DUALITY AND NONLEPTONIC HYPERON DECAY

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Duality reproduces most current-algebra results and leads to octet enhancement for S-wave nonleptonic decays of baryons. Reasons for its inapplicability to P-wave decays are given.

In this Letter we discuss nonleptonic decays of baryons using the spurion formalism, duality,[1] and the lack of exotic states. We obtain essentially all of the results of Sugawara and Suzuki[2] for S-wave decays without assuming partial conservation of axial-vector currents or validity of current algebra.[3] We also find that the contribution of the $\underline{27}$ part of the spurion to these decays is naturally suppressed.[4]

We consider the decay process

$$B(p)\to B'(p')+\pi(q) \tag{1}$$

of a baryon B of mass m and momentum p into another baryon B' of mass m' and momentum p' and a pion of mass μ and momentum q. The related process involving the spurion is

$$S(k)+B(p)\to B'(p')+\pi(q), \tag{2}$$

where S denotes a $J^P=0^-$ or 0^+ strangeness-carrying spurion of mass zero ($k^2=0$). We define $s=(p+k)^2$, $t=(q-k)^2$, $u=(p'-k)^2$, and $\nu=(s-u)/2$, so that $s+t+u=m^2+\mu^2+m'^2$. Evaluation of the matrix element for process (2) at $s=m^2$ and $t=\mu^2$ [corresponding to $\nu=\nu_0\equiv(m^2-m'^2)/2$] then gives the matrix element for the desired process (1),[5] when multiplied by a suitable factor describing the weak transition, vacuum $\to S$.

Our basic assumption is that amplitudes of the form (2) possess several properties usually associated with hadronic amplitudes[6]:

(i) Asymptotic Regge behavior. When S is a 0^- spurion, for example, only natural-parity trajectories contribute in the t channel, and

$$A(\nu,t)\underset{(\nu\to\infty)}{\simeq}\beta_T(t)\left[-\cot\left(\frac{\pi\alpha_T(t)}{2}\right)+i\right]\left(\frac{\nu}{\nu_0}\right)^{\alpha_T(t)}+\beta_V(t)\left[\tan\left(\frac{\pi\alpha_V(t)}{2}\right)+i\right]\left(\frac{\nu}{\nu_0}\right)^{\alpha_V(t)}, \tag{3}$$

where $\alpha_T(t)$ and $\alpha_V(t)$ are the respective trajectories of the strange members of the tensor (T) and vector (V) nonets. A related limit holds for $\nu\to-\infty$.

(ii) Analyticity properties, allowing one to write the following Cauchy representation for $A(\nu,t)$:

$$A(\nu,t)=\frac{1}{\pi}\int_{\nu_s}^{N}d\nu'\frac{\mathrm{Im}A_s(\nu',t)}{\nu'-\nu}+\frac{1}{\pi}\int_{\nu_u}^{N}d\nu'\frac{\mathrm{Im}A_u(\nu',t)}{\nu'+\nu}+(\text{pole contributions below threshold})+\frac{1}{2\pi i}\int_C d\nu'\frac{A(\nu',t)}{\nu'-\nu}. \tag{4}$$

In Eq. (4), $\mathrm{Im}A_s(\nu',t)$ is the absorptive part above threshold ν_s for process (2). $\mathrm{Im}A_u(\nu',t)$ is the corresponding absorptive part above threshold ν_u for the crossed process $\bar{\pi}(-q)+B(p)\to B(p')+\bar{S}(-k)$. The contour C consists of two semicircles of radius N in the ν plane, above and below the real axis. We choose N sufficiently large to allow the use of Eq. (3) in the last term of the representation (4).

(iii) Duality in the original sense of Dolen, Horn, and Schmid,[1] whereby the average contribution of s-channel resonances to $\mathrm{Im}A(\nu,t)$ behaves as a Regge term considerably below N: say, above a convenient low cutoff L.[7]

(iv) Lack of exotic states: No exotic resonances or exotic t-channel trajectories contribute to $\mathrm{Im}A(\nu,t)$.

The algebraic set of constraints implied by (iii), (iv), and SU(3) symmetry has been discussed in detail for meson-meson and meson-baryon scattering.[8] If S is an octet member, a factorizable solution exists, containing exchange-degenerate T and V trajectories and having a simple interpretation in terms of quark diagrams.[9]

Considering Eq. (4) at the kinematical point of interest ($\nu=\nu_0, t=\mu^2$), we distinguish two types of contributions: (a) pole and continuum contributions for $\nu' \leqslant L$, and (b) contributions describable by Regge exchange on the average for $L \leqslant \nu' \leqslant N$ and more or less exactly around the circle of radius N.

