pected to predominate over the pulling. Van Loef[9] has shown that the sublattice magnetization of various anti-, ferro-, and ferrimagnets fits a simple power law $\approx(T_c-T)^\beta$ with $\beta=0.28$-0.35 over the temperature range $0.5T_c < T < 0.99T_c$, where T_c is the critical temperature. The data of Fig. 2 appear indeed to be asymptotically approaching a $(T_N-T)^{1/3}$ dependence at the highest temperatures reached. The measurements are presently being extended to higher temperatures. Between 14 and 20 K two distinct resonant absorptions, as indicated in Fig. 2, were observed. The two absorptions differed in the dependence on φ of their intensities. The temperature dependence of the new absorption signal does not fit that predicted for the ν_2 mode. We have no adequate explanation at this time of the appearance and subsequent disappearance of the additional resonant absorption.

The observed angular dependence of the NAR frequency at 4.3 K agrees well with that calculated from Eqs. (1) and (2): The observed shift in frequency between $\varphi=0°$ and $\varphi=45°$ was (8 ± 2) MHz; the calculated value, 10 MHz. The angular behavior of the NAR line intensity at 4.3 K indicates a $\sin^2 2\varphi$ dependence. This is the same as the dependence calculated by Silverstein[10] in his theory of NAR in an antiferromagnet. The apparent agreement may be more fortuitous than real, however, since, as contrasted to cubic $RbMnF_3$, Silverstein considered a uniaxial system.

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THERMAL FLUCTUATIONS AND THE JOSEPHSON SUPERCURRENT*

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The effects of thermal fluctuations on the dc Josephson effect are examined experimentally through measurements of the current-voltage characteristic. The results are compared with a calculation due to Ambegaokar and Halperin which uses an analogy with the Brownian motion of a particle in a field of force.

The effects of thermal fluctuations on superconducting systems have recently been the subject of intensive study. Fluctuations play an important role in the dc and ac Josephson effects[1,2] where they may be understood using relatively simple models. In this Letter we report measurements on a Josephson junction in a regime in which the current-voltage characteristic is rounded by thermal fluctuations.[3,4] The results are found to be in qualitative agreement with a theory due to Ambegaokar and Halperin.[5,6]

There are two methods which can be used to reduce the coupling energy across a Josephson junction to the order of thermal energies. One can apply sufficient magnetic field so as to bias the junction in the neighborhood of one of the minima[3] of the $I_1(H)$ curve, or one can set the temperature to a value just below the transition temperature T_c. For a uniform junction of transverse dimensions $L < \lambda_J$ these procedures produce essentially identical results. In this Letter, however, we present only data obtained near T_c. Results obtained at minima of the $I_1(H)$ curve will be discussed in a later publication.

The present experiments were conducted on an almost ideal junction in a carefully controlled environment. The apparatus was completely housed in an all-metal rf-shielded room.[7] The cryostat was surrounded by a Mumetal enclosure which reduced the static magnetic field to less than 10^{-3} Oe. The substrate, a calibrated germanium thermometer, and a heater were tied to a copper block which was suspended in a can containing a small amount of He^4 exchange gas. The temperature of the block was regulated to $\pm 10^{-5}\,{}^\circ K$. The absolute accuracy of the thermometer was $\pm 2 \times 10^{-3}\,{}^\circ K$. Current-voltage characteristics were obtained using an audio technique in which the current was supplied from a high-impedance source and the voltage measured with a high-impedance voltmeter. The measuring system was capable of resolving changes in current and voltage smaller than 10^{-8} A and 10^{-8} V, respectively. In all measurements the junction was current biased with the largest peak-to-peak low-frequency noise current substantially smaller than 5×10^{-9} A.

Figure 1 shows the I-V characteristic at low temperature, the magnetic field dependence of the maximum zero-voltage current away from T_c, and the temperature dependence of the maximum zero-voltage current.[8] These curves agree with the theory of the Josephson effect and are the evidence for the sample being a nearly ideal Josephson junction with a relatively homogeneous oxide layer. The extrapolated value of the zero-voltage current at low temperatures is 97% of the theoretical value.[9] The latter was calculated using a quasiparticle resistance of 1.3 Ω and an energy gap 2Δ of 1.05×10^{-3} V. The transverse dimensions of the junction were much smaller than the Josephson penetration depth λ_J at the temperatures at which measurements were made. The capacitance of the junction was calculated from $C = \epsilon A/4\pi l$. Here A is the junction area and l the thickness of the oxide. The junction measured 1.4×10^{-2} cm $\times\, 1.38\times 10^{-2}$ cm. The quantity ϵ/l was obtained by computing the wave velocity $\bar{c}$ from the spacing of the junction self-resonant or Fiske modes at low temperatures.[10,11] Taking the bulk penetration depth of tin to be 510 Å, and using the measured junction width of 1.4×10^{-2} cm and the measured step separation of 1.79×10^{-4} V, the value of $\bar{c}/c$ was 8.4×10^{-2}; the capacitance was 245 pF.

