TEST OF (ρ, A_2) EXCHANGE DEGENERACY, DUALITY, AND EVIDENCE FOR SECONDARY TRAJECTORIES OBTAINED FROM $(KN, \bar{K}N)$ CHARGE-EXCHANGE REACTIONS*

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An analysis of the difference between the charge-exchange reactions $K^+n \to K^0p$ and $K^-p \to \bar{K}^0n$ over the momentum range 1-5.5 GeV/c provides strong evidence for exchange degeneracy of the (ρ, A_2) trajectories and shows the importance of the exchange of secondary trajectories. The energy dependence of this difference is shown to be inconsistent with simple cut models. A new test of the duality principle is proposed and appears to be in good agreement with the present data.

In this note we show that the difference between the angular distributions of the charge-exchange reactions

$$K^+n \to K^0p, \tag{1}$$

$$K^-p \to \bar{K}^0n \tag{2}$$

can be utilized to provide a sensitive test of several currently discussed theoretical conjectures, namely, (ρ, A_2) exchange degeneracy, the existence of secondary trajectories, and the Dolen-Horn-Schmid duality principle.

We begin by discussing the Regge-pole–model t-channel description of reactions (1) and (2).[1] Denoting the amplitudes for the exchange of the ρ and A_2 quantum numbers by A_ρ and A_R, respectively, the spin-flip or the spin-nonflip amplitudes are given by

$$A(K^+n \to K^0p) = (\tfrac{1}{2})^{1/2}A_\rho + (\tfrac{3}{2})^{1/2}A_R, \tag{3}$$

$$A(K^-p \to \bar{K}^0n) = -(\tfrac{1}{2})^{1/2}A_\rho + (\tfrac{3}{2})^{1/2}A_R. \tag{4}$$

Suppressing the spin dependence of the amplitudes, the resulting differential cross sections are given by

$$\frac{d\sigma(K^+n \to K^0p)}{dt} = \tfrac{1}{2}|A_\rho|^2 + \tfrac{3}{2}|A_R|^2 + \sqrt{3}\,\mathrm{Re}(A_\rho{}^*A_R), \tag{5}$$

$$\frac{d\sigma(K^-p \to \bar{K}^0n)}{dt} = \tfrac{1}{2}|A_\rho|^2 + \tfrac{3}{2}|A_R|^2 - \sqrt{3}\,\mathrm{Re}(A_\rho{}^*A_R). \tag{6}$$

In addition, assuming SU(3) invariance, these reactions are related to the reactions

$$\pi^-p \to \pi^0n, \tag{7}$$

$$\pi^-p \to \eta n. \tag{8}$$

The differential cross sections for (7) and (8) can be written as[1]

$$\frac{d\sigma(\pi^-p \to \pi^0n)}{dt} = |A_\rho|^2, \tag{9}$$

$$\frac{d\sigma(\pi^-p \to \eta n)}{dt} = |A_R|^2. \tag{10}$$

Reactions (7) and (8) both show evidence for large spin-flip amplitudes and, therefore, it is expected that both the ρ[2] and A_2[3] contribute appreciably in the spin-flip amplitudes for reactions (1) and (2). Thus if both ρ and A_2 trajectory exchanges are important, a difference between the cross sections for reactions (1) and (2) is expected.

In the Regge-pole model the A_ρ and A_R amplitudes are given by[4]

$$A_\rho = \gamma_\rho(t)\frac{[e^{-i\pi\alpha_\rho(t)} - 1]}{\sin\pi\alpha\rho(t)}s^{\alpha_\rho(t)-1}, \tag{11}$$

$$A_R = \gamma_R(t)\frac{[e^{-i\pi\alpha_R(t)} + 1]}{\sin\pi\alpha_R(t)}s^{\alpha_R(t)-1}, \tag{12}$$

where the γ's denote the residue functions and the α's denote the Regge trajectories.

