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LIGHT-CONE COMMUTATOR AND HIGH-ENERGY LEPTON-HADRON INELASTIC SCATTERING*

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An extension of the chiral $[SU(3)\otimes SU(3)]_{\gamma_5}$ equal-time commutation relations to the light cone is proposed based on a universality defined by the $[SU(3)\otimes SU(3)]_\beta$ algebra. Consequences concerning asymptotic lepton-hadron inelastic scattering are derived and found to be in good agreement with experiment.

The Fubini–Dashen–Gell-Mann sum rule has been shown[1] to be essentially[2] equivalent to the relation [we consider only forward ($t=0$) amplitudes[3]]

$$\tfrac{1}{2}\int dx_+ [J^a(x), J^b]\delta(x_-) = if^{abc}J^c\delta(\vec{x})\delta(x_-), \tag{1}$$

where $J^a(x) = n^\mu J_\mu{}^a(x)$ [$J \equiv J(0)$] with $J_\mu{}^a$ ($\mu = 0$-4, $a = 1$-8) the octet vector currents ($\frac{1}{2}\bar{\psi}\gamma_\mu\lambda^\alpha\psi$ in the quark model), and $n^\mu=(1,0,0,-1)$, $x_\pm=x_0\pm x_3$, $\vec{x}=(x_1,x_2)$. Equation (1) requires that [between rest states[3] with momentum $p^\mu=(1,0,0,0)$, so that $p\cdot x=x_0$]

$$[J^a(x),J^b]\sim 2\pi^{-1}i\delta(x^2)[f^{abc}J^c f(2x_0)+R^{ab}(x)] \tag{2}$$

for $x^2\sim 0$, where

$$\tfrac{1}{2}\int dx_+ f(x_+)=1,\ f(-\lambda)=f(\lambda);\quad \int dx_+ R^{ab}(x)\delta(x_-)=0,\ R^{ab}(-x)=-R^{ba}(x). \tag{3}$$

In this paper we shall explore some simple consequences of the assumption that the form of R^{ab} is such as to give (2) a universal structure maximally symmetric in f- and d-type octet couplings. Thus we propose that for $x^2\sim 0$,[4]

$$[J^a(x),J^b]\sim 2\pi^{-1}i\delta(x^2)f(2x_0)[f^{abc}J^c-d^{abc}S^c\epsilon(x_0)]. \tag{4}$$

Here S^c is the octet scalar current density given by $\frac{1}{2}\bar\psi\lambda^a\psi$ in the quark model. The quark model $[\mathrm{SU}(3)\otimes\mathrm{SU}(3)]_\beta$ commutation relations,

$$[J_0{}^a(x),J_0{}^b]\delta(x_0)=if^{abc}J_0{}^c\delta(x),\ [J_0{}^a(x),S^b]\delta(x_0)=if^{abc}S^c\delta(x),\ [S^a(x),S^b]\delta(x_0)=if^{abc}J_0{}^c\delta(x), \tag{5}$$

establish the scale of S relative to J_0[5] and make precise the universality implicit in (4) in exactly the same way[6] that the $[\mathrm{SU}(3)\otimes\mathrm{SU}(3)]_{\gamma_5}$ commutation relations establish the scale of $J_0{}^5$ relations to J_0 and make precise weak-interaction universality. We shall see that (4) gives results usually taken to imply a composite structure[7] for the nucleon.

In addition to the weak V-A universality analogy, some motivation for (4) comes from the following formalisms which have been applied in other contexts: (a) The Cabibbo, Horwitz, Ne'eman[8] proposal applied to vector-meson–nucleon scattering gives an operator structure similar to that of (4) in the $[\mathrm{U}(3)\otimes\mathrm{U}(3)]_\beta$ symmetry limit if one takes all Regge trajectories to cross $j=1$ at $t=0$. (b) Okubo[9] has suggested that the pseudoscalar-meson source commutator for $x^2=0$ involves (unspecified) unitary singlets and octets. (c) Given the presence of the first term in (4), its relation to nonsense right-signature $j=1$ fixed poles,[10] and its analogy with odd-signature vector-meson Regge-pole exchange, the presence of the second term can be inferred from a (very) generalized interpretation of exchange degeneracy.[11]

Parts (a)-(c) are concerned with the on-shell Regge limit whereas, as we shall see, the light cone should only describe the Regge limit far off the mass shell. They also all involve only Regge poles, whereas (4) involves (perhaps only) fixed poles. Exchange degeneracy, for example, essentially equates the odd-signature vector-meson Regge-pole residues and trajectories with those of the even-signature tensor-meson Regge poles. We assume the same relation between the right-signature fixed poles and the wrong-signature singularities generalized by SU(3) from the combined singular residue[12] and Pomeranchuk singularity (or any other mechanisms) which give rise to the forward coupling of the Pomeranchuk to photons.

