CALCULATION OF THE COULOMB ENERGY SHIFT BETWEEN Sc^{41} AND Ca^{41} †

E. H. Auerbach, S. Kahana, and J. Weneser
Brookhaven National Laboratory, Upton, New York 11973
(Received 3 October 1969)

A straightforward, purely single-particle analysis of the Coulomb energy shift in Sc^{41}, Ca^{41} leads to an abnormally small radius for the $f_{7/2}$ valence neutron. Presented in this note is a correlational correction that may help to increase this radius. The calculation of this correction is stated in as general a fashion as is possible, and an estimate is made which leans heavily on a parametrization of the proton and neutron densities.

I. Introduction. —Recently Nolen and Schiffer[1] have pointed to a serious difficulty which arises in the extraction of neutron radii from the Coulomb energy separation between analogue states. This energy difference can be divided conveniently into a direct single-particle Coulomb term and various corrections of both correlational and single-particle nature. The size of the direct term, from which one obtains a neutron radius, may be determined by adding the corrections to the measured energy separation. The single-particle corrections[1,2] consist of the Coulomb exchange energy, an electromagnetic spin-orbit energy, and a kinetic energy generated by the n-p mass difference. For the ground state of Ca^{41} and its analogue in Sc^{41} the measured energy separation is 7.28 MeV and the resultant direct Coulomb energy is ~7.6 MeV. Assuming that only the $f_{7/2}$ valence level is involved in the transition $Ca^{41} \rightarrow Sc^{41}$, one can then extract as a radius for this "excess" neutron orbit a value $\langle r^2 \rangle^{1/2} = 3.50$ F. Since the rms radius for the charge distribution of Ca^{40} is also ~3.50 F, one concludes that the $f_{7/2}$ valence orbit lies spatially on top of the core 2s and 1d orbits. This result is so extremely difficult to fit into even a shell model with correlations that it was thought necessary to examine the basis of its derivation.

The underlying assumption in the above analysis is the strict use of an independent-particle model: that is, in transforming from Ca^{41} to Sc^{41} the valence neutron becomes a valence proton and nothing else changes. In this note we consider how a small natural excursion beyond the single-particle model modifies the conclusions about orbit radii. The effect that seems most important is, for the specific case of Ca^{41}-Sc^{41}, that due to the admixture of a small component of ($T = 1$ core, overall $T = \frac{1}{2}$) into the dominant ($T = 0$ closed core, $T = \frac{1}{2}$) wave function. This admixture can be characterized as equivalent to that produced by a small isovector effective charge for the valence particles in a single-particle model. In the work of Refs. 1 and 2 the Coulomb energy displacement is stated as a general expression involving the difference between proton and neutron densities in Ca^{41}. To see the difficulty discussed above and to analyze it one must take apart this general expression in terms of models.

II. Derivation of the core-polarization correction. —The energy-difference correction under study may be represented to lowest order in the Coulomb and nuclear interactions among core particles by the two graphs, Figs. 1(a) and 1(b). Such graphs occur for either a neutron or proton propagating along the external valence lines; the eventual energy difference is obtained by taking the difference between the neutron and proton terms. A finite contribution is obtained only if the particle-hole pair excited out of the core is coupled to isospin $T = 1$. The final result is clearly of the form of an effective charge contribution; only the isovector charge is of interest here. The third graph, Fig. 1(c), is one in the same order of perturbation theory and should be included as well. However, we find this graph to be small when estimated via the shell model or collective models. Moreover, this graph represents a change in the valence–core-particle Coulomb energy due to correlations. Included in these are the short-range correlations which should be taken into account together with the finite proton size. The effect of the latter is to re-

