[15]See for example, G. J. Wagner, "Proton Removal from the 1$p$ and 2$s$-1$d$ Shells," Princeton University, Report No. PUC-937-354, 1969 (to be published).

[16]A. B. Migdal, Theory of Finite Fermi Systems and Applications to Atomic Nuclei (John Wiley & Sons, Ltd., London, England, 1967).

# LARGE-ANGLE ELASTIC PROTON-PROTON SCATTERING FROM 1.5 TO 3.5 GeV/$c$ †

B. B. Brabson, R. R. Crittenden, R. M. Heinz, R. C. Kammerud, H. A. Neal,* H. W. Paik, R. A. Sidwell, and K. F. Suen‡
Department of Physics, Indiana University, Bloomington, Indiana 47401
(Received 15 November 1969)

We present $p$-$p$ elastic differential cross-section results at nine momenta in the range 1.5 to 3.5 GeV/$c$ for $40° \lesssim \theta_{c.m.} \leq 90°$. No strong evidence of secondary diffraction-like behavior in this momentum region is observed. Rapid changes in the slope of the 90° cross section are seen near $-t = 0.7$ and 3 (GeV/$c$)$^2$.

An experiment has been conducted to study, with high statistical accuracy, the angular distribution in elastic proton-proton scattering at 13 momenta in the range 1.5 to 5.5 GeV/$c$, for the center-of-mass angular region $40° \lesssim \theta_{c.m.} \leq 90°$. We report in this Letter the results at momenta up to and including 3.5 GeV/$c$.

In the past few years several developments have created considerable interest in the behavior of the $p$-$p$ angular distribution at large angles and intermediate energies:

(1) Measurements made by Longo and Neal have indicated an anomalous behavior of the polarization in $p$-$p$ elastic scattering near $|t| = 0.7$ (GeV/$c$)$^2$ for incident proton momenta above ~2.0 GeV/$c$.[1] This behavior has been substantiated in subsequent experiments by other groups.[2] A model has been proposed to explain the observed effects,[3] but the angular-distribution data have been too sparse to make a detailed analysis feasible.

(2) An experiment by Akerlof et al.[4] resulted in the discovery of a break in the 90° c.m. $p$-$p$ elastic cross section at ~8-GeV/$c$ incident proton momentum. This finding enhanced speculation concerning the exact nature of the rapid slope change that had been assumed to occur near 2-GeV/$c$ incident momentum. Evidence for the latter has been based principally on the results obtained by extrapolating the 90° cross-section data into the 1.0- to 2.5-GeV/$c$ region from lower and higher momenta.

(3) Precision measurements of the large-angle $p$-$p$ elastic-scattering distribution at momenta above 8 GeV/$c$ by Allaby et al.[5] have revealed diffractionlike structure. It is of interest to know to what minimum momentum this behavior persists.

The measurements reported here were carried out in the external proton septum beam at the Argonne National Laboratory zero-gradient synchrotron. The momentum resolution of the incoming beam was approximately 1%, with an equal uncertainty in the absolute value of the central momentum. Beam fluxes varied from 15 000 to 100 000 protons per 600-msec pulse. A gas threshold Cherenkov counter ($C_G$) was used to separate the protons from the pions and kaons in the beam. The protons constituted more than 30% of the beam at 1.5 GeV/$c$ and more than 50% of the beam at 3.5 GeV/$c$.

The experimental layout is shown in Fig. 1. The proton beam was incident on a 12-in.-long liquid-hydrogen target. Scintillation counters $B_1$ and $B_2$ detected the incoming protons, and the outgoing protons were detected by one of the fast-arm counters ($F_1$, $F_2$, or $F_3$) and one of the slow-arm counters ($S_1$, $S_2$, or $S_3$). Veto counters $A_1$, $A_2$, $A_3$, and $A_4$ (the last two are not shown in Fig.

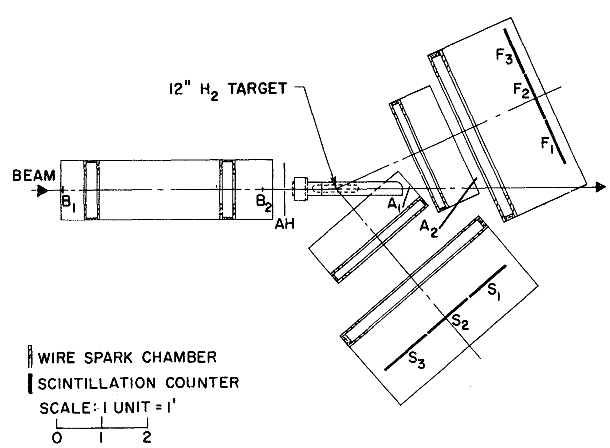


FIG. 1. The experimental layout.