We proceed to outline derivations of some of the implications of (4). Details will be given elsewhere.[13] We consider the forward spin-averaged current-proton scattering amplitude

$$T_{\mu\nu}{}^{ab}=i\int dx\, e^{iq\cdot x}\theta(x_0)\langle p|[J_\mu{}^a(x),J_\nu{}^b]|p\rangle+\mathrm{poly}=p_\mu p_\nu T_2{}^{ab}(\kappa,\nu)+\cdots, \tag{6}$$

and its absorptive part

$$W_{\mu\nu}{}^{ab}=(1/\pi)\,\mathrm{Im}T_{\mu\nu}{}^{ab}=(1/2\pi)\int dx\, e^{iq\cdot x}\langle p|[J_\mu{}^a(x),J_\nu{}^b]|p\rangle=p_\mu p_\nu W_2{}^{ab}(\kappa,\nu)+\cdots. \tag{7}$$

Here $\kappa=q^2$ and $\nu=q\cdot p$. Choosing $n\cdot q=0$, say $q^\mu=[\nu,(-\kappa)^{1/2},0,-\nu]$, we can project out T_2 and W_2 with n^μ:

$$T_2{}^{ab}=i\int dx\, e^{iq\cdot x}\theta(x_0)\langle p|[J^a(x),J^b]|p\rangle, \tag{8}$$

$$W_2{}^{ab}=(1/2\pi)\int dx\, e^{iq\cdot x}\langle p|[J^a(x),J^b]|p\rangle. \tag{9}$$

In (8) and (9) we can write $\langle p|[J^a(x),J^b]|p\rangle\equiv\hat W_2{}^{ab}(x^2,x_0)$ since the allowed quadratic dependence on $n\cdot x=x_-$ is absent because $n\cdot q=0$. Our proposal (4) then tells us that for $x^2\sim 0$,

$$\hat W_2{}^{ab}(x^2,x_0)\sim L^{ab}(x^2,x_0)\equiv 2\pi^{-1}i\delta(x^2)f(2x_0)[if^{abc}F^c-d^{abc}D^c\epsilon(x_0)], \tag{10}$$

where we have written

$$iF^c = \langle p|J^c|p\rangle, \quad D^c = \langle p|S^c|p\rangle. \tag{11}$$

Some applications of (10) are given below.

Electron scattering.—We take $a=b=Q$, with $J^Q = J^3 + 3^{-1/2}J^8$ the electromagnetic current. We are interested in the Bjorken[14] deep inelastic limit $\nu \to +\infty$, $\rho \equiv -\nu/\kappa > 0$ fixed, which we call the A limit. By rotational invariance, we can change q^μ to $k^\mu = [\nu, 0, 0, -(\nu^2-\kappa)^{1/2}]$ in (9) (even though $n\cdot k \neq 0$). Since

$$k\cdot x \underset{A}{\to} \nu x_- + (x_- - x_+)/4\rho, \tag{12}$$

the behavior of (9) in the A limit is determined by the $x_- \to 0$ behavior of $\hat{W}$, and since the large x_+ contributions are oscillated away, this (by causality) is the light cone $x^2 = x_+x_- - \vec{x}^2 \to 0$. Thus the leading light-cone singularity controls the A limit and we have

$$W_2^{QQ} \underset{A}{\to} (1/2\pi)\int dx\, e^{ik\cdot x} L^{QQ},$$
$$\underset{A}{\to} -d^{QQc}D^c(i/2\pi^2)\int dx_+ dx_- d\vec{x}\, e^{ix_-\nu - ix_+/4\rho}\,\delta(x_+x_- - \vec{x}^2)f(x_+)\epsilon(x_+). \tag{13}$$

Now, $\int d\vec{x}\,\delta(x_+x_- - \vec{x}^2) = \pi\theta(x_+x_-)$ and, since the leading asymptotic behavior is given by the leading singularity at $x_- = 0$, we can put $\theta(x_+x_-) \sim \theta(x_-)\epsilon(x_+)$ in (13) to obtain

$$\nu W_2^{QQ} \underset{A}{\to} d^{QQc}D^c(1/2\pi)\int dx_+\, e^{-ix_+/4\rho}f(x_+) \equiv F_2(\rho). \tag{14}$$

Thus (4) implies the Bjorken scaling law[15] and with (3) gives the asymptotic behavior

$$F_2(\rho) \xrightarrow[\rho\to\infty]{} \pi^{-1}d^{QQc}D^c \equiv w_2. \tag{15}$$

This constant asymptotic behavior is experimentally indicated.[16,17] Since $d^{QQc}D^c = \frac{2}{3}D^Q$, to obtain the value of the constant limit w_2 we need to know D^3 and D^8. As a first approximation, we assume $[SU(3) \otimes SU(3)]_\beta$ is an exact rest symmetry and obtain $D^Q = 1$ and $w_2 \sim 0.21$. A better approximation is to only assume SU(3) symmetry. Then there is a d-type admixture in D^c which has been determined from mass-shift calculations[18,19] (which assume $H_I \propto S^8$) from the Cabibbo-Horwitz-Ne'eman[8] theory,[20] and from low-lying saturation of (5).[18,19] These determinations are roughly consistent (giving $d/f \sim \frac{1}{3}$) and we find now that $w_2 \sim 0.27$. This seems close enough to the observed[16,17] value of ~0.3 to render plausible our universality assumption. The above values should be compared with weak-interaction V-A universality, which gives $g_A \equiv |G_A/G_V| = 1$ in the chiral-symmetry limit, $g_A = 5/3$ or $\frac{3}{2}$ with low-lying saturation, and the observed value of $g_A \sim 1.2$ from the exact Adler-Weisberger relation.