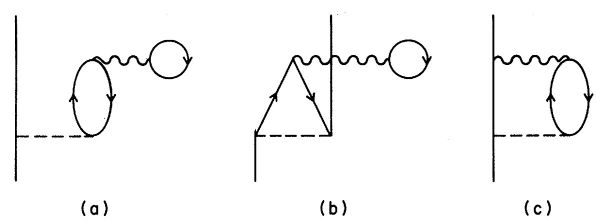

FIG. 1. Corrections to the Coulomb energy difference in Sc^{41}-Ca^{41}. (a) and (b) are effective charge corrections and (c) is a correlation correction. The dashed line represents a residual core-valence interaction and the wavy line, a core-core Coulomb interaction.

duce any increase in the two-particle Coulomb energy due to correlation. There are additional irreducible graphs of first order in the Coulomb energy but of higher order in the nuclear interaction between core and valence particles; these we do not consider here. There are also graphs that are of higher order in the core-core nuclear interaction, for example the random phase approximation extensions of Figs. 1(a) and 1(b), which are included in the schematic estimate made below.

It is also possible to view the graphs of Figs. 1(a) and 1(b) as alterations in the single-particle fields. In previous analyses of the analogue-state Coulomb-energy separation one assumed the existence of a nuclear average potential $U(1)$ common to both protons and neutrons. Including the core-core Coulomb interactions in a calculation of the core energy will induce a small separation, $\Delta U = U_p - U_n$, between the average proton and neutron single-particle potentials. By using a core nucleon–valence nucleon interaction

$$K = K_0 + K_\sigma \vec{\sigma}\cdot\vec{\sigma}_c + K_\tau \vec{\tau}\cdot\vec{\tau}_c + K_{\sigma\tau}\vec{\sigma}\cdot\vec{\sigma}_c\vec{\tau}\cdot\vec{\tau}_c, \tag{II.1}$$

which may be viewed as an appropriate two-body reaction matrix, one may calculate the neutron-core and proton-core interaction energies from

$$(\gamma|U_\gamma|\gamma) = \textstyle\sum_c\{(\gamma c|K|\gamma c) - (\gamma c|K|c\gamma)\}; \qquad \gamma = n, p. \tag{II.2}$$

If the core-proton states $|c_p)$ include the Coulomb interactions then one will find a nonvanishing contribution to the Sc^{41}-Ca^{41} Coulomb-energy difference

$$\Delta E_{\text{core}} = \langle p|(U_p - U_n)|p\rangle. \tag{II.3}$$

A more familiar shell-model terminology can be used to discuss this additional energy. The wave functions for Ca^{41}-Sc^{41} are of the form

$$\psi(T=\tfrac{1}{2}) = \alpha|\gamma, T_c = 0; T=\tfrac{1}{2}\rangle + \textstyle\sum\beta_{\text{ph}}|\gamma, \text{ph}\,T_c = 1, T = \tfrac{1}{2}\rangle + \cdots. \tag{II.4}$$

Here γ is a neutron or proton valence orbit; the closed core is denoted simply by $(T_c = 0)$ and the additional isovector particle-hole core excitation by $(\text{ph}\,T_c = 1)$. The terms proportional to β_{ph} represent a small improvement over the strict individual particle model. There are $T = 0$ core-polarization corrections which should also enter, but these are suppressed. The Coulomb energies are to be evaluated with the improved wave function and there results an additional energy difference due to core polarization:

$$\begin{aligned}\Delta E_{\text{core}} &= \langle Sc^{41}|V_C^{\text{core}}|Sc^{41}\rangle - \langle Ca^{41}|V_C^{\text{core}}|Ca^{41}\rangle \\ &= 2\sum_{\gamma,\text{ph}}\langle\gamma, T_c = 0; T = \tfrac{1}{2}|V_C^{\text{core}}\tau_3^{(\gamma)}|\gamma, \text{ph}\,T_c = 1; T = \tfrac{1}{2}\rangle\beta_{\text{ph}}.\end{aligned} \tag{II.5}$$