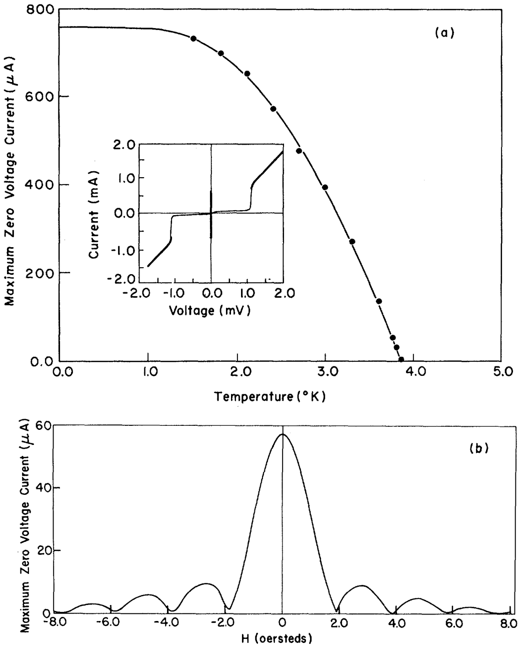

FIG. 1. (a) Temperature dependence of the maximum zero-voltage Josephson current fitted by the Ambegaokar-Baratoff theory, Ref. 8. In this curve the quasiparticle resistance was 1.0 Ω. The resistance changed with time as the junction was stored at 77°K for nine months and frequently cycled between 77 and 4°K. For the curves in Fig. 2 the resistance was 1.3 Ω and the zero-temperature maximum Josephson current was 638 μA or 97% of the theoretical value. The inset is a tracing of the current-voltage characteristic at $T = 2.1\,{}^\circ K$. (b) Magnetic field dependence of the maximum Josephson current at $T = 3.8\,{}^\circ K$. This tracing was obtained using a servomechanism with a 10% error.

Both the theory of Ivanchenko and Zil'berman[6] and that of Ambegaokar and Halperin[5] are based on the simple model of the dc Josephson effect due to Anderson,[1] where the junction is assumed to be connected in series with a constant-current generator. This combination is then described by a phase-dependent potential energy of the form

$$U(\varphi) = (\hbar/2e)(I_1 \cos\varphi + I\varphi). \qquad (1)$$

Here φ is the relative phase of the order parameters, I_1 is the maximum zero-voltage current that can flow through the junction, and I is the current supplied by the generator. The first term of the potential is the phase-dependent coupling energy of the junction and the second term takes account of the current source. With a current I flowing through the junction, the phase adjusts to keep the system in one of the local minima of the potential. In the absence of fluctuations the current may be increased without the appearance of voltage until $I = I_1(T)$. At this point there cease to be minima in $U(\varphi)$ and the system slides along the potential. When this happens, the voltage[9] $V = (\hbar/2e)d\varphi/dt$ across the junction is no longer zero.

Ivanchenko and Zil'berman[6] use a simple kinetic model to estimate the effect of fluctuations on the I-V characteristic. The system is assumed to oscillate in one of the minima of the potential at the Josephson plasma[11] frequency ω_J with a probability of jumping to an adjacent minimum proportional to the product of the attempt frequency ω_J and a factor $\exp(-\Delta E/kT)$. Here ΔE is the barrier height, k is the Boltzman constant, and T is the temperature. Assuming the process of jumping between barriers to be Markoffian, they then calculate the average value of $V=(\hbar/2e)\times d\varphi/dt$ and find a finite, current-dependent voltage for $I<I_1$.

In the theory of Ambegaokar and Halperin, which is more detailed than that of Ivanchenko and Zil'berman, the dynamical equations resulting from (1) which relate the phase jump across the junction, the voltage difference V, and the current through the junction are supplemented by a fluctuating noise current. The resulting equations are recognized to be equivalent to the dynamical problem of Brownian motion of a particle in an external potential of the form of Eq. (1).[12] The action of the fluctuations is to perturb the system so that it diffuses along the potential resulting in a nonzero value of the phase slippage and the voltage across the junction for values of current $I<I_1$. The voltage is given by

$$V=I_1R\left(\frac{4\pi}{\gamma}\right)\left[(e^{\pi\gamma x}-1)^{-1}\int_0^{2\pi}d\theta\, f(\theta)\int_0^{2\pi}d\theta'\,\frac{1}{f(\theta')}+\int_0^{2\pi}d\theta\int_\theta^{2\pi}d\theta'\,\frac{f(\theta)}{f(\theta')}\right]^{-1}. \tag{2}$$

