ISOSPIN STRUCTURE OF PAIRING COLLECTIVE MOTION

B. F. Bayman, D. R. Bes, and R. A. Broglia
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

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The isospin degree of freedom of the pairing force is studied in the framework of the rotational ($\Delta \neq 0$) and vibrational ($\Delta = 0$) models. The magnitude Δ is the BCS gap parameter. The predictions of the two models concerning two-nucleon transfer reactions and energy levels are compared and contrasted. Some of the available experimental material around Ni^{56} is discussed in this framework. Further experiments are suggested which would help decide between the coupling schemes.

In medium-weight nuclei, where neutrons and protons fill the same shells, it is important to take account of the isospin properties of the $T=1$ pairing force. The character of the spectrum that results depends upon the strength G of the pairing force. Let G_c be the smallest value of G for which there exists a solution of the BCS equations. Then if $G<G_c$, the system can be described in terms of pairing vibrations; if $G>G_c$ the system can be described in terms of pairing rotations.

(1) Pairing vibrations (nonsuperconducting systems).[1,2]–First a choice must be made of the reference state $|0\rangle$, which will serve as the "vacuum" of the vibrations. In the following, we choose this to be the Ni^{56} ground state. We then define boson creation operators $a_1{}^+$, $a_0{}^+$, $a_{-1}{}^+$ for addition quanta. When $a_\tau{}^+$ acts it adds two nucleons in a highly correlated $T=1$, $T_z=\tau$, $I^\pi=0^+$ paired state, predominantly composed of orbits outside $|0\rangle$. Similarly, the boson creation operators $r_1{}^+$, $r_0{}^+$, $r_{-1}{}^+$ remove two nucleons in a highly correlated paired state, predominantly composed of orbits within $|0\rangle$. It is sometimes useful to think of the nucleons added by $a_\tau{}^+$ as being in the $2p_{3/2}$ shell, and the nucleons removed by $r_\tau{}^+$ as being in the $1f_{7/2}$ shell.

If N_a operators $a_\tau{}^+$ and N_r operators $r_\tau{}^+$ are applied to $|0\rangle$, there results a state with $56+2(N_a-N_r)$ nucleons. It is convenient to characterize this state by the set of quantum numbers $(N_rT_r;N_aT_a;TT_z)$. Here T_r is the total isospin of the N_r removal quanta. It can take on the values $T_r=N_r$, N_r-2, $\cdots$, 0, or 1. (The situation is fully analogous to the angular momenta possible for N_r quanta in the isotropic three-dimensional harmonic oscillator.) Similarly T_a is the total isospin of the N_a addition quanta. The total isospin T can assume all values consistent with the triangle conditions with T_r and T_a. We sometimes omit its z component T_z. Naturally, all these states are 0^+.

In general, the lowest states will be those with fewest quanta. As an example, we consider pairing vibrations in ${}_{24}\mathrm{Cr}_{28}{}^{52}$. Here $T_z=2$, so the lowest value of T is 2. Since $2\times(N_a-N_r)=-4$, the fewest quanta would be achieved with $N_a=0$, $N_r=2$. Thus the Cr^{52} ground state would be (22; 00; 22). The next Cr^{52} state would have $N_a=1$, $N_r=3$, and would be of the form (31; 11; 22) or (33; 11; 22) or (33; 11; 32) or (33; 11; 42). In a pure harmonic description, all these $N_a=1$, $N_r=3$ states would have the same energy. However, we expect that states of lowest T_r, T_a, and T will lie lowest. States in the nucleus ${}_{24}\mathrm{Cr}_{30}{}^{54}$ must involve at least two nucleons beyond $|0\rangle$. Thus, $N_a\geqslant 1$, and the ground state would be (22; 11; 33). The cross section for the addition of an r quantum in the pickup reaction $(\frac{1}{2},t_z)+(N_rT_r;N_aT_a;TT_z)\to(\frac{1}{2},t_z')+(N_r+1,T_r';N_aT_a;T'T_z')$ is given by

$$\sigma_r=\alpha_r(T1T_z,T_z'-T_z|T'T_z')^2(1\tfrac{1}{2},t_z'-t_z,t_z|\tfrac{1}{2}t_z')^2U^2(T_aT'T_r1;T_r'T)$$
$$\times\left\{\delta_{T_r',T_r+1}\frac{(T_r+1)(N_r+T_r+3)}{2T_r+3}+\delta_{T_r',T_r-1}\frac{T_r(N_r-T_r+2)}{2T_r-1}\right\}. \qquad (1)$$

