucleon and the spectator pair, but include final-state interactions between the two spectator particles.

The choice of the nuclear ground-state wave function is of particular importance since we want to learn about the structure of the isotopic doublet, ^{3}He and ^{3}H. The dominant component of the trinucleon ground state is fully symmetric under exchange of the spatial coordinates, and as a first approximation we consider only this part. In this approximation, the ground state results from an average of the singlet and triplet interactions. Since we take nucleon-nucleon interactions to be represented by separable s-wave potentials of the Yamaguchi form,[8] only two parameters, the strength and range, need be specified to calculate the ground-state wave function. The two sets of parameters which we use in this work are given in Table I.[9] The first set of parameters is based on an effective-range analysis of the n-p s-wave interaction and we use them only in the coincidence calculations.[10] The second set of parameters are those of Tabakin's model 2 which are based on a fit to the s-wave phase shifts to 340 MeV.[11] We use Tabakin's parameters in both the coincidence and inelastic cross-section calculations. Thus we can compare two descriptions of the ground state in the coincidence calculations, but not in the inelastic calculations due to the inapplicability of the first set for the final-state interactions.

The final-state wave functions depend on the process in which we are interested. For the coincidence data, we are concerned with processes (1a), (1b), and (2b). Since the coincidence data are at the quasielastic peak, the laboratory energy of the detected proton is always ~100 MeV and the relative energy of the spectator pair is ~10 MeV for Johansson's experiment. These conditions are the justification for two aspects of the model, namely, the neglect of final-state interactions between the ejected proton and the spectator pair, and the assumption of only s-wave interactions between the spectator particles. Then the final state for process (1a) is simply a free proton plus deuteron, and for process (1b), a free proton plus a nucleon pair with either singlet or triplet interaction and parameters as givin in Table I. These results are useful when we calculate the inelastic cross sections above the elastic peak and below the pion threshold. However, here we note the relative energy of the spectator pair can be as large as 100 MeV, which explains why we use only Tabakin's parameters for the inelastic calculations. Yet, the relative energy of the spectator-pair center of mass and the ejected nucleon remains great enough to justify neglecting their interactions for excitation energy above ~50 MeV.

When we use the wave functions described above to compute the cross sections, it is a straightforward but lengthy task. The form of the cross sections for the coincidence processes are

$$d^3\sigma/dE_f d\Omega_e d\Omega_p = \tfrac{3}{2}\sigma_0|I_1|^2, \tag{1a'}$$

$$\frac{d^3\sigma}{dE_f d\Omega_e d\Omega_p} = \tfrac{1}{2}\int_0^{k_{\max}} d^3k\,\sigma_0(\vec{k})|I_{0,1}(\vec{k})|^2, \tag{1b'}$$

$$\frac{d^3\sigma}{dE_f d\Omega_e d\Omega_p} = \int_0^{k_{\max}} d^3k\,\sigma_0(\vec{k})|I_0(\vec{k})|^2, \tag{2b'}$$

where σ_0 is a function of nucleon form factors and kinematical expressions, and for (1a′)

$$I_1(\vec{P}_l) \equiv (2\pi)^{3/2}\int d^3p\,\varphi_B^*(\vec{p})\Psi^s(\vec{p}, \vec{P}_l). \tag{3}$$

In Eq. (3), $\Psi^s(\vec{p}, \vec{P}_l)$ is the symmetric ground-state function in the momentum representation

Table I. Parameters for separable interactions.

	Range (fm^{-1})	Strength (fm^{-3})	BE (MeV)	Range (fm^{-1})	Strength (fm^{-1})	Range (fm^{-1})	Strength (fm^{-3})
	Ground state			Final-state triplet		Final-state singlet	
Effective Range	1.450	0.353	12.55	1.450	0.415	1.304	0.211
Tabakin	1.150	0.182	9.33	1.150	0.220	1.150	0.148

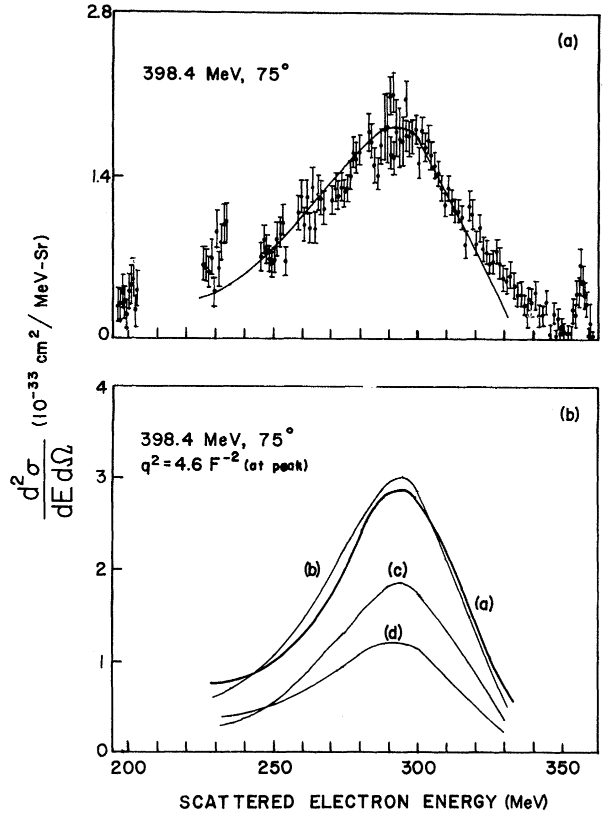

FIG. 1. (a) Inelastic cross section for electrodisintegration of ^{3}H. The solid curve is the theoretical result with the Tabakin parameters. The data points are from Hughes et al. from Ref. 1. (b) Inelastic cross section for electrodisintegration of ^{3}He. Curve *a* represents the experimental data. Curve *b* is the theoretical result with the Tabakin parameters. Curves *c* and *d* are the two-body and three-body contributions to the theoretical result.

