by

$$M \cong -\frac{1}{4\pi}\frac{H_c(T)-H_0}{n}, \tag{12}$$

where H_0 is the external field and n is the diamagnetization coefficient.

In the dilute limit of the vortex lines [i.e., $B \ll H_c(T)$], Eq. (8) reduces to

$$\varphi(0) = \lim_{B\to 0}\varphi(B) = \frac{1}{4mD} \tag{13}$$

which is identical to Eq. (11).

Finally we would like to conclude this note with a remark on the Hall effect in a pure type-II superconductor. The kinetic-energy term in the chemical potential gives rise to only a negligible Hall effect in a pure type-II superconductor. Therefore we are still far from understanding the Hall effect observed by Fiory and Serin[12] in a pure niobium sample.

In conclusion the author would like to thank Dr. Weijsenfeld for informing him of his result prior to publication. He is also grateful to Professor Muto and Dr. Noto and Dr. Mori for informative discussions on their experimental data of the Hall effect in Nb-Mo alloys.

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TEST OF DYNAMIC SCALING BY NEUTRON SCATTERING FROM $RbMnF_3$ †

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Measurements are presented of the inelastic magnetic scattering of neutrons from $RbMnF_3$ in the vicinity of the critical point. The data are analyzed quantitatively in terms of the dynamic scaling proposal of Halperin and Hohenberg and provide strong confirmation of the theory.

In a recent paper[1] Halperin and Hohenberg proposed a generalization of the static scaling laws to dynamic phenomena by making assumptions about the behavior of time-dependent correlation functions in the vicinity of the critical point of "second-order" phase transitions. They write the Fourier transform of the symmetrized space-time correlation function for the operator A, in terms of wave vector $\vec{q}$, frequency ω, and inverse range parameter κ, in the general form

$$C_\kappa{}^A(\vec{q},\omega) = [2\pi/\omega_\kappa{}^A(\vec{q})]C_\kappa{}^A(\vec{q})f_{\vec{q},\kappa}{}^A[\omega/\omega_\kappa{}^A(\vec{q})],$$

where the characteristic frequency $\omega_\kappa{}^A(\vec{q})$ is defined by the condition

$$[\omega_\kappa{}^A(\vec{q})]^{-1}\int_{-\omega_\kappa{}^A(\vec{q})}^{\omega_\kappa{}^A(\vec{q})} f_{\vec{q},\kappa}{}^A[\omega/\omega_\kappa{}^A(\vec{q})]d\omega = \tfrac{1}{2},$$

and where the spatial transform $C_\kappa{}^A(\vec{q})$ is assumed to obey static scaling.[2] The dynamic scaling assumptions are (i) that the characteristic frequency $\omega_\kappa(\vec{q})$ is a homogeneous function of q and κ,

$$\omega_\kappa(\vec{q}) = q^E\Omega(q/\kappa),$$

and (ii) that the form of the frequency-dependent function $f_{q,\kappa}$ depends only on the ratio q/κ and not on $\vec{q}$ and κ separately. These scaling assumptions are used to relate the behavior of the correlation function in the "critical" region ($q/\kappa \gg 1$) to that calculated for the two "hydrodynamic" cases ($q/\kappa \ll 1$, with $T < T_N$ or $T > T_N$). Using this matching procedure, and the theory of spin waves for $T < T_N$, Halperin and Hohenberg predict that $\omega_\kappa(\vec{q}) \sim q^{1.5}$ at T_N and $\omega_\kappa(0) \sim \kappa^{1.5}$ for $T > T_N$.

We have measured the inelastic neutron scattering from $RbMnF_3$ in the "critical" region and have analyzed our data quantitatively, including resolution and instrumental effects. Our results strongly support the concept of dynamic scaling and agree well with the specific predictions of the theory.