In this equation $\gamma=\hbar I_1/ekT$, $x=I/I_1$, R is the quasiparticle resistance, and

$$f(\theta)=e^{\gamma(x\theta+\cos\theta)/2}. \tag{3}$$

We compare the data with the work of Ambegaokar and Halperin[5] as it is more detailed than that of Ivanchenko and Zil'berman.[6] There are difficulties in making this comparison as Ambegaokar and Halperin[5] perform the calculation for the case of zero capacitance which amounts to neglecting hysteresis in the I-V characteristic.[13,14] The parameter which characterizes hysteresis is $r=RC(2eI_1/\hbar C)^{1/2}$. In the limit $r=0$ there is no hysteresis. Equation (2) gives no means of interpolating for finite r; however, if r is less than one, the hysteresis is small and some comparison with experiment may be made. For the junction of the present investigation $C=245$ pF and $R=1.3\ \Omega$, so that $r<1$ only if $I_1<8\times10^{-7}$ A. A definite comparison of the Ambegaokar-Halperin theory with the theory of Ivanchenko and Zil'berman comes in the limit $\gamma\gg1$ where the voltage obtained by Ivanchenko and Zil'berman for the same I/I_1 is $1/r$ times that obtained by Ambegaokar and Halperin. The capacitance of our junction is such that if $r<1$, the quantity $\gamma\sim6$, a value which does not satisfy the condition $\gamma\gg1$. Attempts to produce junctions of significantly smaller capacitance which would permit a definite comparison are under way.

The solid curves in Fig. 2 are the experimental I-V characteristics in the vicinity of T_c. The dashed lines are the results of a two-parameter fit of the data to the theory over the first 100 nV of each curve, and an extrapolation over the rest of the range using Eq. (2). The temperature T and the maximum Josephson current I_1 are taken as independent, fitted parameters. Effectively, this amounts to treating T as a noise temperature. It was possible to fit all of the curves with $T=10$°K. The fact that the effective noise temperature is greater than the actual temperature suggests that some external noise is present in the apparatus.

The anticipated consequence of having $r>0$ in the experiment and equal to zero in the calcula-

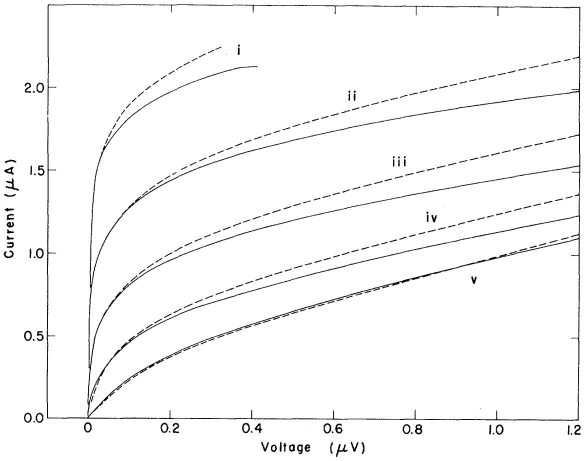

FIG. 2. Current-voltage characteristics of the junction at several temperatures near T_c. The solid lines are data and the dashed line is a two-parameter fit to the theory of Ambegaokar and Halperin, Ref. 5, with an effective noise temperature of 10°K for each curve. The relevant parameters for the various curves are (i) $T=3.855$°K, $\gamma=14.7$, $I_1=3.09\times10^{-6}$A; (ii) $T=3.857$°K, $\gamma=11.0$, $I_1=2.30\times10^{-6}$A; (iii) $T=3.859$°K, $\gamma=8.04$, $I_1=1.68\times10^{-6}$A; (iv) $T=3.861$°K, $\gamma=5.58$, $I_1=1.17\times10^{-6}$ A; (v) $T=3.863$°K, $\gamma=3.59$, $I_1=0.75\times10^{-6}$ A.

tion is for the calculation to underestimate the voltage for a given value of I/I_1.[15] This appears to occur in our analysis. Only for curve (v) in Fig. 2 is the extrapolation using Eq. (2) and the fitted parameters in good agreement with the data. This curve is the only one for which $r < 1$. For the other curves, $r > 1$ and extrapolation by the theory yields voltages less than those observed.[16,17] A more definitive comparison of experiment and theory would be possible if the calculations were extended to include the case $r > 0$, as the quasiparticle resistance, the capacitance, and the temperature, which completely parametrize the theory, have all been measured.

In conclusion, this Letter reports measurements of the current-voltage characteristics of an almost ideal Josephson junction as influenced by thermal fluctuations. The observed current-voltage characteristics are found to be in qualitative agreement with the calculation of Ambegaokar and Halperin.

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Note added in proof.—It has been brought to our attention that Ivanchenko and Zil'berman, in a second paper,[18] have also calculated the effects of thermal fluctuations on the dc I-V characteristics of junctions by finding approximate solutions to the Fokker-Planck equation.

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[17] If one plots the values of I_1 obtained through the analysis of the data in Fig. 2 as a function of T, one obtains a curve which extrapolates to a higher T_c than that obtained by extrapolating the data of Fig. 1. It is not known whether this is due to a geometric inhomogeneity in the junction or is a consequence of thermal fluctuations near T_c. It should be noted that the I-V characteristics of Fig. 2 are modulated by a magnetic field in a fashion qualitatively similar to the modulation of Fig. 1(b).

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