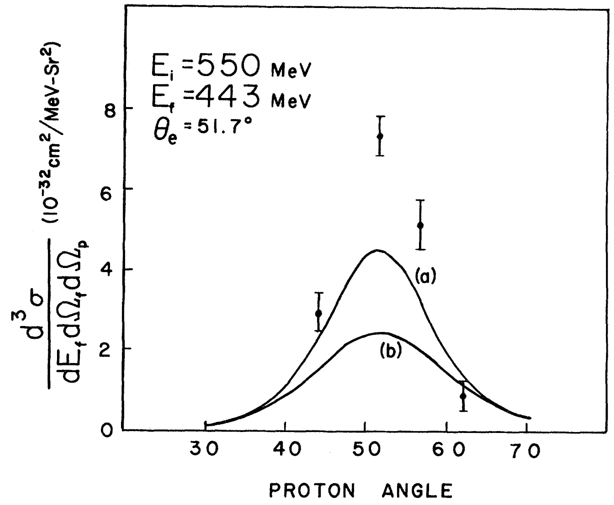

FIG. 2. Coincidence cross section for electrodisintegration of ^{3}He. Curve *a* is the theoretical result with Tabakin's parameters and curve *b* with the effective range parameters. The data points are from Ref. 2.

with $\vec{P}_i$ equal to the negative of the spectator-pair center-of-mass momentum and $\varphi_B(\vec{p})$ is the deuteron wave function. Equation (3) applies to the $I(\vec{k})$ in Eqs. (1b′) and (2b′), if $\varphi_B(\vec{p})$ is replaced by the appropriate scattering state $\varphi_k^{(-)}(\vec{p})$. The value of $k_{\max}$, the maximum relative momentum of the spectator particles, is determined kinematically from the smallest value of proton energy in the experiment. By use of these results, we obtain the inelastic cross sections by integrating over the nucleon angles. The results of interest we compute numerically, integration done by appropriate Gaussian quadrature.

The results obtained with this model are encouraging and interesting. First let us consider the inelastic cross sections. In Fig. 1(a) we give the result for electrodisintegration of tritium superimposed on the Hughes et al. data for initial electron energy equal to 398.4 MeV. The agreement with the experimental results (which have not been adjusted for radiative effects[12]) is excellent over a range of final electron energies corresponding to excitation energies from ~40 to ~130 MeV. Figure 1(b) contains similar results for ^{3}He, but we show the contributions from the two-body and three-body processes. We obtained similar results for the data at $E_i = 248.8$ MeV and $\theta = 90°$. Now we look at the coincidence results with consideration of both the Tabakin and effective-range parameters. We obtain excellent agreement with the Johansson data for process (2b) when we use the Tabakin parameters, but the effective-range set gives a result which is a factor of 2 too small. However, for processes (1a) and (1b), agreement is poor. If we plot the sum of these processes against the Johansson data as shown in Fig. 2, we see the result lies 40% below the data at the peak for the Tabakin parameters. When we consider only process (1a) and compare it with both the Johansson and Gibson-West analyses, the Tabakin parameters give a result ~30% below the Gibson-West peak which is equivalent to 40% below the Johansson peak. Again the effective-range parameters give a result roughly 50% smaller than the Tabakin case.

We therefore conclude that this fairly simple model description of the electrodisintegration of ^{3}H and ^{3}He yields good results for the inelastic cross sections and points to the need for better coincidence data. Disagreement with the inelastic data is expected at both low and high q^2 values. In the low q^2 region, three-particle aspects of the final-state interactions are expected to be important[13] and these are neglected in impulse approximation. At high q^2, the cross section should be more sensitive to the trinucleon structure and to meson-exchange effects. However within these two extremes, the approximation of a spatially symmetric ground state corresponding to Tabakin's model 2 appears to be adequate within experimental errors. Thus, unfortunately, we cannot say anything about other components in the ground state nor about other aspects of the nucleon-nucleon interaction. But the point should be made that nowhere is three-body data used to determine parameters. All parameters are determined from two-nucleon data. The disagreement of the model with the ^{3}He coincidence data can be attributed to two possibilities. Firstly, it is possible that the model is not adequate for explaining the coincidence data and that when we compute the inelastic cross section the sensitivity to the model is decreased and agreement with experiment is accidental. Secondly, it is possible that the model is adequate, but the experimental results of Johansson could be incorrect by as much as 40% rather than his estimate of 20% in addition to statistics. Resolution of these alternatives could be achieved by new coincidence measurements with higher reliability and more points. Finally, the apparent inadequacy of the pole model could be traced to neglect of final-state interactions between the spectator pair as well as the ambiguity in determining the vertex parameter for process (1a).

A detailed account of this work including results not shown here will be submitted for publication elsewhere.

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