$RbMnF_3$ has the simple cubic perovskite structure in the paramagnetic state and becomes antiferromagnetic below 83°K. In the ordered state it exhibits negligible magnetic anisotropy, no measurable distortion from cubic symmetry, and appears to be an ideal Heisenberg antiferromagnet.[3] Measurements were made near the $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ point of reciprocal space using longitudinal constant-$\vec{q}$ scans ($\vec{q} \parallel 2\pi\vec{\tau}$) and incident neutron energies of 6.6, 13.0, and 47.0 meV. The zone boundary in this direction is 0.644 $Å^{-1}$. The sample temperature was regulated to better than 10 mdeg using an ac resistance bridge[4] and a calibrated platinum resistance thermometer. The inverse correlation range κ was obtained as a function of temperature from the analysis of our two-axis data.[5]

The calculated scattered intensity at the instrumental setting $(\vec{q}_0, \omega_0)$ is obtained by convolution of the cross section with the resolution function:

$$I(\vec{q}_0, \omega_0) = P(\omega_0)\int \frac{d^2\sigma}{d\Omega d\omega}(q, \omega)R(\vec{q}-\vec{q}_0, \omega-\omega_0)d\vec{q}d\omega,$$

where P contains an instrumental constant and a slowly varying correction for the energy dependence of the counter sensitivity and the reflectivity of the analyzing crystal. $\vec{q}$ is the momentum transfer measured from the $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ spot to a general point in reciprocal space, and ω is the energy transfer. The resolution function was calculated analytically[6] using experimentally determined values of mosaic spread and collimation parameters. The cross section for $T \geq T_N$ can be written[7]

$$\frac{d^2\sigma}{d\Omega d\omega}(\vec{q}, \omega) \propto \frac{k_f}{k_i}|f(\vec{Q})|^2 \frac{1}{1+e^{-(\omega/kT)}} C_\kappa(\vec{q}, \omega),$$

where $\vec{k}_i$ and $\vec{k}_f$ are the initial and final neutron wave vectors, and $f(\vec{Q})$ is the magnetic form factor.

At $T = T_N$ and for values of q greater than about 0.1 $Å^{-1}$, where resolution corrections are small, the uncorrected energy spectra of scattered neutrons exhibit three unresolved peaks. We have chosen, therefore, to analyze the data at T_N for all q in terms of the dynamic correlation function

$$C_{\kappa=0}(\vec{q}, \omega) \propto \frac{A}{q^{2-\eta}}\left[\frac{\Gamma_1}{\Gamma_1^2+\omega^2} + \frac{B\Gamma_2}{\Gamma_2^2+(\omega+\omega_s)^2} + \frac{B\Gamma_2}{\Gamma_2^2+(\omega-\omega_s)^2}\right],$$

where $A/q^{2-\eta}$ is the static part, B is an arbitrary constant, and the energy widths Γ_1, Γ_2, and ω_s (all in meV) are taken to be proportional to q^E with proportionality constants C, G, and D, respectively. This choice for the energy widths automatically ensures that $C_\kappa(\vec{q}, \omega)$ satisfies the assumptions of dynamic scaling along the line $\kappa = 0$. Inserting this form of the correlation function into the expression for the cross section using $\eta = 0.067$,[5] we have performed least-squares fittings of the data for the range 0.05 $Å^{-1} \leq q \leq 0.25$ $Å^{-1}$, which is experimentally accesible with an incident energy of 13 meV. A very good fit to the data is obtained with an exponent $E = 1.4 \pm 0.2$, in agreement with the predicted value of 1.5. Figure 1 shows the observed data taken at 13 meV and calculated curves based on the least-squares parameters $B = 1.82$, $C = 5.00$, $G = 9.66$, $D = 16.0$, and $E = 1.4$. As can be seen from the scale in Fig. 1, there is a factor of 100 between the peak intensities for $q = 0.05$ $Å^{-1}$ and $q = 0.25$ $Å^{-1}$. This fit has been obtained using a single normalization constant to place all the intensity data on an absolute basis.

A second test of dynamic scaling was made in the hydrodynamic region ($q \ll \kappa$) above T_N, where $C_\kappa(\vec{q}, \omega)$ is assumed to have the "diffusion" form

$$C_\kappa(\vec{q}, \omega) \propto \frac{1}{(\kappa^2+q^2)^{1-\eta/2}} \frac{\Gamma(q, \kappa)}{[\Gamma(q, \kappa)^2+\omega^2]}.$$