1) covered much of the solid angle that was not subtended by the detectors. Counter $AH$ was a veto counter which contained a 1-in.-diam hole through which the beam passed. A trigger of the system occurred when the following logic signature was satisfied: $B_1B_2\overline{AH}\,\overline{C}_G F_i S_j \overline{A}_k$, where, for example, $F_i$ means any of $F_1$ or $F_2$ or $F_3$, etc.

The trajectories of the incoming proton and the two scattered protons were determined by 12 wire spark chambers with magnetostrictive readout. The digitized spark-location information was transferred directly from storage scalers to a continuous-write magnetic-tape transport. Trigger rates varied from four to ten events per pulse. The experiment was monitored off-line using an Advanced Scientific Instruments Model No. 6020 computer.

Analysis of the data was carried out on the Indiana University Pattern Recognition Project's Sigma 5 computer. In the master analysis program each event, after being filtered to remove any spurious sparks, was reconstructed in three dimensions. The standard deviation of sparks from the reconstructed trajectories was approximately 1.0 mm. Elastically scattered events were separated from a small background of other processes by imposing kinematic restrictions on the coplanarity and opening angle.

A typical coplanarity distribution is shown in Fig. 2(a). The distribution is composed of all events that passed the filter program and satisfied the fiducial volume requirements. A three-standard-deviation coplanarity cut was imposed and the accepted events were sorted into the appropriate 2° bin in $\theta_{c.m.}$. Then, for each bin, a histogram was made of the difference between the measured slow-proton laboratory angle and the angle predicted from the measured fast-proton angle, using elastic-scattering kinematics. Such a distribution at 3.0 GeV/$c$ for $50° \leq \theta_{c.m.} < 52°$ is shown in Fig. 2(b). These distributions were used to determine the background beneath the elastic peak. The subtraction varied from ~1% at 1.5 GeV/$c$ to a maximum of ~9% at 3.5 GeV/$c$.

Corrections were applied to the data to account for the following effects: beam attenuation in the hydrogen target and counter $B_1$ [$(5\pm3)$%], counter inefficiency [$(1\pm1)$%], wire-chamber inefficiency [$(2\pm2)$%], nuclear interactions of the final-state protons [$(\sim6\pm2)$%], and kaon and pion contamination in the beam [$(3\pm2)$%]. The angular dependence of the correction was taken into account where appropriate. The normalization uncertainty is ±7%.

Our results are presented in Fig. 3. The error bars shown represent only the statistical error, and are typically ±4%. Results from other experiments[6-9] in this momentum region agree with our data, within the combined statistical and

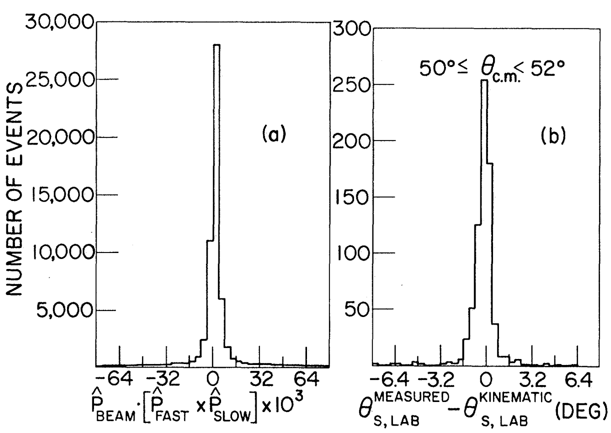


FIG. 2. (a) Coplanarity distribution for all reconstructable events at 3.00 GeV/$c$. $\hat{P}_{\text{beam}}$, $\hat{P}_{\text{fast}}$, and $\hat{P}_{\text{slow}}$ are unit vectors in the direction of the beam, fast, and slow protons, respectively. (b) The distribution of $\theta_{s,\text{lab}}^{\text{meas}}-\theta_{s,\text{lab}}^{\text{kin}}$ for one $\theta_{c.m.}$ bin at 3.00 GeV/$c$, illustrating the resolution of the system and relative magnitudes of the background and elastic signal. $\theta_{s,\text{lab}}^{\text{meas}}-\theta_{s,\text{lab}}^{\text{kin}}$ is the difference in the measured slow proton laboratory angle and the angle predicted from the measured fast proton angle using elastic scattering kinematics.