The corresponding expression for the addition of an a quantum in the stripping reaction $(\frac{1}{2},t_z)+(N_rT_r;N_aT_a;TT_z)\to(\frac{1}{2},t_z')+(N_rT_r;N_a+1,T_a';T'T_z')$ is obtained from (1) by interchanging the subscripts r and a. The constants α_r and α_a depend upon the microscopic structure of the r and a pairs.

(2) Pairing rotations (superconducting systems).[1,3]–If $G>G_c$ a permanent pairing distortion exists, and the description of the isospin degree of freedom is similar to the description of the rotational degree of freedom in deformed nuclei. States in a given nucleus are grouped into bands built upon an intrinsic state. Assuming axial symmetry in isospin space, we associate a K quantum number with each

intrinsic state. The possible isospins in the corresponding band are $T=K$, $K+1$, $K+2$, $\cdots$. If $K=0$ we have the additional restriction that $|T-\frac{1}{2}A|$ must be even. As in quadrupole-deformed nuclei, we expect $K=0$ for the lowest band. The intrinsic state is characterized by two vibrational quantum numbers, n_Δ and n_Γ. The Δ oscillations are vibrations of the gap parameter. They preserve axial symmetry, whereas the Γ oscillations cause deviations from axial symmetry. $K=n_\Gamma$ if $n_\Gamma \leq 1$. Thus the quantum numbers needed to specify a state in the rotational scheme are $(A, n_\Delta, n_\Gamma, K, T, T_z)$. A random phase approximation calculation indicates that Δ and Γ oscillations are degenerate, $\omega_\Gamma = \omega_\Delta$. We illustrate again with the nuclei Cr^{52} and Cr^{54}. The ground state of the former has $n_\Delta = n_\Gamma = 0 = K$. It is the $T=2$ state of a band with $T=0$, 2, 4, 6, $\cdots$. The $T_z=2$, $T=4$, 6, $\cdots$ members of this band are excited states in Cr^{52}. Other excited states are members of an $n_\Delta=1$, $n_\Gamma=0$ band ($T=2$, 4, 6, $\cdots$), or an $n_\Delta=0$, $n_\Gamma=1$ band ($T=2$, 3, 4, $\cdots$). The situation is similar in ${}_{24}\mathrm{Cr}_{30}{}^{54}$, except that members of the $K=0$ bands have odd T.

The cross section for the two-particle transfer reaction $(\frac{1}{2}, t_z) + (A, 0, 0, 0, T, T_z) \rightarrow (\frac{1}{2}, t_z') + (A \pm 2, n_\Delta, n_\Gamma, K, T'T_z')$ is given by

$$\sigma = \frac{2T+1}{2T'+1}(T1T_z, T_z'-T_z|T'T_z')^2(1\tfrac{1}{2}, t_z'-t_z, t_z|\tfrac{1}{2}t_z')^2(T10K|T'K)^2 \times [1+\delta_{K,1}][\delta_{n_\Delta,0}\delta_{n_\Gamma,0}\alpha_z + (\delta_{n_\Delta,0}\delta_{n_\Gamma,1} + \delta_{n_\Delta,1}\delta_{n_\Gamma,0})\alpha_y]. \tag{2}$$

Here the constants α_z and α_y depend upon the details of the reaction, as do the α_r and α_a in (1).

There is a remarkable similarity between the structures of the pairing vibration and rotation level schemes. These schemes are compared and contrasted in Table I. We now discuss specific experiments that could distinguish between the two schemes. We ignore Q-dependent effects.