As noted earlier, dynamic scaling theory predicts that the width $\Gamma(q, \kappa)$ (or characteristic frequency) should vary as $\kappa^{1.5}$ for $q = 0$. In making a measurement at a nominal setting of $q = 0$, nonzero values of q are simultaneously sampled because of finite instrumental resolution and hence the dependence of Γ on q is required in order to perform the convolution of the cross section with the resolution function. To accord with the theory of dynamic scaling and the expected analyticity for $q = 0$, $\Gamma(q, \kappa)$ has been taken to have the form

$$\Gamma(q, \kappa) = \Gamma_0(\kappa)[1 + c(q/\kappa)^2],$$

where $\Gamma_0(\kappa)$ is the characteristic frequency for $q = 0$. The insert in Fig. 2 shows a typical least-squares fit from which $\Gamma_0(\kappa)$ is obtained at a given value of κ. Figure 2 gives the observed $\Gamma_0(\kappa)$ in meV as a function of $\kappa^{1.4}$ together with the line $\Gamma_0(\kappa) = d\kappa^E$ drawn for the best values of d and E which are 10.8 and 1.4. The exponent E agrees well, within the estimated error of ±0.2, with the

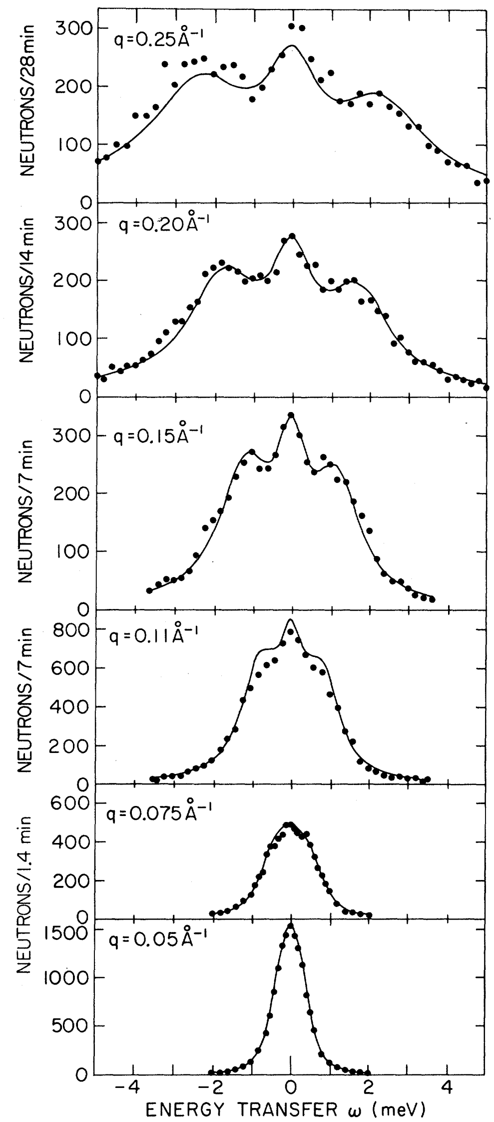

FIG. 1. Calculated and observed intensities as a function of energy transfer at T_N for different momentum transfers q. Observed data taken with incoming neutron energy of 13 MeV.

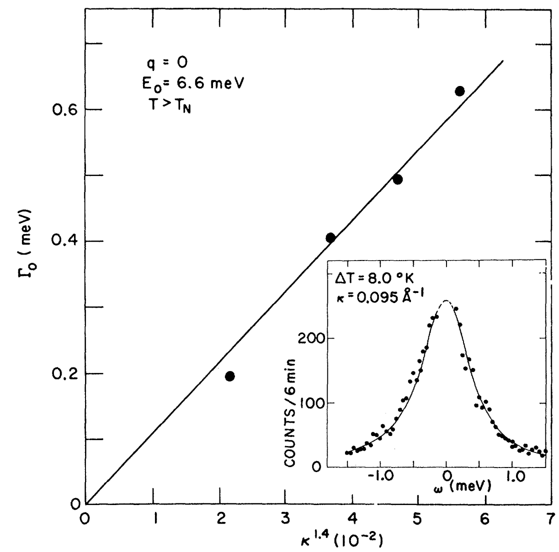

FIG. 2. Plot of $\Gamma_0(\kappa)$, the characteristic frequency at $q=0$, as a function of $\kappa^{1.4}$. The line corresponds to $\Gamma_0(\kappa)=d\kappa^E$ for the best values of d and E, which are 10.8 and 1.4. The insert shows a typical fit of calculated and observed intensities, for $q=0$ and $T_N=8°K$. The central points have been deleted because of the interference of a weak nuclear reflection coming from a higher-order contamination.

predicted value of 1.5.