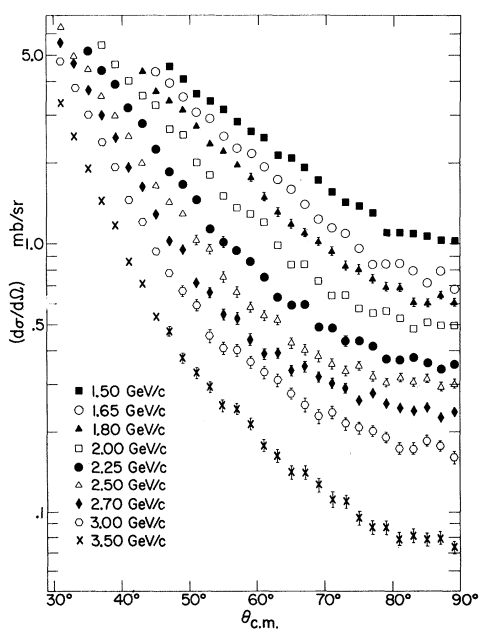


FIG. 3. Angular distribution for $p$-$p$ elastic scattering from 1.50 to 3.50 GeV/$c$. The errors shown are the statistical errors only.

normalization uncertainties.

At the angles and momenta explored in this experiment we find no significant evidence for secondary diffractionlike behavior. This is in contrast to the pronounced structure observed by Allaby et al.[5] in measurements above 8 GeV/$c$. Our distributions are in general quite smooth and exhibit the characteristic flattening at large angles attributed, at least in part, to the symmetry of the two-proton system.

Attempts have been made in recent years to obtain a simple parametrization applicable to all high-energy elastic proton-proton scattering data. In one analysis,[4] for example, it has been suggested that the cross section can be represented as a sum of Gaussian terms in $\beta P_\perp$, where $\beta$ is the velocity of the protons in the c.m. system and $P_\perp$ is the transverse momentum:

$$\frac{d\sigma}{dt}=\sum_{i=1}^{3} c_i \exp(-\beta^2 P_\perp^2/a_i). \qquad (1)$$

Each of the three terms refers to a corresponding interaction region of the proton. It has been shown that $(\beta P_\perp)^2$ is the proper variable for a diffraction model in which the Lorentz transformation contracts by a factor $\gamma$ a spherically symmetric interaction probability density.[4] Good fits to a large body of data at momenta above 5 GeV/$c$ were obtained with this parametrization. We have investigated the applicability of this predicted angular dependence in the momentum region 2-3.5 GeV/$c$. As an example of the excellent agreement obtained, a fit to our large-angle data at 3.5 GeV/$c$ is shown in Fig. 4, where the form

$$d\sigma/d\Omega = A \exp(-CP_\perp^2) \qquad (2)$$

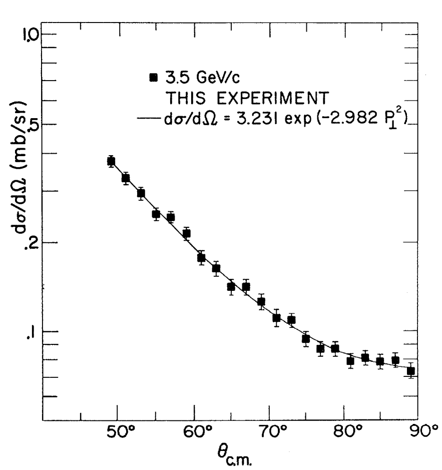


FIG. 4. A fit to our large-angle 3.50-GeV/$c$ data using the form $A\exp(-CP_\perp^2)$ proposed in Ref. 4.

was assumed.