Electron-positron annihilation.—Since the structure function $\overline{W}_2$ for $e^+ + e^- \to$ hadrons satisfies $\overline{W}_2(\kappa, \nu) = -W_2(\kappa, -\nu)$,[21,22] we easily find from (10) that $\overline{F}_2(\rho) \equiv \lim_A \nu\overline{W}_2(\kappa,\nu)$, $\rho = +\nu/\kappa$, is also given by (14) and hence, as has been suggested,[21] is the analytic continuation of $F_2(\rho)$ from $\rho > \frac{1}{2}$ to $\rho < \frac{1}{2}$.

Neutrino scattering.—With $J^a = J^W$, the Cabibbo current $[\bar{p}'\gamma_\mu(1-\gamma_5)(n'\cos\theta + \lambda'\sin\theta)$ in the quark model], and $J^{W\dagger} \equiv J^{\overline{W}}$, (9) is a structure function for $\bar{\nu} + p \to \mu^+ +$ hadrons. Defining[14] $\vec{F}_2(\rho) = \lim_A \nu W_2^{W\overline{W}}$, and assuming (4) to be valid for both vector and axial-vector currents, we find as above that

$$\vec{F}_2(\rho) \xrightarrow[\rho\to\infty]{} \pi^{-1}[\sqrt{3}(\tfrac{4}{3} - 2\sin^2\theta)D^8 + 2\sin^2\theta D^3] \equiv \vec{w}_2. \tag{16}$$

$[SU(3)\otimes SU(3)]_\beta$ symmetry gives $\vec{w}_2 \sim 0.59$ and only SU(3) gives $\vec{w}_2 \sim 0.9$. Experimentally, $\vec{F}_2$ is observed[23] to approach a constant asymptotic value, rather like F_2. Assuming that $\vec{F}_2(\rho) \propto F_2(\rho)$, and that the other[14] structure functions are as large as possible,[24] we find from the observed[23] total cross section that $\vec{w}_2 \sim 0.85 \pm 0.2$. Our prediction agrees well with this value.

Writing the above $\vec{w}_2 = w_2(\bar{\nu}p)$, and noting that $d^{W\overline{W}c} = d^{\overline{W}Wc}$, we predict that $w_2(\bar{\nu}p) = w_2(\nu p)$. Further, since $\langle p|S^8|p\rangle = \langle n|S^8|n\rangle$, we predict in the $\theta \to 0$ limit that $w_2(\bar{\nu}p) = w_2(\bar{\nu}n)$. With the correct θ, the two should differ by ~5%. These relations should be testable soon at CERN.

Regge limit.—The Regge limit $\nu \to \infty$, with $-\kappa$ fixed and $\gg 1$ (R' limit), is similarly determined from the light-cone commutator. [$\nu \to \infty$ corresponds to $x_- \to 0$, and $-\kappa \gg 1$ keeps $x_+ \ll 1/x_-$, so that the light cone $x_+x_- \to 0$ is relevant.[13]] As above, we find from (4) that[25]

$$T_2^{ab} \underset{R'}{\to} \nu^{-1}[f^{abc}F^c + id^{abc}D^c], \tag{17}$$

$$W_2^{ab} \underset{R'}{\to} (\pi\nu)^{-1}d^{abc}D^c. \tag{18}$$

Thus even for W_2 we do not obtain the usual Regge behavior (with singular residues) which says that only the Pomeranchuk contribution goes as $1/\nu$.[26] Assuming the usual Regge trajectory behavior, (18) requires, for example, additional fixed nonsense wrong-signature (double) poles.[27] Alternatively, our proposal can be easily altered to be consistent with the usual Regge picture, but then universality, as we have formulated it, would be lost. In any case, according to (15) and (18), (4) is seen to incorporate the suggestion[28] that the leading Regge contribution continues to dominate in the A limit.

Although the above numerical results, if not accidental, support the validity of (4), we conclude by listing some alternative, but related, possibilities: (i) Sum c from 0 to 8. (ii) Maintain consistency with the usual Regge picture by keeping only S^0 and S^8. (iii) Multiply the second term in (4) by a universal constant. (iv) Give up universality and allow $f^{abc}J^c + g^{abc}S^c$, with $g^{abc} \neq -d^{abc}$. What is certain is that the experimental nonvanishing of w_2 implies the nontrivial nature of R_{ab} in (2), and (4) appears to be algebraically the most appealing possibility.

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