(1) The forbiddenness described in Table I(e) allows us to test the vibrational scheme. This test is especially meaningful if the final state in question can be reached by an allowed reaction, in addition to the forbidden reaction. For instance,[4] (τ, p) on $\mathrm{Ti}^{46}(51;00;1)$ populates the $(51; 11; \frac{1}{2})$ states in V^{48}. These cannot be reached by (p, τ) on $\mathrm{Cr}^{50}(31;00;1)$. The state $(51;11;2)$ in Ti^{48} can be populated by (t,p) on $\mathrm{Ti}^{46}(51;00;1)$ but not by (p,t) on $\mathrm{Ti}^{50}(33;00;3)$. Moreover, the $T_z=0$ analog of $(51;11;2)$ in Cr^{48} cannot be populated by (p,t) on $\mathrm{Cr}^{50}(31;00;1)$. The stripping and pick-up reactions to the rotational counterparts of these states are allowed.

(2) A similar comparison is afforded by the reactions $\mathrm{Cr}^{50,52,54}(t,p)$. In $\mathrm{Cr}^{50}(t,p)$, the vibrational scheme predicts transitions to the ground state and to an excited state (both having $T=2$). But only the ground-state transitions are allowed in $\mathrm{Cr}^{52,54}(t,p)$. In the rotational scheme, ground and excited state transitions should be seen in all reactions, with comparable intensities. The data seem to favor the vibrational scheme, since the ratios of the excited to ground-state intensities in these three reactions are 1.2, 0.12, and 0.04, respectively.[5]

(3) See Table I(c). The experimental, vibrational, and rotational ground-state cross sections for $\mathrm{Fe}^{54,56}(t,p)$ are, respectively, 1/1.6,[6] 1/2, and 1/0.9. For the ground-state $\mathrm{Ni}^{58-64}(p,t)$ reactions, they are 1/1.3/1.4/1.3,[7] 1/2/3/4, and 1/1.2/1.3/1.3, respectively. For the ground-state Zn^{64-70} reactions, they are 1/1.0/0.9/0.9,[7] 1/1.4/1.7/2.1, and 1/1.1/1.1/1.14. Thus, the Ni and Zn data favor the rotational scheme. The Ni^{60}, Ni^{58}, Fe^{56}, Fe^{54}, and $\mathrm{Cr}^{52}(\tau,p)$ reactions to the ground-state analogs have cross sections[8] in the ratios 3.83/4.80/2.56/1.17/1. The vibrational ratios would be 3/3/2/1.5/1, whereas the rotational ratios would be 1/1.4/1/1.4/1. Thus these data favor the vibrational scheme. The corresponding numbers for $\mathrm{Ti}^{48,46}(\tau,p)$ are 1/2.2 (experimental), 1/2.8 (vibrational), and 1/1.4 (rotational), again favoring the vibrational scheme.

(4) See Table I(d). In some cases where two reactions populate a state, the ratio of cross sections in the vibrational scheme involves α_r/α_a, whereas in the rotational scheme this ratio is independent of α_y or α_z (if both reactions involve α_y, or both involve α_z). For instance, (p,τ) on $\mathrm{Ni}^{58}[(00;11;1)/(58,0,0,0,1)]$ and (τ,p) on $\mathrm{Fe}^{54}[(11;00;1)/(56,0,0,0,1)]$ may populate the $[(11,11,1)/(56,0,1,1,1)]$ and the $[(11;11;2)/(56,0,0,0,2)]$ states in Ni^{56}. The ratio between stripping and pickup cross section to the same level are proportional to α_a/α_r in the vibrational coupling scheme and to $\alpha_y/\alpha_y = \alpha_z/\alpha_z = 1$ in the rotational coupling scheme. Analogous states of the above-mentioned $T=2$ state in ${}^{56}\mathrm{Ni}$ may be reached by $\mathrm{Fe}^{54}(t,p)$ or $\mathrm{Ni}^{58}(p,t)$. The ratio between these latter is $6\alpha_a/\alpha_r$ in the vibrational scheme while

Table I. Similarities and differences between the vibrational and rotational isospin pairing coupling schemes. All arguments are symmetric with respect to r and a quanta. For instance, the discussion in (a) also applies to the state $(00;NT;T)$.