Other models, in agreement with an earlier empirical fit by Orear, predict an angular dependence in large-angle $p$-$p$ elastic scattering which for fixed $P_{\text{c.m.}}$ goes as $\exp(\text{const}\sin\theta_{\text{c.m.}})$ rather than $\exp(\text{const}\sin^2\theta_{\text{c.m.}})$ as above. The thermodynamical model for compound elastic scattering,[10] for example, leads to the following transverse momentum distribution:

$$W(P_\perp) \sim P^{3/2}\exp(-P_\perp/T_0) \quad \text{for } P_\perp \gg T_0 \approx 0.16 \text{ GeV}/c. \qquad (3)$$

This type of behavior is also predicted by the incoherent-droplet model of Huang.[11] Fits have been made to our large-angle data in the 2- to 3.5-GeV/$c$ range with the hypothesis that

$$d\sigma/d\Omega = A\exp(-CP_\perp) \qquad (4)$$

with $A$ and $C$ being the free parameters. The fits thus obtained are as acceptable as those in Eq. (2). This same ambiguity persists in the analysis of high-energy data.

As pointed out earlier, there has been reason to believe that a break in the $(90°)_{\text{c.m.}}$ $p$-$p$ cross section occurs in the region of 1.0-2.5 GeV/$c$ $[0.4 \le |t| \le 1.6\ (\text{GeV}/c)^2]$. We present in Fig. 5 our

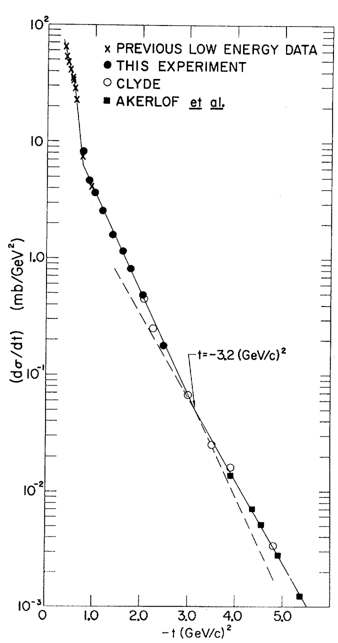


FIG. 5. The $p$-$p$ elastic differential cross section at 90°. The low-energy data shown above are taken from Ref. 6. The open circles are from Ref. 7 and the solid squares from Ref. 4. The break at $|t| \approx 0.7\ (\text{GeV}/c)^2$ is clearly visible. Another change in slope is noted to occur at $|t| \approx 3\ (\text{GeV}/c)^2$ (see Ref. 14).

data at 90° along with those from other experiments.[4,6,7] We observe evidence for a rapid change of slope near $t = -0.75$ (GeV/$c$)$^2$ ($P_L \approx 1.5$ GeV/$c$).[12] This occurs roughly at the $t$ value at which an anomalous behavior of the $p$-$p$ polarization is observed. A study of possible correlations between breaks in the differential cross section and minima (or maxima) in the polarization has been recently reported by Neal and Predazzi.[13]

In Fig. 5 we also present evidence for the existence of a change in the slope of the 90° cross section near $t = -3$ (GeV/$c$)$^2$.[14] This evidence is based on a straight-line extrapolation of our data and those of Akerlof et al.[4] The absence of accurate, high-density, low-energy data precluded an earlier observation of this feature. It is of interest that at least three models[15-17] predict the existence of a break between the one discovered by Akerlof et al. at $-t \approx 6.5$ (GeV/$c$)$^2$ and the one suspected around $-t = 0.7$ (GeV/$c$)$^2$ and discussed above. In the model proposed by Fleming, Giovannini, and Predazzi[15] the cross section is represented as an infinite sum of Gaussian terms in $\beta P_\perp$. This expansion has the property that the Cerulus-Martin bound is not violated, as it is in the fit of Ref. 4, and that the exponential behavior proposed by Orear[18] follows automatically at large angles. Shoulders are predicted at the transition points where one goes from the region in which a particular Gaussian dominates to the region where the successive Gaussian dominates. Such a transition point is predicted near $|t| = 3.0$ (GeV/$c$)$^2$.

Kokkedee and VanHove[16] assume that the known breaks are associated with the opening of various inelastic channels. The break discovered by Akerlof et al.[4] would correspond to the baryon-antibaryon threshold, while the first slope change, near $|t| = 0.1$ (GeV/$c$)$^2$, would correspond to the single-pion production threshold. Another break would then be expected near $|t| = 1.6$ (GeV/$c$)$^2$, corresponding to the $K\Lambda$ threshold.