If the trajectories are now assumed to be exchange degenerate,

$$\alpha_\rho(t) = \alpha_R(t), \tag{13}$$

the interference term $\mathrm{Re}(A_\rho{}^*A_R)$ in (5) and (6) vanishes and we are left with the equality

$$\frac{d\sigma(K^+n \to K^0p)}{dt} = \frac{d\sigma(K^-p \to \bar{K}^0n)}{dt}. \tag{14}$$

Although a complicated conspiracy between residues and trajectory parameters without (ρ, A_2) exchange degeneracy could also lead to the realization of (14) it seems unlikely that such a conspiracy would be maintained over a wide range of s and t values. Therefore, a measurement of the difference

$$\Delta(s,t)=\frac{d\sigma(K^+n\to K^0p)}{dt}-\frac{d\sigma(K^-p\to\bar{K}^0n)}{dt}, \tag{15}$$

which equals $2\sqrt{3}\,\mathrm{Re}(A_\rho{}^*A_R)$ for (ρ, A_2) exchange dominance, is expected to be very sensitive to departures from relation (13). In addition, if the interference term in (5) and (6) is zero, the SU(3) sum rule[6]

$$\frac{d\sigma(K^-p\to\bar{K}^0n)}{dt}=\frac{1}{2}\frac{d\sigma(\pi^-p\to\pi^0n)}{dt}+\frac{3}{2}\frac{d\sigma(\pi^-p\to\eta n)}{dt} \tag{16}$$

is obtained. This can be tested by measurement of only one of the charge-exchange reactions (1) and (2).

We now turn to the available experimental data on reactions (1) and (2). Figures 1(a)-1(d) show the differential cross sections for $(K^\pm)$ mean momenta of 1.22, 2.3, 3.0, and 5.5 GeV/c.[7,8] In order to extract the most reliable values of $\Delta(s,t)$ from the present data an empirical formula with six parameters simultaneously fitting both reactions was used.[9] The fitted curves are shown in Fig. 1. The trend of the data shows that $\Delta(s,t)$ is large at small s and decreases rapidly, becoming consistent with zero for all $|t|$ values out to ~1 (GeV/c)^2 at 5.5 GeV/c. Furthermore Δ appears to be consistent with zero at small $|t|$ values for all s. Although no data on reaction (1) presently exist for beam momenta above 5.5 GeV/c, the success of the (ρ, A_2) exchange-degenerate SU(3) sum rule at momenta of $\simeq 7$ and 9.9 GeV/c indicates that Δ is likely to be very small above 5.5 GeV/c.[1,6] This small value of Δ for momenta $\gtrsim 5.5$ GeV/c is strong evidence for the exchange degeneracy given by (13). This conclusion is at odds with other direct Regge-pole analyses of reactions (7) and (8).[10] However, the interference technique suggested here is very likely more sensitive to a direct