Treating the core polarization in perturbation theory yields

$$\begin{aligned}\Delta E_{\text{core}} = 2\sum_{\gamma,\text{ph}}&\langle\gamma, T_c = 0; T = \tfrac{1}{2}|V_C^{\text{core}}\tau_3^{(\gamma)}|\gamma, \text{ph}\,T_c = 1; T = \tfrac{1}{2}\rangle \\ &\times(1/\Delta E_{\text{ph}})\langle\gamma, \text{ph}\,T_c = 1; T = \tfrac{1}{2}|K|\gamma, T_c = 0, T = \tfrac{1}{2}\rangle.\end{aligned} \tag{II.6}$$

It is apparent that only terms diagonal in overall isospin will contribute if K is an isoscalar. This perturbation form can be interpreted as the polarization of the core by the isovector part of the core Coulomb force followed by an interaction of the valence nucleon with this core polarization. Thus one returns to Eq. (II.2).

As a useful simplification we introduce zero-range forces

$$K_I = V_I\delta(\vec{r} - \vec{r}_c) \tag{II.7}$$

which are characterized by the singlet and triplet strengths V_S, V_T. Then one obtains from Eq. (II.3)

$$\Delta E_{\text{core}} = -(\tfrac{3}{4}V_T - \tfrac{1}{4}V_S)\int d\tau|\psi_f(\vec{r})|^2 \times[\rho_p(\vec{r}) - \rho_n(\vec{r})], \tag{II.8}$$

where ρ_n, ρ_p are the neutron and proton densities of the Ca^{40} core obtained in the presence of core Coulomb interaction, and ψ_f is the wave function of a valence proton in the $f_{7/2}$ orbit. The neutron-core potential derived on a similar basis is

$$U_n = (\tfrac{3}{4}V_T + \tfrac{3}{4}V_S)\rho_n + (\rho_p - \rho_n)(\tfrac{3}{4}V_T + \tfrac{1}{4}V_S). \tag{II.9}$$

Before proceeding to estimate the magnitude of ΔE_{core} let us emphasize that the treatment of the Ca^{40} core in Eq. (II.8) is essentially exact, provided one uses the exact densities. The finite-range forces, aside from modifying the exchange terms, would only lead to a small change in the meanings of the parameters in this basic equation.

III. Numerical estimate of ΔE_{core}. —To use the expression (II.8) for ΔE_{core} we need the parameters V_S, V_T and the important function $\rho_p(\vec{r}) - \rho(r)$. We can use (II.9) to compare U_n with the empirical neutron potential energy and so obtain a value for the combination V_S+V_T. The energy $\langle n|U_n|n\rangle = -32$ MeV was employed in this determination. To get at another combination of the force strengths we noted the similarity of ΔE_{core} to the nuclear symmetry energy. The known strength of the $\vec{t}\cdot\vec{T}$ part of the optical potential in p-n reactions suggests for the ratio $(\frac{3}{4}V_T-\frac{1}{4}V_S)/(\frac{3}{4}V_T+\frac{3}{4}V_S)$ a value ~0.6. On the other hand current realistic interaction calculations imply a value ~0.5. We will use the 0.6 value.

The more interesting quantity $\rho_p(\vec{r})-\rho_n(\vec{r})$ is not so easy to reach. Although in principle experimentally measurable, this difference in densities is not now known. A calculation with single-particle wave functions, whether self-consistent or not, may very likely be inadequate.[3] Instead, we take for $\rho_p(\vec{r})$ the ad hoc form

$$\rho_p(\vec{r}) = (A/2)\{(1-\beta)\kappa^3\pi^{-3/2}e^{-\kappa^2r^2} + (\beta/4\pi R^2)\delta(r-R)\}. \quad \text{(III.1)}$$

Thus, the smooth part of the charge distribution is diminished by a small amount and the latter then concentrated into a bump at radius R. The neutron density function, $\rho_n(r)$, is of the form (III.1) with $\beta \to -\beta$ and hence

$$\rho_p-\rho_n = A\beta\{(\tfrac{1}{4}\pi R^2)\delta(r-R) - \kappa^3\pi^{-3/2}e^{-\kappa^2r^2}\}. \quad \text{(III.2)}$$

The parameter κ is obtained by requiring the rms radius of $\rho_p(\vec{r})$ to be fixed at the experimental value of 3.50 F. The valence wave function is taken to be that of a harmonic oscillator with the size parameter $\mu^2 = \hbar/m\omega$ chosen to ensure a reasonable nuclear radius.