The question of interest is to what extent the dual algebraic structure of the amplitude at higher ν is exhibited at low energies, i.e., in $A(\nu_0, \mu^2)$. Obviously this depends on the importance of contribution (b)—which has the dual structure—relative to that of part (a)—which may deviate from the extrapolated Regge expression in the presence of large individual contributions from low-lying resonances or poles. In such a case we would not expect the part (a) to have the above-mentioned dual algebraic structure.

In particular, pole contributions to parity-conserving (P-wave decay) amplitudes are very large. The net experimental values of such amplitudes depend critically on delicate cancelations involving broken SU(3) for masses and coupling constants.[10] Contributions describable by Regge exchange are expected to be comparatively small as a result of the low intercept of the relevant abnormal-parity (K and K_A) trajectories.

For parity-nonconserving (S-wave) decays the situation is exactly the opposite. The octet pole contributions are small—typically 15% of the amplitudes[10]—as a result of kinematic factors involving mass differences arising in the numerators. Similar mass difference factors, of order $M_{10}-M_8$, occur as well in the contributions of the decimet intermediate state, making these contributions small as well.[11] In contrast, the Regge exchanges involved are those of the K^* and K^{**} trajectories, whose effects are quite important at high energies.

For processes (2) contributing to S-wave decays, we thus have no prominent local fluctuations in $A(\nu, t)$ below or near threshold. The deviation of the overall contribution (a)+(b) from the extrapolated Regge term will then be small. No such conclusion can be drawn for P-wave decays.

The S-wave amplitudes will therefore be proportional to $\gamma_{K^*S\pi}\,\gamma_{K^*BB'}$, where the γ's denote factorized residue functions, and $\gamma_{K^*S\pi}{}^2=\gamma_{K^{**}S\pi}{}^2$, $\gamma_{K^*BB'}{}^2=\gamma_{K^{**}BB'}{}^2$. Comparing ratios of various S-wave amplitudes, the ratios of the $\gamma_{K^*S\pi}$ for various processes will be fixed by isospin alone, while those of the $\gamma_{K^*BB'}$ will depend only on a ratio F/D. We thus recover most of the results of the Sugawara-Suzuki analysis,[2] including the predictions

$$\begin{aligned} \Sigma_+{}^+ &\equiv A(\Sigma^+ \to n\pi^+)=0,\\ \Sigma_-{}^- &\equiv A(\Sigma^- \to n\pi^-)=(2F-1)x,\\ \Lambda_- &\equiv A(\Lambda \to p\pi^-)=-(2F+1)x/\sqrt{6},\\ \Xi_-{}^- &\equiv A(\Xi^- \to \Lambda\pi^-)=(4F-1)x/\sqrt{6}, \end{aligned} \tag{5}$$

with other relations following from the assumed $\Delta I=\frac{1}{2}$ nature of the spurion. (Here x is a common factor whose magnitude we do not specify, and $F+D=1$.) We thus identify F as the same parameter appearing in fits to high-energy scattering data.[12] This identification is indeed borne out by experiment.[13] A well-known corollary of Eqs. (5) is the Lee-Sugawara relation for S waves,[14] $2\Xi_-{}^- + \Lambda_- = (\frac{3}{2})^{1/2}\Sigma_-{}^-$.[15]

The conditions (iii) and (iv) cannot be satisfied simultaneously if S transforms as a member of 27. To see this it suffices to consider the process

$$S(I=2)+B(I=1)\to B'(I=1)+\pi(I=1) \tag{6}$$

which has only $I=1, 2$ in the s and t channels. If the $I=2$ amplitudes in s and t are set equal to zero, the $I=1$ amplitudes must vanish as well.[16] A similar argument holds for the u and t channels. This suggests that even if $H_{\rm wk}$ has parts transforming as a 27 of SU(3) their contribution to the above decay processes will be suppressed by the requirement of duality.[17]

One can have amplitudes of the form (2) which lack exotics in s and u when $S\in 27$ (cf. Ref. 4). The existence of such a solution may be visualized in terms of duality graphs,[9] and has been verified by explicit calculation. Amplitudes lacking exotics in s and u are irrelevant to the present approach, however.