Marshall Libby[17] has predicted the values of $|t|$ at which the previously known slope changes occur by relating the differential elastic-scattering cross section to the interaction energy of two protons using Fermi's "Golden Rule No. 2," and solving for the values of $t$ which minimize the interaction energy. One heretofore unobserved slope change is predicted to occur near $|t| = 4$ (GeV/$c$)$^2$. A more thorough analysis of our data in light of the above predictions will be presented in a subsequent paper.

We are very grateful to Professor Kenneth Galloway, Professor Jack Mott, Mr. Alan Barnes, and Mr. Harry Farmer, and to the zero gradient synchrotron staff for their very valuable assistance during the setup and execution of the experiment. We wish to thank Dr. Richard Lundy and Dr. Donald Meyer and members of their experimental group for the use of their trailer and several pieces of electronics instrumentation. We thank Professor Hugh Martin for use of the Pattern Recognition Group's computer for the analysis of our data. In addition we wish to acknowledge several useful discussions with Professor Donald Lichtenberg, Professor Hugh Martin, and Professor Enrico Predazzi. Finally, we wish also to express our gratitude to the Indiana University Foundation and the Alfred P. Sloan Foundation for financial support during the initial phases of the experiment.

†Work supported in part by the Atomic Energy Commission, Contract No. AT(11-1)-2009.

*Alfred P. Sloan Foundation Fellow.

‡Present address: Department of Physics, University of Manitoba, Winnipeg, Manitoba, Canada.

[1]H. A. Neal and M. J. Longo, Phys. Rev. 161, 1374 (1967).

[2]For example, see N. E. Booth, G. Conforto, R. J. Esterling, J. Parry, J. Scheid, D. Sherden, and A. Yokosawa, Phys. Rev. Letters 21, 651 (1968).

[3]K. Huang and S. Pinsky, Phys. Rev. 174, 1915 (1968), and private communications.

[4]C. W. Akerlof, R. H. Hieber, A. D. Krisch, K. W. Edwards, L. G. Ratner, and K. Ruddick, Phys. Rev. 159, 1138 (1967).

[5]J. V. Allaby, A. N. Diddens, A. Klovning, E. Lillethun, E. J. Sacharidis, K. Schlüpmann, and A. M. Wetherell, Phys. Letters 27B, 49 (1968); J. V. Allaby, F. Binon, A. N. Diddens, P. Duteil, A. Klovning, R. Meunier, J. P. Peigneux, E. J. Sacharidis, K. Schlüpmann, M. Spighel, J. P. Stroot, A. M. Thorndike, and A. M. Wetherell, Phys. Letters 28B, 67 (1968).

[6]R. Wilson, The Nucleon-Nucleon Interaction, Experimental and Phenomenological Aspects (Interscience Publishers, Inc., New York, 1963), and references cited therein.

[7]A. R. Clyde, University of California Radiation Laboratory Report No. 16275, 1966 (unpublished).

[8]C. M. Ankenbrandt, A. R. Clark, B. Cork, T. Elioff, L. T. Kerth, and W. A. Wenzel, Phys. Rev. 170, 1223 (1968).

[9]J. L. Friedes, R. Sutter, H. Palevsky, G. Bennett, G. Igo, W. D. Simpson, R. L. Stearns, and D. M. Corley, Nucl. Instr. Methods 54, 1 (1967).

[10]R. Hagedorn, Nuovo Cimento Suppl. 3, 147 (1965).

[11]K. Huang, Phys. Rev. 146, 1075 (1966).

[12]Evidence of structure near $|t| = 1$ (GeV/$c$)$^2$ at 19.2 and 21.1 GeV/$c$ has been reported in Ref. 5.

[13]H. A. Neal and E. Predazzi, Nuovo Cimento 62A, 275 (1969).

[14]The location of the conjectured change in slope is presently questionable. The nominal value we report is based upon straight-line extrapolation of the data and is quite sensitive to the slope of these lines.

[15]H. Fleming, A. Giovannini, and E. Predazzi, Ann. Phys. (N.Y.) 54, 62 (1969).

[16]J. J. J. Kokkedee and L. VanHove, Phys. Letters 25B, 228 (1967), and CERN Report No. TH-801, 1967 (unpublished).

[17]L. Marshall Libby, Phys. Letters 29B, 348 (1969); L. Marshall Libby and E. Predazzi, to be published.

[18]J. Orear, Phys. Letters 13, 190 (1964).