	Similarities		Differences
a)	In both schemes there is a subset of states (henceforth called the Z-subset such that only ground states and their isobaric analogues belong to it. In the vibrational scheme, this subset consists of the states $(NT;00;T)$, with $N=\vert A-A_0\vert/2$, A_0 being the number of nucleons in $\vert 0\rangle$. In the rotational scheme, all states $(A,0,0,0,T)$ constitute this subset. In both schemes, the rule $T-\frac{A}{2}$ even is valid for states of the subset.	a)	In the vibrational scheme, states in the Z-subset satisfy $T\leqslant N\equiv\vert A-A_0\vert/2$. In the rotational scheme, the T values are unbounded. Thus all ground states of even A nuclei are in the Z-subset of the rotational scheme, whereas some even A ground states are not in the Z-subset of the vibrational scheme. For instance, the ground state of Ca^{50} is $(A,0,0,0,5)$ in the rotational scheme, but is $(44;11;5)$ in the vibrational scheme. The state $(44;11;5)$ is not in the Z-subset. This occurs whenever $Z<28$ and $N>28$.
b)	In both schemes it is useful to define a Y subset of states. This subset has two parts: $Y_1: \vert T-\frac{A}{2}\vert$ odd $(N+1\ T;11;T)$ or $(A,0,1,1,T)$ $Y_2: \vert T-\frac{A}{2}\vert$ even $(N+1\ T\pm1;11;T)$ or $(A,{1 \atop 0},{0 \atop 1},{0 \atop 1},T)$	b)	If $T>N+1$, there is no vibrational member of the Y_1 subset. The state $(56,0,1,1,3)$ in Mn^{56} corresponds to $(22;22;3)$, which is not in the Y_1 subset. This is the only example that can be reached by a two-nucleon transfer reaction. If $T\geqslant N+2$, there is no vibrational member of the Y_2 subset. (The $(N+1,N+1;11;N+2)$ state corresponds to a state belonging to the Z-subset in the rotational scheme (see a above)).
c)	Transition rates of two-nucleon transfer reactions are proportional to geometrical factors depending on initial and final values of T and T_z.	c)	In the vibrational scheme, the transition rates may increase with $\vert A-A_0\vert$, as they depend upon the number of phonons present in the initial and final states (see eq. (1)). In the rotational scheme, they are independent of A (see eq. (2)).
d)	The transition rates depend upon two constants in both schemes, α_r and α_a in the vibrational scheme, and α_z any α_Y in the rotational scheme.	d)	If two or more states with the same T exist in the final nucleus, the transition to the lowest state is proportional to α_z in the rotational scheme. In the vibrational scheme, this transition can be proportional to either α_r or α_a.
e)	In both cases, transitions populate members of the Z and Y subsets.	e)	In the rotational scheme, all $Z\to Y$ transitions are allowed. In the vibrational scheme, transitions of the form $(NT;00;T)\to(N+2\ T;11;T')$ are forbidden.
f)	Correction terms must be added to the energies given in the opposite paragraph. In the vibrational scheme, we find diagonal matrix elements of the Hamiltonian proportional to $T(T+1)$, A^2, $(N_r+N_a)^2$ and $\vec{T}_r\cdot\vec{T}_a$ if we maintain symmetry between addition and removal quanta and confine ourselves to anharmonic terms with 4 phonon operators. In the rotational scheme we may introduce different moments of inertia for the Δ and Γ oscillations, which is essentially equivalent to the $\vec{T}_r\cdot\vec{T}_a$ term in the vibrational scheme. In the rotational scheme, the degeneracy of the Y_2 states is lifted by a term involving T_-, while a quadrupole isospin term plays a similar role in the vibrational scheme. Thus, the corrected energy spectra produced by the two models are essentially the same.	f)	In the vibrational scheme, the lowest order energies are linear in the number of quanta ($E=\hbar\omega(N_a+N_r+1)$). In the rotational scheme, we have a three-parameter expression in lowest order ($E=\frac{\hbar^2}{2\mathcal{J}}\cdot T(T+1)+\frac{\hbar}{2B}(A-A_0)^2+(n_\Delta+n_\Gamma+1)\hbar\omega$).