The basic homogeneity assumption of dynamic scaling was further investigated by measurements at and above T_N for general points in the (q, κ) plane where resolution corrections to the characteristic frequency are negligible. Characteristic frequencies were determined directly from the uncorrected data by integration. Following Halperin and Hohenberg,[1] we assumed a simple homogeneous form for the characteristic frequency:

$$\omega_\kappa(q) = (aq^6 + bq^4\kappa^2 + c'q^2\kappa^4 + d'\kappa^6)^{E/6},$$

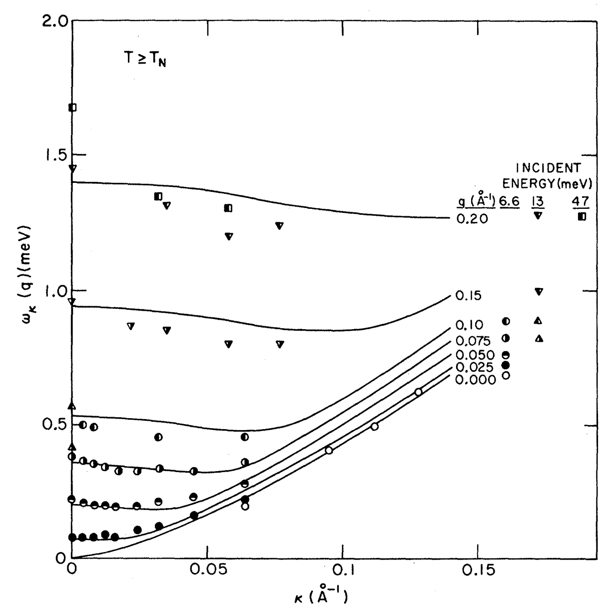

FIG. 3. Comparison of observed and calculated characteristic frequencies $\omega_\kappa(q)$ for $T \geq T_N$. Observed frequencies are half-area values, uncorrected for resolution, and are labeled by different symbols, according to momentum transfer and incident neutron energy. Calculated curves are labeled according to the value of q.

where a, b, c', d', and E are constants. This expression was used to fit 51 observed characteristic frequencies in the range $0<q \leq 0.2$ Å^{-1} and $0 \leq \kappa \leq 0.128$ Å^{-1}. The observed frequencies in meV are shown in Fig. 3 together with calculated curves based on the best values of the constants: $a=6.7\times10^4$, $b=-1.14\times10^5$, $c'=1.27\times10^5$, $d'=2.69\times10^4$, and $E=1.4$. In addition to providing a satisfactory fit to the observed characteristic frequencies, these constants are in good agreement with those previously obtained in fitting the data for $q=0$ and for $\kappa=0$.

In the present experiment three separate tests of dynamic scaling have been performed. In all cases quantitative agreement was obtained with the predictions of the theory. In addition, the three sets of results show a mutual consistency required by the theory, thus providing further confirmation of the concept of dynamic scaling.

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FREQUENCY-PULLING EFFECTS IN JOSEPHSON RADIATION

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Contrary to recent suggestions, cavity detuning per se does not modify the Josephson relation $\omega_J(V)=2eV/\hbar$, although it can produce measurable differences between the electrostatic and electrochemical potentials in a Josephson junction.

Recent theoretical results by Stephen[1] and by Scully and Lee[2] suggest that the radiation emitted from a Josephson junction mounted in a resonant cavity and biased at a particular dc potential difference V can be shifted from the Josephson frequency

$$\omega_J(V)=2eV/\hbar \tag{1}$$

by cavity detuning. This particular interpretation is not correct but stems from improper use of the electrostatic potential V_0 for the electrochemical potential V in the Josephson formula. Although the shifts computed for the specific structures studied are small ($\lesssim 1$ in 10^8), this is an important point to understand because the ac Josephson effect has been used for precision measurements of $e/\hbar$ and may have important applications in secondary voltage standards.[3,4]

If a Josephson junction coupling two bulk superconductors is sufficiently weak that the separate superconductors remain internally in thermal equilibrium, it is well known[5,6] that the order-parameter phase difference $\Delta\varphi(t)$ across the junction develops in time according to

$$\partial\Delta\varphi/\partial t=2[\mu_1(t)-\mu_2(t)]/\hbar, \tag{2}$$

where $\mu_j(t)$ is the electrochemical potential (electron Fermi energy) in superconductor $j=1$ or 2, and that the supercurrent $i_s(t)$ flowing through