# PHYSICAL REALIZATION OF NONCOMPACT DYNAMICAL SYMMETRIES

J. Katriel and G. Adam
Nuclear Research Centre-Negev, P.O.B. 9001, Beer-Sheva, Israel
(Received 8 October 1969)

The dynamical symmetry of a charged spinless harmonic oscillator in a constant magnetic field is identified. For low fields it is SU(3), for high fields SU(2,1), and for a certain definite intermediate field SU(2) ⊗ H.

The problem of a charged particle in a magnetic field was recently discussed, with new interest in the infinite degeneracy of the continuous eigenenergies.[1] It was also noticed that when a harmonic-oscillator potential is added to the Hamiltonian, one can, with a certain choice of parameters, get a system with an infinitely degenerate discrete spectrum.[2] The extent to which the "accidental" degeneracy of a quantum system is understandable in terms of irreducible representations of the dynamical symmetry group of the corresponding Hamiltonian is of current active interest.[3] Dynamical symmetry groups hitherto considered—e.g., $O(n+1)$ for the $n$-dimensional Kepler problem, $SU(n)$ for the $n$-dimensional isotropic harmonic oscillator—are all compact. Noncompact groups which entered the field of dynamical symmetries are the noninvariance groups of these systems. An attempt is made to relate the above-mentioned infinite degeneracy to a noncompact invariance group.

The Hamiltonian for a charged spinless particle in a constant magnetic field $F=2Mc\omega/e$ directed towards the $z$ axis and a potential $\frac{1}{2}M\omega^2z^2$ can be shown to be[2]

$$\mathcal{H}=p^2/2M+M\omega^2r^2/2+\omega L_z. \tag{1}$$

Defining $H=\mathcal{H}/\hbar\omega$ and introducing the notation of second quantization we get

$$H=a_x^\dagger a_x+a_y^\dagger a_y+a_z^\dagger a_z+\tfrac{3}{2}+i(a_y^\dagger a_x-a_x^\dagger a_y). \tag{2}$$

Let

$$A=(a_x+ia_y)/\sqrt{2},\quad A^\dagger=(a_x^\dagger-ia_y^\dagger)/\sqrt{2};$$
$$B=(a_x-ia_y)/\sqrt{2},\quad B^\dagger=(a_x^\dagger+ia_y^\dagger)/\sqrt{2};$$
$$C=a_z,\quad C^\dagger=a_z^\dagger. \tag{3}$$

These operators satisfy boson commutation relations analogous to the Cartesian creation and annihilation operators introduced in Eq. (2). With this notation it follows straightforwardly that

$$H=2B^\dagger B+C^\dagger C+\tfrac{3}{2}. \tag{4}$$

This is seen to be equivalent to a two-dimensional anisotropic harmonic oscillator with $\omega_B=2$, $\omega_C=1$. The dynamical symmetry group of this Hamiltonian was shown to be SU(2).[4] With this observation, any degeneracy associated with the four operators $B$, $B^\dagger$, $C$, $C^\dagger$, has been taken into account. It is clear, however, SU(2) being compact and thus having finite-dimensional representations, that no infinite degeneracy has been introduced. The additional degeneracy present is associated with the third coordinate, represented by $A$ and $A^\dagger$. These two operators generate the noncompact Heisenberg group H.[5] As they commute with the four operators generating SU(2) and with the Hamiltonian it follows that the dynamical symmetry group of the Hamiltonian is SU(2) ⊗ H.

For the more general system having a potential $k'(x^2+y^2)+kz^2$ and in a magnetic field directed towards the $z$ axis but of strength $F=2c[(k-k')M]^{1/2}/e$, we get[2]

$$\mathcal{H}=p^2/2M+\tfrac{1}{2}kr^2+eFL_z/2Mc$$

or, using the notation of Eq. (3),

$$H=\alpha A^\dagger A+(2-\alpha)B^\dagger B+C^\dagger C+\tfrac{3}{2}, \tag{5}$$

where $\alpha=1-eF/2Mc\omega$. For the case $F=2Mc\omega/e$ we get $\alpha=0$ which brings us back to Eq. (4). Reversing the magnetic field we get a Hamiltonian $H=2A^\dagger A+C^\dagger C+\frac{3}{2}$ with similar consequences. For $0<\alpha<2$, i.e., $|F|<2Mc\omega/e$, the Hamiltonian, Eq. (5), is that of a three-dimensional an-