The integral in (II.8) is easily performed and yields

$$\Delta E_{\text{core}} = -1.23\langle n|U_n|n\rangle\{(8/105\kappa^3\mu^6) \times(\mu^2+\kappa^2)^{9/2}(\mu R)^6 e^{-(\mu R)^2}-1\}. \quad \text{(III.3)}$$

If the position of the proton bump in (III.1) is deliberately placed at the maximum of $|\psi_f(\vec{r})|^2$ there results

$$\Delta E_{\text{core}} \sim 31\beta \text{ MeV}.$$

Now a value $\Delta E_{\text{core}} = 700$ keV would reduce the direct Coulomb energy to ~6.9 MeV and raise the valence rms radius to 4.1 or 4.2 F, removing the anomaly. This situation requires $\beta \sim 0.0225$, i.e., a $2\frac{1}{4}\%$ enrichment of protons and a $2\frac{1}{4}\%$ depletion of neutrons in the neighborhood of the maximum in the valence neutron distribution. This seems a not unreasonable state of affairs. It is interesting that the isovector effective charge implied by the above is only of the order of $\frac{1}{20}$ of the proton charge.

Very importantly, we can use the parameters (β, R), now determined, to calculate ΔE_{core} for a $1d_{3/2}$ hole orbit and hence to predict the ground-state Coulomb energy difference for Ca^{39}-K^{39}. It turns out that

$$\Delta E_{\text{core}}(1d_{3/2})/\Delta E_{\text{core}}(1f_{7/2}) \sim \tfrac{2}{3},$$

which appears to be just right. The $2p_{3/2}$ particle state in Ca^{41} is more difficult to treat because its wave function has a node in the region of interest. A fit can probably be achieved by altering the available parameters slightly. In particular it helps to dissociate the parameter κ of (III.2) from that appearing in (III.1) since in any case the rms charge radius is not appreciably altered. A larger value of κ simultaneously permits one to handle the $2p_{3/2}$ level and to reduce the size of β required for the $1f_{7/2}$ and $1d_{3/2}$ levels.

IV. Discussion.—Of course the foregoing must be regarded as schematic and speculative; more analysis is required. An attempt can be made to calculate $\rho_p - \rho_n$ from a nuclear model; at the very least any single-particle model should be improved by including in it a neutron-proton residual nuclear interaction. The purely independent-particle estimates of $\rho_p-\rho_n$ seem to produce a more than sufficient excess of protons but at radii past the maximum in $|\psi_f|^2$. The n-p residual interaction will help in this regard. Nor is it clear that ΔE_{core} must provide the full 700 keV or so required to remove the radius anomaly. One might try to absorb some of the radius anomaly in the core wave function. Perhaps the best route to a knowledge of the crucial density difference is through measurement, difficult as this may prove to be.

We conclude with a comment on nuclei possessing a definite neutron excess. In such nuclei ΔE_{core}, like the entire Coulomb energy shift, is a result of averaging over all the levels occupied by the excess nuetrons. For example, the "p"$_{3/2}$ state in Sc^{49} which is the analogue of the Ca^{49} ground state is $\frac{8}{9}$ an $f_{7/2}$ state and only $(\frac{1}{9})$ $p_{3/2}$ and the analysis for ΔE_{core} is much the same as above.

---

†Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]J. A. Nolen and J. P. Schiffer, Phys. Letters 29B, 396 (1969); J. P. Schiffer, in Proceedings of the Second Conference on Nuclear Isospin, Asilomar-Pacific Grove, 1969, edited by J. D. Anderson, S. D. Bloom, J. Cerny, and W. W. True (Academic Press, Inc., New York, 1969).