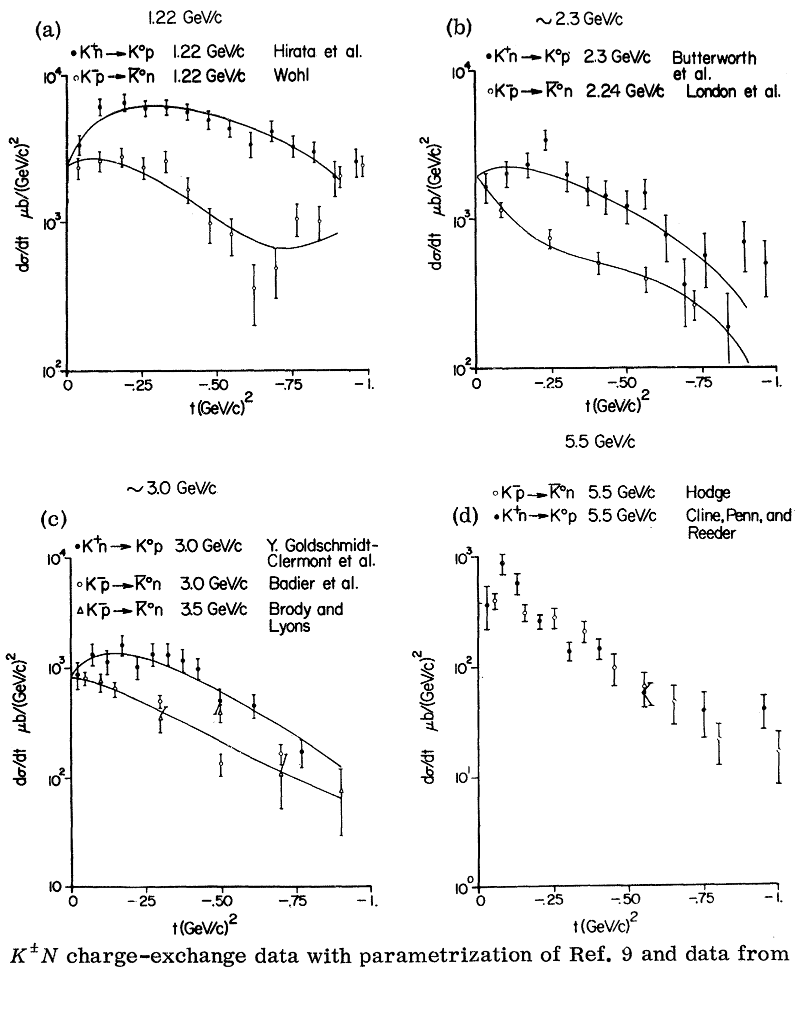

FIG. 1. Fits to $K^\pm N$ charge-exchange data with parametrization of Ref. 9 and data from Refs. 7 and 8.

comparison of the (ρ, A_2) trajectories.[11] The large value of Δ for low beam momenta is also interesting and may arise from interference between the (ρ, A_2) amplitude and the amplitude for secondary trajectory exchange or from interference with the amplitude arising from cuts.[1]

In view of the above discussion we take the ρ and A_2 trajectories to be exchange degenerate and attempt to account for the observed s dependence of the $\Delta(s,t)$ at intermediate energies. The existence of secondary trajectories like the ρ' and the A_2' has been suggested by numerous recent analyses of high-energy data.[12] Making the amplitude replacements $A_\rho \to A_\rho + A_{\rho'}$ and $A_R \to A_R + A_{R'}$ in (3) and (4) leads to the expression for $\Delta(s,t)$:

$$\Delta(s,t) = 2\sqrt{3}\,\mathrm{Re}(A_\rho^* A_R + A_\rho^* A_{R'} + A_{\rho'}^* A_R + A_{\rho'}^* A_{R'}). \qquad (17)$$

Assuming further that the ρ' and A_2' trajectories are exchange degenerate, Eq. (17) becomes

$$\Delta(s,t) = 2\sqrt{3}\,\mathrm{Re}(A_\rho^* A_{R'} + A_{\rho'}^* A_R). \qquad (18)$$

For the case that $\Delta(s,t)$ may be due to simple cuts we use the Mandelstam formula[13] for the effective trajectory of the leading edge of the cut and assume the amplitude replacements $A_\rho \to A_\rho + A_{\text{cut}}$ and $A_R \to A_R + A_{\text{cut}}$. In either case, since we are interested in the s dependence of $\Delta(s,t)$, we can write[14]

$$\Delta(s,t) = a(t) s^{\alpha_\rho(t) + \alpha_{\text{eff}}(t) - 2}, \qquad (19)$$

where α_{eff} is either the exchange-degenerate (ρ', A_2') trajectory or the effective-cut trajectory or a combination of these.

Figure 2 shows the values of $\alpha_{\text{eff}}(t)$ determined by fitting to Eq. (19) the values of $\Delta(s,t)$ at fixed t obtained from Fig. 1 and assuming a (ρ, A_2) trajectory of the form $0.5+t$.[15] It is seen that these data can be interpreted as arising from the exchange of a low-lying trajectory which has an intercept consistent with an intercept one unit down from the (ρ, A_2) trajectory intercept. However, given the present data, this interpretation is not unique. The simple cut models seem to be ruled out (see Fig. 2). With the accumulation of better data on these reactions it will be possible to obtain more explicit information on the existence of the ρ' and A_2' trajectories and their exchange degeneracy.

An important characteristic of the angular distributions shown in Fig. 1 is the tendency for the equality (14) to be realized for small $|t|$ at intermediate momenta. This suggests that the secondary trajectories or cuts do not contribute appreciably at $|t|=0$. Furthermore, the strong exchange-degeneracy relation,