it is $6(\alpha_z/\alpha_z)$ in the rotational coupling scheme. Conversely, if two reactions populate states T, $T+1$ in the same final nucleus, it may be that σ_T/σ_{T+1} is proportional to α_y/α_z in the rotational scheme, but is independent of α_r/α_a in the vibrational scheme. The following are some experimental, vibrational, and rotational ratios: $Ca^{48}(\tau,p)Sc^{50}$, 1.4, 4, $8.8\alpha_y/\alpha_z$[9]; $Cr^{52}(\tau,p)Mn^{54}$, 0.4, 2, $4.7\alpha_y/\alpha_z$[10]; $Fe^{54}(\tau,p)Co^{56}$, 0.6, 1, $2.5\alpha_y/\alpha_z$.[10] The ratio α_y/α_z can reasonably be expected to lie between 0.5 and 1. Thus both schemes predict too large a value of the ratio σ_T/σ_{T+1}. The Q dependence of the (t,p) reaction might have an important effect in this comparison. It is difficult to draw definite conclusions from distorted wave Born approximation calculations because the Q values of σ_T and σ_{T+1} are so different. However, it is unlikely that Q-dependent effects would enhance σ_{T+1} compared with σ_T. Thus, the absence of a strong σ_T transition represents a failure of both schemes.

(5) The ratios between cross sections for reactions populating levels in different nuclei may also distinguish between the two schemes. In $\mathrm{Cr}^{52,54}(p,t)$ and $\mathrm{Ca}^{46,48}(t,p)$, $\sigma_A(0)/\sigma_{A+2}(0)$ should be near unity according to the rotational scheme, and should be near α_r/α_a according to the vibrational scheme. The experimental ratios are $0.1(\mathrm{Cr})^{11}$ and $0.3(\mathrm{Ca})$.[12]

(6) See Table I(f). Because of the $T(T+1)$ term, the comparison of energies favors the lowest order version of the rotational scheme. However, we have seen that with the correction terms, the two models become very similar. Since first-order corrections to the vibrational energies are of the same magnitude as the lowest order energies, and since the energies of Δ and Γ oscillations are comparable with the rotational energies, it seems that it is necessary to solve the complete collective Hamiltonian, as has been done for the quadrupole force and for the pairing force between identical particles.

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REARRANGEMENT EFFECTS AND THE PARAMETRIZATION OF THE EFFECTIVE FIELD IN NUCLEI*

H. Meldner†
Institute for Pure and Applied Science, University of California, La Jolla, California 92037

and

C. M. Shakin‡
Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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A simple parametrization of the nonlocal effective field in nuclei is provided. The parameters as determined by fitting the properties of nuclear matter are quite similar to those which have been shown to yield good results for the size and binding energies of finite nuclei throughout the periodic table. It is shown that the self-consistent single-particle energies are significantly different from the separation energies (rearrangement effect) and consequences for structure calculations and particle knock-out experiments are discussed.

In this work we avoid the introduction of an explicit form for the two-body force and attempt to make a simple self-consistent parametrization of the effective field in nuclei. We assume that the nonlocal potential for a single particle in nuclear matter is given by[1]

$$V(\vec{r}-\vec{r}\,') = v\,\frac{\exp[-|\vec{r}-\vec{r}\,'|/a]}{|\vec{r}-\vec{r}\,'|/a}\,\frac{\rho}{2}\left[1-\left(\frac{\rho}{2\rho_1}\right)^{2/3}\right], \tag{1}$$