[2]N. Auerbach, J. Hufner, A. K. Kerman, and C. M. Shakin, Phys. Rev. Letters 23, 484 (1969).

[3]A. Bohr, J. Damgaard, and B. R. Mottelson, in Nuclear Structure, edited by A. Hossain, Harun-Ar-Rashid, and M. Islam (North Holland Publishing Co., Amsterdam, The Netherlands, 1967), p. 1.

TOTAL PHOTOABSORPTION CROSS SECTIONS UP TO 18 GeV AND THE NATURE OF PHOTON INTERACTIONS*

D. O. Caldwell, V. B. Elings, W. P. Hesse, G. E. Jahn, R. J. Morrison, and F. V. Murphy
Physics Department, University of California, Santa Barbara, California 93106

and

D. E. Yount†
Stanford Linear Accelerator Center, Stanford University, Stanford, California
(Received 13 October 1969)

The smallness of nucleon photoabsorption would imply total photonucleus cross sections which vary as the number of nucleons in the nucleus, but if ρ dominance is correct the resulting strong interactions would take place primarily with the surface nucleons. At Stanford Linear Accelerator Center we have measured the photoabsorption in H, D, C, Cu, and Pb and have found cross sections which vary as $\sim A^{0.9}$ indicating that the photon interacts, at least partially, as a strongly interacting particle.

Total cross sections for the photoproduction of hadrons from hydrogen, deuterium, carbon, copper, and lead have been measured at Stanford Linear Accelerator Center (SLAC) at energies up to 18.3 GeV. The high-energy portion of these measurements has been analyzed and yields fundamental information about the nature of the photon interaction with nuclear matter. On the basis of the hydrogen photoabsorption cross section, which implies a photon mean free path of about 800 F, one would expect that the total cross section on a nucleus of nucleon number A should be simply proportional to A; i.e., $\sigma_{\gamma A}=A\sigma_{\gamma p}$. As first pointed out by Stodolsky,[1] however, vector dominance[2] implies that photons interact mainly via the ρ meson, which has a mean free path ~3 F, and consequently the surface nucleons would shadow the rest. If the mean free path were zero, the shadowing would be complete and the cross section would be proportional to the nuclear area. We find cross sections which have an A dependence of about $A^{0.9}$ indicating that the photon interacts in nuclear matter, at least partially, as a strongly interacting particle. The interpretation of this result as a test of ρ dominance depends upon which ρ-production data are used in the comparison.[3-9]

The photoabsorption cross sections were measured by sending photons of known energy into a target and determining whether they were absorbed with the production of hadrons. As shown in Fig. 1, the tagged-photon beam[10] utilized a 1-cm-diam positron beam of energy $E_0 \pm 0.4\%$ which was incident on a thin radiator. Those positrons which radiated a photon of energy E_γ such that $0.74E_0 \leq E_\gamma \leq 0.94E_0$ were deflected by a magnet into one of four counter telescopes which determined the final energy of the positron, E_+. A coincidence count from any tagging telescope indicated that a photon of energy $E_\gamma = E_0 - E_+$, known to $\pm 2.5\%$ of E_0, had entered the target. The tagging system therefore determined the energy of each photon and, with associated anti counters, monitored the photon flux to better than $\frac{1}{2}\%$ accuracy.

The main experimental problem was to separate photon interactions producing hadrons from those yielding electron pairs, which were 100 to 2000 times more numerous, depending on the target. Our solution was primarily geometric, taking advantage of the small opening angles of high-energy pair and Compton-scattering events as compared with the larger angles of hadronic events. Nearly all the background electromagnetic products passed with the beam through holes in the hadron counters $S2a$ and $S2b$ and were vetoed by the lead-scintillator, total-energy shower counter $S1$. Additional discrimination against background was provided by the hadron detectors, which were made of four layers of