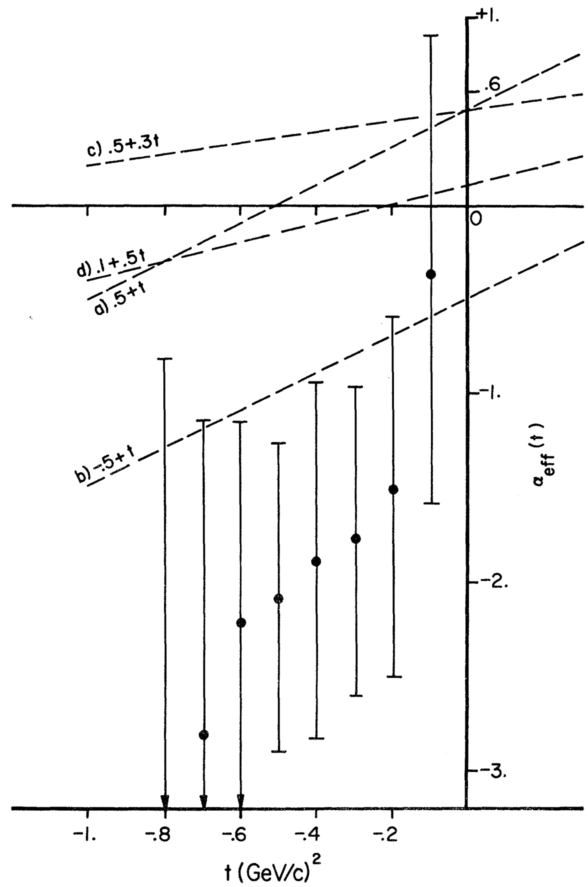

FIG. 2. Effective trajectory from fits to $\Delta(s,t)$ [cf. Eq. (20) and Ref. 9]. The dashed curves are (a) a ρ trajectory of $0.5+t$; (b) a ρ' trajectory of $-0.5+t$; (c) the effective trajectory for a P-ρ cut obtained using the Mandelstam formula with Pomeranchuck and ρ trajectories of $1+0.4t$ and $0.5+t$, respectively; and (d) the effective trajectory for a P'-ρ cut using P' and ρ trajectories of $0.6+0.9t$ and $0.5+t$, respectively.

$$\gamma_\rho(t) = \gamma_R(t), \qquad (20)$$

is known to hold at $t=0$.[4] It is of interest to see if relation (14) is true at $t=0$ even at low energies. Figure 3 shows an experimental test of relation (14) for $|t|\sim 0$ using the presently available data.[16] Although the data are not adequate for a detailed test it appears that the cross section for reaction (1) oscillates around that of reaction (2) over the momentum range 1-5.5 GeV/c. It should be noted that over this same momentum range the total charge-exchange cross sections have changed from ~5 mb to ~150μb – a factor of

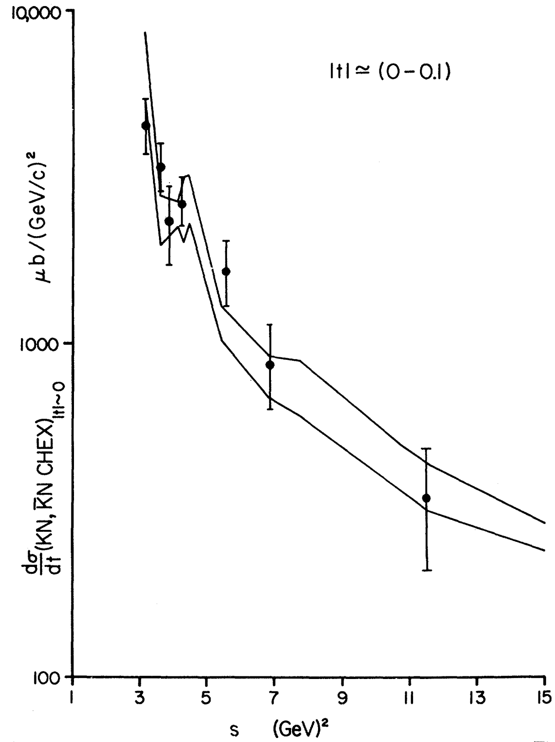

FIG. 3. Comparison of the near-forward KN and $\bar{K}N$ charge-exchange data of Refs. 7, 8, and 15. The data points shown are the smallest measured $|t|$ value at each energy for the reaction $K^+n \to K^0p$ and the curves represent the limits of the smallest measured $|t|$ values for the $K^-p \to \bar{K}^0n$ data.

30 ! At low energies it is known that reaction (2) is dominated by direct-channel resonances whereas reaction (1) is not. Therefore, the approximate validity of (14) can probably be understood within the context of the Dolen-Horn-Schmid duality principle which relates the average of direct-channel resonance amplitudes to the Regge-pole exchange amplitudes.[17] It should be noted that this test does not rely on the extrapolation of Regge amplitudes into the low-energy region, and the uncertainties due to the effects of secondary trajectories are most likely not important.

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