To sum up: We have found that duality explains most features of S-wave nonleptonic decays of baryons without appealing to current algebra or the hypothesis of partially conserved axial-vector currents.[18] We have also shown why we expect it to fail for P waves, for which individual pole contributions play a much more dominant role.[19]

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[1]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968). For an excellent review, see M. Jacob, CERN Report No. Th-1010, 1969 (to be published).

[2]H. Sugawara, Phys. Rev. Letters 15, 870, 997(E) (1965); M. Suzuki, Phys. Rev. Letters 15, 986 (1965).

[3]The suggestion that duality might be useful in explaining nonleptonic processes was first made by M. Suzuki, Phys. Rev. Letters 22, 1217, 1413(E) (1969), and 23, 109(E) (1969). Unfortunately the original application, the derivation of $P(\Sigma_-{}^-)=0$, was apparently wrong (see above errata).

[4]This was observed independently by K. Kawarabayashi and S. Kitakado, Phys. Rev. Letters 23, 440 (1969). These authors, however, overlook a singularity in the s-u crossing matrix that requires modification of their arguments.

[5]For each process (2) there are two amplitudes. However, at the kinematical point of interest only the amplitude with no t-channel helicity flip contributes. We denote this amplitude by $A(\nu,t)$.

[6]Such assumptions are often made for cases involving only one weak or electromagnetic vertex, such as ρ photoproduction. We assume the weak Hamiltonian to act in such a way in process (1) that these assumptions are valid as well for the present case.

[7]Such an approach was applied successfully by R. Gatto, Phys. Rev. Letters 18, 803 (1967), for example.

[8]See, e.g., C. Schmid, Phys. Rev. Letters 20, 628 (1968); C. Schmid and J. Yellin, Phys. Letters 27B, 19 (1968); C. B. Chiu and J. Finkelstein, Phys. Letters 27B, 510 (1968); J. Rosner, Phys. Rev. Letters 21, 950 (1968); H. J. Lipkin, Nucl. Phys. B9, 349 (1969); M. Kugler, Phys. Rev. 180, 1538 (1969).

[9]H. Harari, Phys. Rev. Letters 22, 562 (1969); J. Rosner, Phys. Rev. Letters 22, 689 (1969).

[10]Expressions and values for pole terms are given, for example, by S. A. Bludman, Cargèse Lectures in Physics (Gordon and Breach Publishers, Inc., New York, 1967), and by A. Kumar and J. C. Pati, Phys. Rev. Letters 18, 1230 (1967).

[11]This follows from the explicit form of the spin-$\frac{3}{2}$ propagator and the coupling $\bar{\psi}_\mu \psi \partial^\mu \varphi$.

[12]V. Barger and M. Olsson, Phys. Rev. 146, 1080 (1966); Rosner, Ref. 9 (and references contained therein).

[13]M. Suzuki, Phys. Rev. 171, 1791 (1968). Our approach is a concrete demonstration of how the relation proposed by Suzuki between Regge trajectories and spurion amplitudes may arise, and may be valid for semistrong and electromagnetic mass shifts as well.

[14]B. W. Lee, Phys. Rev. Letters 12, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) 31, 213 (1964).

[15]The (current algebra + partially conserved axial-vector current) analysis can go beyond our results in prescribing the value of x in Eq. (5). Various attempts were made to calculate this term, with some measure of success. See, e.g., Y. Hara, Progr. Theoret. Phys. (Kyoto) 37, 710 (1967); and S. Nussinov and G. Preparata, Phys. Rev. 175, 2180 (1968).

[16]The present argument resembles that of Lipkin, Ref. 8, for $\bar{\Delta}\Delta \to \bar{\Delta}\Delta$. Note that only the 27 need be considered in a $J_\mu J^\mu$ Hamiltonian which is symmetric in the SU(3) indices of the currents.

[17]For other mechanisms giving octet dominance, see S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964); and R. Dashen, S. C. Frautschi, and D. Sharp, Phys. Rev. Letters 13, 777 (1964).

[18]The suggestion that much of the pattern of current algebra is contained in pure hadron dynamics was made by H. R. Rubinstein and G. Veneziano, Phys. Rev. 160, 1286 (1967), and is implied by J. J. Sakurai, Phys. Rev. Letters 17, 102 (1966).

[19]See, e.g., Ref. 10, as well as B. W. Lee and A. R. Swift, Phys. Rev. 136, B228 (1964); L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters 16, 751 (1966); C. Itzykson and M. Jacob, Nuovo Cimento 48A, 655 (1967).